University of Nottingham

School of Mathematical Sciences

MTHS2007 Advanced Mathematics and Statistics for Mechanical Engineers

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Applications of Fourier Series for ODEs Problem Sheet 3

1. The current I in an electrical circuit satisfies the equation

$$L\frac{dI}{dt} + RI = E.$$

If E is constant and equal to E_0 when t lies between 0 and T, $2T$ and $3T$, $4T$ and $5T$, etc., and is zero at other times, find the current in the circuit.

[*Hint: Express E as a Fourier series.*]

2. If

$$y''(t) + ky'(t) + 25y(t) = f(t),$$

where k is a constant and

$$f(t) = \begin{cases} 0 & \text{for} \quad -\pi \leq t < 0 \\ 1 & \text{for} \quad 0 \leq t < \pi, \end{cases} \qquad f(t + 2\pi) = f(t),$$

what is the solution when (a) $0 < k < 10$, (b) $k = 0$?

3. The function $f(x)$ is periodic with period 2π, is defined in $0 < x \leq \pi$ by $f(x) = \pi - x$ and is an odd function. Sketch the graph of $f(x)$ in the range $-3\pi \leq x \leq 3\pi$.

Find the Fourier coefficients of $f(x)$.

To what value does the Fourier series for $f(x)$ converge when

(i) $x = 2\pi$, (ii) $x = \frac{1}{2}\pi$.

Deduce that

$$\frac{\pi}{4} = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}.$$

Find a particular integral in the form of a Fourier series

$$\sum_{n=1}^{\infty} (A_n \cos nx + B_n \sin nx)$$

for the ordinary differential equation

$$\frac{d^2y}{dx^2} + 2y = f(x),$$

where $f(x)$ is defined above.