1. An unrestrained steel bar of length 80 mm is heated from 20 °C to 50 °C, determine the change in length of the bar. α = 11 × 10^{-6} °C^{-1} for steel.
   [Ans.: 0.0264 mm]

   The free thermal extension of a bar is given by $\delta l_{themal} = l\alpha\Delta T$, therefore for the bar in question, this can be determined as:

$$\delta l = l\alpha\Delta T = (80 \times 10^{-3}) \times (11 \times 10^{-6}) \times (50 - 20)) = 2.64 \times 10\text{^} - 5 \text{ m}$$
$$= 0.0264 \text{ mm}$$

2. If the bar in Q1 has a Young's modulus of 200 GPa and is restrained from expanding axially, determine the stress in the bar.
   [Ans.: -66 MPa]

   For combined thermal and mechanical loading, the total change in length in a uniaxial bar can be determined using:

$$\delta l_{total} = \delta l_{therma} + \delta l_{mechanical} = l\alpha\Delta T + \frac{Fl}{AE}$$

   In this case, the overall extension $\delta l_{total} = 0$ due to the restraint, therefore the stress in the bar can be determined using:

$$\frac{F}{A} = -\frac{l\alpha\Delta TAE}{lA} = -\alpha\Delta TE = (11 \times 10^{-6}) \times 30 \times (200 \times 10^{9}) = -66000000 \text{ Pa}$$
$$= -66 \text{ MPa}$$

3. The bolt and sleeve assembly shown in Figure Q3 is initially tightened so that there is no pre-stress at a temperature of 20 °C. The temperature of the assembly is increased to 70 °C. Determine the total extension of the assembly and the stress in the sleeve and the bolt if the bolt is made of steel with a cross-sectional area of 85 mm^2 and the sleeve of aluminium with a cross-sectional area of 235 mm^2. α = 11 × 10^{-6} °C^{-1} and E = 200 GPa for steel and α = 23 × 10^{-6} °C^{-1} and E = 70 GPa for aluminium.
   [Ans.: extension: 0.084 mm; bolt stress: 59 MPa; sleeve stress: -21 MPa]

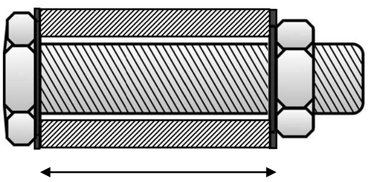


Figure Q3

For a uniaxial bar

$$\delta l_{total} = \delta l_{thermal} + \delta l_{mechanical} = l\alpha\Delta T + \frac{Fl}{AE}$$

In this case, the total deformation of the sleeve and the bolt must be equal, giving

$$\delta l_{bolt} = \delta l_{sleeve}$$

or

$$l\alpha_{bolt}\Delta T + \frac{Fl}{A_{bolt}E_{bolt}} = l\alpha_{sleeve}\Delta T - \frac{Fl}{A_{sleeve}E_{sleeve}}$$

As the sleeve must be in compression and the bolt in tension.
We can cancel through by the length leaving

$$\alpha_{bolt}\Delta T + \frac{F}{A_{bolt}E_{bolt}} = \alpha_{sleeve}\Delta T - \frac{F}{A_{sleeve}E_{sleeve}}$$

and rearrange,

$$F = \frac{(\alpha_{sleeve} - \alpha_{bolt})\Delta T}{\frac{1}{A_{sleeve}E_{sleeve}} + \frac{1}{A_{bolt}E_{bolt}}} = 5016\ \text{N}$$

We can then calculate the overall extensions as

$$l\alpha_{bolt}\Delta T + \frac{Fl}{A_{bolt}E_{bolt}} = 100 \times 11 \times 10^{-6} \times 50 + \frac{5016 \times 100}{85 \times 200 \times 10^3} = 0.084\ \text{mm}$$

And the stress in the bolt is

$$\sigma_{bolt} = \frac{F}{A_{bolt}} = \frac{5016}{85} = 59\ \text{MPa}$$

while the stress in the sleeve is

$$\sigma_{sleeve} = \frac{-F}{A_{sleeve}} = \frac{-5016}{235} = -21\ \text{MPa}$$

4. The 50-mm-diameter central cylinder shown in Figure Q4 is made from aluminium (α = 23 × 10^{-6} °C^{-1} and E = 70 GPa) and is placed in the clamp when the temperature is T_1 = 20° C. If the two steel (α = 11 × 10^{-6} °C^{-1} and E = 200 GPa) bolts of the clamp each have a diameter of 10 mm, and hold the cylinder snug with negligible force against the rigid jaws at T_1, determine the stress in the cylinder when the temperature rises to T_2 = 100° C.
[Ans.: -6.83 MPa]

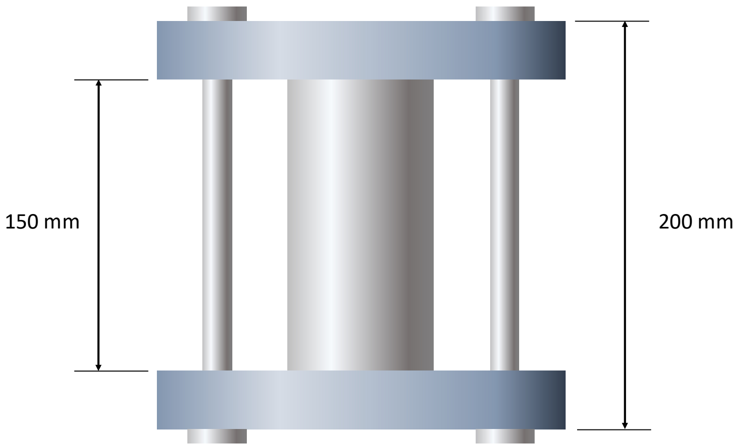


Figure Q4

For a uniaxial bar

$$\delta l_{total} = \delta l_{thermal} + \delta l_{mechanical} = l\alpha\Delta T + \frac{Fl}{AE}$$

In this case, the total deformation of the cylinder and the bolt must be equal, giving

$$\delta l_{bolt} = \delta l_{sleeve}$$

or

$$l\alpha_{bolts}\Delta T + \frac{Fl}{A_{bolts}E_{bolts}} = l\alpha_{cyl}\Delta T - \frac{Fl}{A_{cyl}E_{cyl}}$$

As the cylinder must be in compression and the bolts in tension.
Therefore:

$$(200 \times 10^{-3}) \times (11 \times 10^{-6}) \times (100 - 20) + \frac{(200 \times 10^{-3})F}{2 \times \pi \times (5 \times 10^{-3})^2 \times (200 \times 10^9)}$$
$$= (150 \times 10^{-3}) \times (23 \times 10^{-6}) \times (100 - 20)$$
$$- \frac{(150 \times 10^{-3})F}{\pi \times (25 \times 10^{-3})^2 \times (70 \times 10^9)}$$

or rearranged

$$F = \frac{(l_{cyl}\alpha_{cyl} - l_{bolts}\alpha_{bolts})\Delta T}{\frac{l_{cyl}}{A_{cly}E_{cyl}} + \frac{l_{bolts}}{A_{bolts}E_{bolts}}}$$
$$= \frac{\left((150 \times 10^{-3})(23 \times 10^{-6}) - (200 \times 10^{-3})(11 \times 10^{-6})\right) \times 80}{\frac{(150 \times 10^{-3})}{\pi \times (25 \times 10^{-3})^2 \times (70 \times 10^9)} + \frac{(200 \times 10^{-3})}{2 \times \pi \times (5 \times 10^{-3})^2 \times (200 \times 10^9)}}$$
$$= 13409 \text{ N}$$

Therefore the stress in the cylinder and be calculated as

$$\sigma_{cyl} = -\frac{F}{A_{cyl}} = -\frac{13409}{\pi \times (25 \times 10^{-3})^2} = -6.83 \times 10^6 \text{ Pa} = -6.83 \text{ MPa}$$

5. An unrestrained rectangular section aluminium beam with the cross-sectional dimensions shown in Figure Q5, has a temperature profile given by:

$$\Delta T = 50\left(1 - \frac{4y^2}{40^2}\right)$$

Plot the stress distribution and determine the maximum tensile stress in the bar. For aluminium, α = 23x 10^{-6} °C^{-1} and E = 70 x 10^9 GPa.
[Ans.: 53.7 MPa]

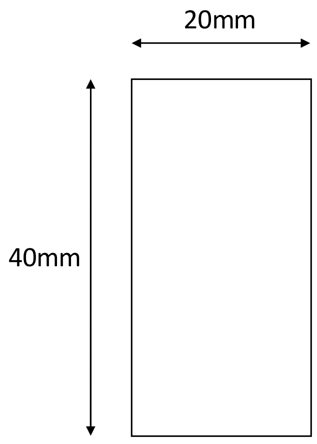


Figure Q5

Applying axial force equilibrium, as there is no applied external force

$$P = 0 = E\bar{\varepsilon}A - E\alpha \int_A \Delta T dA = E\bar{\varepsilon}bd - E\alpha \int_{-\frac{d}{2}}^{\frac{d}{2}} 50\left(1 - \frac{4y^2}{40}\right) dA$$

therefore rearranging for the mean strain gives

$$\bar{\varepsilon} = \frac{50 \times (23 \times 10^{-6})}{40}\left[y - \frac{4y^3}{4800}\right]_{-\frac{d}{2}}^{\frac{d}{2}} = 2.875 \times 10^{-5}(13.333 + 13.333)$$

$$= 2.875 \times 10^{-5} \times 26.666 = 7.6665 \times 10^{-4}$$

From symmetry we can see that $1/R = 0$, therefore $M = 0$

We can then substitute this into the expression for stress

$$\sigma_x = E(\bar{\varepsilon} + \frac{y}{R} - \alpha\Delta T)$$

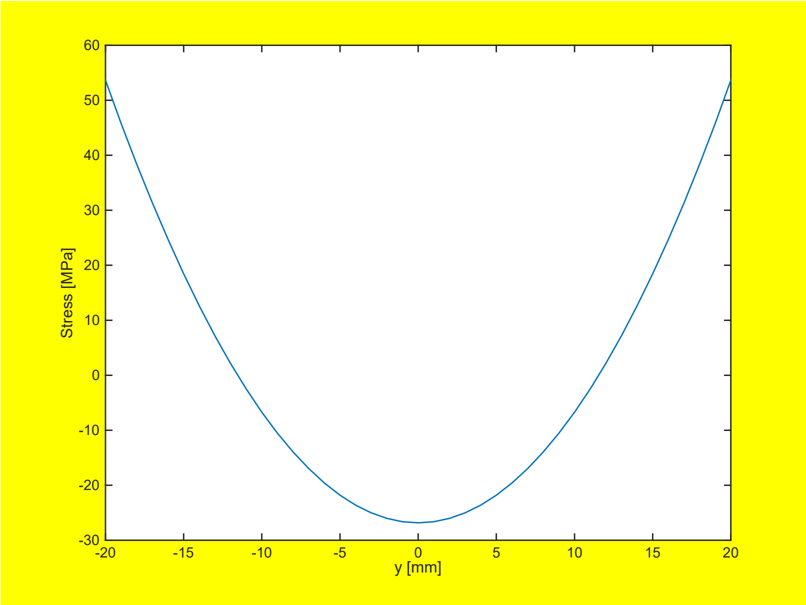
to give us the stress distribution as

$$\sigma_x = E\left(7.6665 \times 10^{-4} + 0 - \alpha \times 50 \times \left(1 - \frac{4y^2}{40^2}\right)\right)$$

which reduces to

$$\sigma_x = 70 \times 10^3\left(7.6665 \times 10^{-4} - 0.0011 + \frac{0.0044y^2}{40^2}\right)$$

This gives the following stress distribution through the thickness of the beam:



Using the expression in the lecture notes, the value of maximum tensile stress can be determined directly using the expression

$$\frac{2E\alpha\Delta T_{max}}{3} = \frac{2 \times 70 \times 10^3 \times 50}{3} = 53.7 \text{ MPa}$$

as shown at -d/2 and d/2 (20 and -20 mm) in the graph.