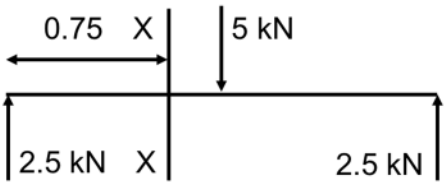
MM2MS3 - Mechanics of Solids 3
Exercise Sheet 4 - Shear Stresses/Shear Centre

1. A simply supported rectangular beam has a depth of 150 mm, a width of 75 mm and a span of 2 m. It carries a load of 5 kN at the centre of the span, find the shear stress and the normal stress at a point 50 mm from the neutral axis, on a sections perpendicular to the axis of the beam at a distance of 0.75 m from one support.
   [*Ans: 0.185 MPa, 4.44 MPa*]

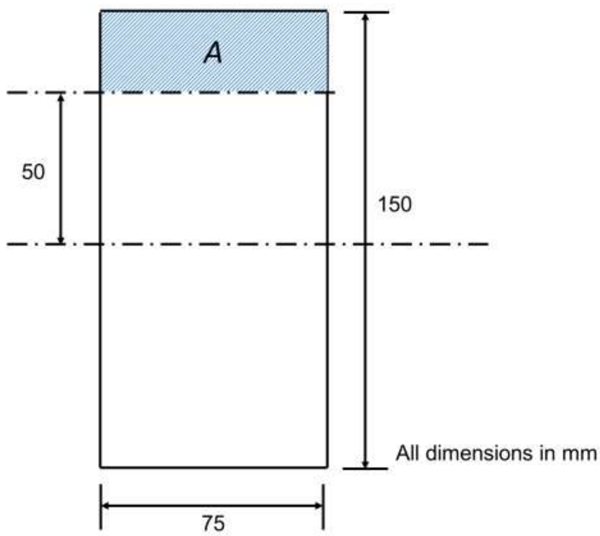
   Assume linear elastic behaviour

$$I = \frac{Bd^3}{12} = \frac{75 \times 150^3}{12} = 21093750 \text{ mm}^4$$



at X-X, S = 2500 N, M = 2500 x 0.75 = 1875 Nm (1875 x 10^3 Nmm)

Shear stress at 50 mm from the N. A.



$$\tau = \frac{SA\bar{y}}{It} = \frac{2500 \times (25 \times 75) \times 62.5}{21093750 \times 75} = \mathbf{0.185\ MPa}$$

Normal stress

$$\sigma = \frac{My}{I} = \frac{1875 \times 10^3 \times 50}{21093750} = \mathbf{4.44\ MPa}$$

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2. Fig. Q2 shows the cross-section of a solid beam which carries a vertical shear force of 100 kN.

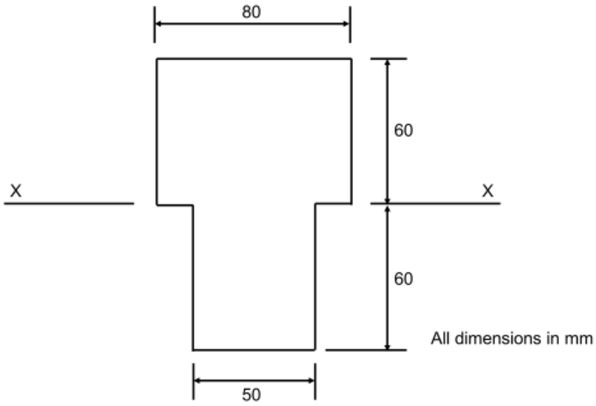


Fig. Q2

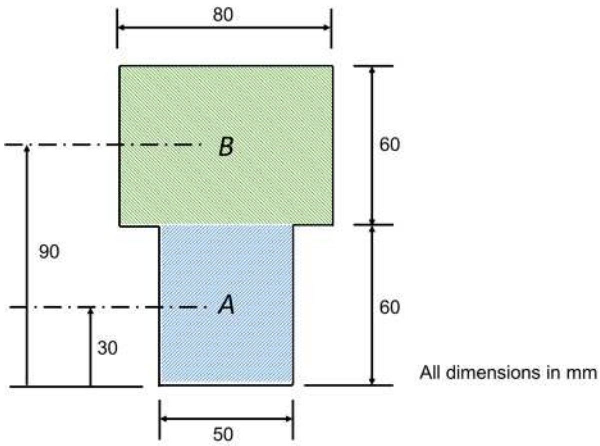
a) Determine the shear stress just above and just below the line X-X
b) Determine the shear stress at the Neutral Axis of the section
c) Sketch the shear stress distribution through the section and state where the maximum shear stress occurs.

[***Ans: a) 15.41 MPa, 24.65 MPa b) 15.68 MPa***]

Determine the location of the N. A. using lower edge as datum.

Total area = (80 x 60) + (50 x 60) = 7800 mm^2

First moments of area of two sub-sections



Area A = (60 x 50) x 30 = 90000 mm^3
Area B = (80 x 60) x 90 = 432000 mm^3

Therefore N. A. is located at

$$y = \frac{\sum A_n y_n}{A_T} = \frac{90000 + 432000}{7800} = \mathbf{66.92\ mm}$$

2nd moment of area

$$I = \sum \left(\frac{BD^3}{12} + A(\bar{y}_n - \bar{y})^2 \right)$$

Area	$\bar{y}_n - \bar{y}$	$A(\bar{y}_n - \bar{y})^2$	$\frac{BD^3}{12}$	$A(\bar{y}_n - \bar{y})^2 + \frac{BD^3}{12}$
A	30 – 66.92 = -36.92	4089259	900000	4989259
B	90 – 66.92 = 23.08	2556895	1440000	3996895
			I	**8986154 mm^4**

a) calculate two shear stresses at X-X due to section change using:

$$\tau = \frac{SA\bar{y}}{It}$$

Using area below X-X:

$$\tau_{XX1} = \frac{SA\bar{y}}{It} = \frac{100000 \times (60 \times 50) \times (66.92 - 30)}{8986154 \times 50} = \mathbf{24.65\ MPa}$$

$$\tau_{XX2} = \frac{SA\bar{y}}{It} = \frac{100000 \times (60 \times 50) \times (66.92 - 30)}{8986154 \times 80} = \mathbf{15.41\ MPa}$$

b) calculate shear stress at the N. A.

Can use area above *or* below N. A.

Using area above N.A.

$$\tau_{NA} = \frac{SA\bar{y}}{It} = \frac{100000 \times (80 \times (120 - 66.92) \times \left(\frac{(120 - 66.92)}{2}\right)}{8986154 \times 80} = \mathbf{15.68\ MPa}$$

Using area below N. A. and considering

$$Q = \sum A(y_n - y)$$

$$\tau = \frac{SQ}{It}$$

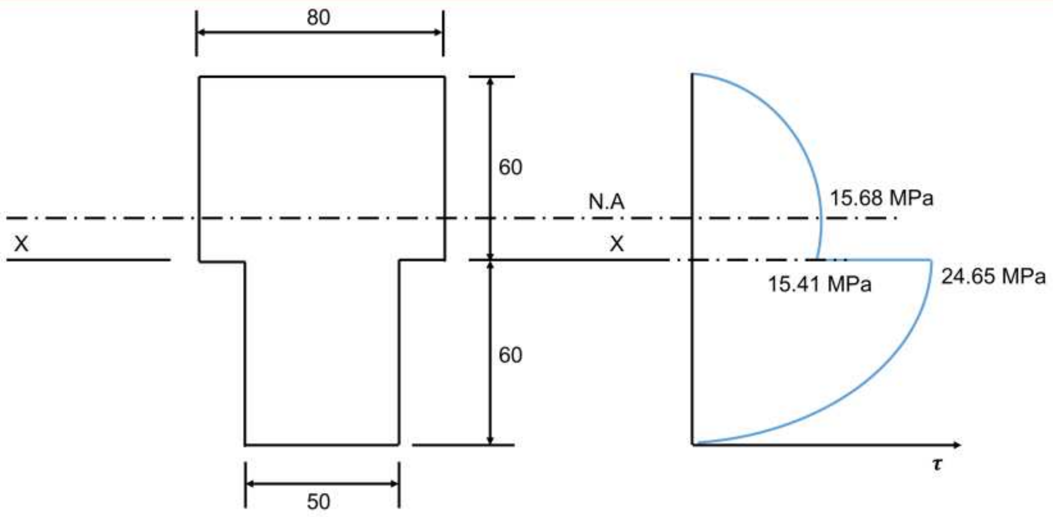
where

$$Q = \big((60 \times 50) \times (62.92 - 30)\big) + \left((80 \times 6.92) \times \left(\frac{6.92}{2}\right)\right) = 112675$$

$$\tau = \frac{SQ}{It} = \frac{100000 \times 112675}{8986154 \times 80} = \mathbf{15.68}\ \mathbf{MPa}$$

which is the identical answer as using the single area previously.

c) Sketch the shear stress distribution



Max shear stress is at X-X

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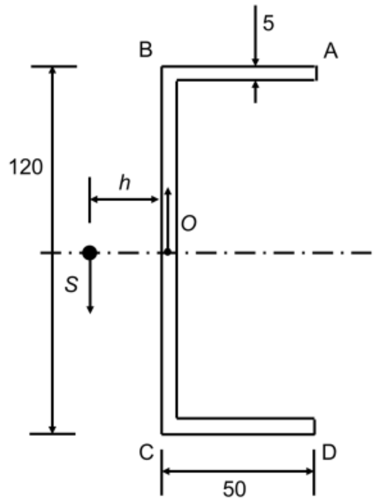
3. The outer dimensions of a channel girder section are 120 mm (web) x 50 mm (flanges); the web of the flanges are 5 mm thick. Determine the position of the shear centre of the section
[*Ans: 16.9mm from the central plane of the web, on the axis of symmetry*]



Calculate 2nd moment of area:

$$I = \frac{BD^3}{12} - \frac{bd^3}{12} = \frac{50 \times 120^3}{12} - \frac{45 \times 110^3}{12} = 2208750\ \text{mm}^4$$

Recalling that the shear force in the web is ≈S

Recalling:

$$\tau = \frac{SA\bar{y}}{Iz}$$

We can determine the shear stress distribution (and hence the shear force in the flange).

At a position x from A,

$$\tau = \frac{S}{I \times 5}[(5 \times x) \times 57.5] = \frac{57.5Sx}{I}$$

We can therefore determine the shear force in the flange AB, F_{AB} by:

$$F_{AB} = \int_0^{47.5} 5\tau dx = \frac{5S}{I} \times 57.5 \left[\frac{x^2}{2}\right]_0^{47.5} = 0.147S$$

which must be equal and opposite to the shear force in flange CD, F_{CD}.

Note: it is reasonable to take the shear stress in AB to extend to x=47.5 mm since this gives a consistent method which will apply to the web.

Taking moments about O gives:

$$S \times h = 2 \times F \times 57.5 = 2 \times 0.147S \times 57.5$$

and rearranging gives

$$\boldsymbol{h = 16.9\ \mathrm{mm}}$$

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4. Show that the difference between the maximum and mean shear stress in the web of an I beam is:

$$\frac{Sd^2}{24I}$$

where *d* is the height of the web.

For an I beam, at any location, *y*, in the web (see lecture notes for geometry notation):

$$\tau = \frac{S}{It}\left(\int_y^{\frac{d}{2}} yzdy + \int_{\frac{d}{2}}^{\frac{D}{2}} yzdy\right)$$

The maximum value of shear stress occurs when *y* = 0:

$$\tau_{max} = \frac{S}{It}\left(\int_0^{\frac{d}{2}} yzdy + \int_{\frac{d}{2}}^{\frac{D}{2}} yzdy\right)$$

i.e.

$$\tau_{max} = \frac{S}{It}\left(\frac{td^2}{8} + B\left(\frac{D^2}{8} - \frac{d^2}{8}\right)\right)$$

In general, the value of the shear stress at any point in the web is given by:

$$\tau_{mean} = \frac{S}{It}\left[t\left(\frac{d^2}{8} - \frac{y^2}{2}\right) + B\left(\frac{D^2}{8} - \frac{d^2}{8}\right)\right]$$

The mean shear stress in the web is given by:

$$\tau_{mean} = \frac{2}{td}\int_0^{\frac{d}{2}} \tau t dy$$

$$\tau_{mean} = \frac{2}{td}\frac{S}{It}t\left[t\left(\frac{d^2y}{8} - \frac{y^3}{6}\right) + By\left(\frac{D^2}{8} - \frac{d^2}{8}\right)\right]_0^{\frac{d}{2}}$$

$$\tau_{mean} = \frac{2}{d}\frac{S}{It}\left[t\left(\frac{d^3}{16} - \frac{d^3}{48}\right) + \frac{Bd}{2}\left(\frac{D^2}{8} - \frac{d^2}{8}\right)\right]$$

$$\tau_{mean} = \frac{S}{It}\left[t\left(\frac{d^3}{8} - \frac{d^3}{24}\right) + B\left(\frac{D^2}{8} - \frac{d^2}{8}\right)\right]$$

So,

$$\boxed{\tau_{max} - \tau_{mean} = \boldsymbol{\frac{Sd^2}{24I}}}$$

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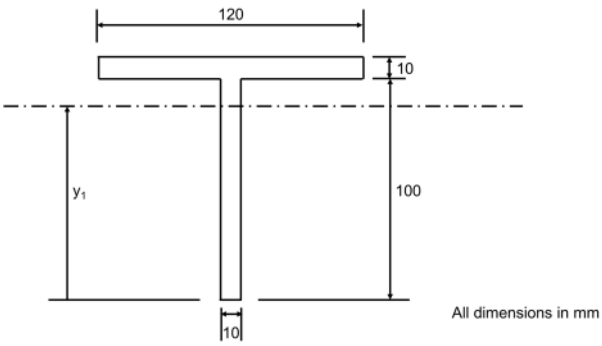
5. For a T-section beam with a flange 120 mm by 10 mm and a web 100 mm by 10 mm, what percentage of the shearing force at any section is carried by the web?
[*Ans: 93.6 %*]



Determine the location of the N. A. using lower edge as datum.

Total area = (100 x 10) + (120 x 10) = 2200 mm^2

First moments of area of two sub-sections as in Q2

Area A = (100 x 10) x 50 = 50000 mm^3
Area B = (120 x 10) x 105 = 126000 mm^3

Therefore N. A. is located at

$$y_1 = \frac{\sum A_n y_n}{A_T} = \frac{50000 + 126000}{2200} = \mathbf{80\ mm}$$

from the base.

At any distance, *y,* from the N.A. the shear stress is given by:

$$\tau = \frac{S}{Iz} A\bar{y}$$

Therefore:

$$\tau = \frac{S}{10I}\left[\int_y^{20} 10y dy + \int_{20}^{30} 120y dy\right]$$

$$\tau = \frac{S}{10I}\left[10\left[200 - \frac{y^2}{2}\right] + 120[450 - 200]\right]$$

$$\tau = \frac{S}{10I}[2000 - 5y^2 + 30000]$$

$$\tau = \frac{S}{I}\left[3200 - \frac{y^2}{2}\right]$$

applies between y = -80 and 20 from the N.A.

The shear force carried by the web, S_{web}, is given by:

$$S_{web} = \int_{-80}^{20} t\tau dy$$
$$S_{web} = \int_{-80}^{20} \frac{10S}{I}\left(32000 - \frac{y^2}{2}\right) dy$$
$$S_{web} = \frac{10S}{I}\left[32000y - \frac{y^3}{6}\right]_{-80}^{20}$$
$$S_{web} = \frac{10S}{I}\left[320000 - \frac{1}{6}(8000 + 512000)\right]$$
$$S_{web} = \frac{10S}{I}[320000 - 86677]$$
$$S_{web} = 2333333\frac{S}{I}$$
$$I = \frac{10 \times 100^3}{12} + 10 \times 100 \times 30^2 + \frac{120 \times 10^3}{12} + 120 \times 10 \times 25^2$$
$$I = 833333 + 900000 + 10000 + 750000$$
$$I = 2493333 \text{ mm}^4$$

Therefore, the proportion of the total shear force carried by the web is given by:

$$S_{web} = \frac{2333333}{2493333}S = 0.936\, S = \mathbf{93.6}\%$$

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6. A length of channel girder from Q3 is loaded as a cantilever by an end load of 1.2kN acting in the plane of the web. What twisting moment acts at a general section of the cantilever?
   [*Ans: 20.3 Nm*]

   Taking moments about O:

$$M = 2 \times .147S \times 57.5 = 2 \times 0.147 \times 57.5 \times 1200 = 20286 \text{ Nmm} = \mathbf{20.3\ Nm}$$

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7. Find the shear centre of the beam cross-section shown in Fig. Q7 (*t* is much smaller than *R*).

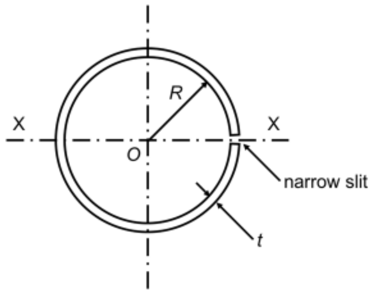
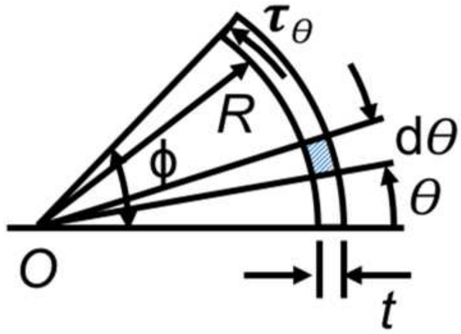


Fig. Q7.

[*Ans: 2R from O along the axis of symmetry X-X, away from the slit*]

By inspection, the shear centre must lie on the X-X axis, to find where we can imagine bending it about the X-X axis with a varying BM along the length.
We know that the shear stress, τ must act tangentially because the inside and outside are free surfaces.



To find the shear stress, τ at ϕ from the slit use the integral form of the shear stress equation:

$$\tau = \frac{S}{Iz}\int_A ydA$$

where z = t and $I = \pi R^3 t$ (for a thin walled tube)

Converting to polar coordinates, substituting and evaluating the integral:

$$\tau = \frac{S}{\pi R^3 t^2}\int_0^{\phi} (R\sin\theta)\, tR\, d\theta = \frac{S}{\pi R t}[-\cos\theta]_0^{\phi} = \frac{S}{\pi R t}(1-\cos\theta)$$

To find the twisting moment associated with this shear stress for the whole section, take moments about *O*:

$$T = \int_0^{2\pi} \tau(R\,d\theta)tR = \frac{R^2tS}{\pi Rt}\int_0^{2\pi}(1-\cos\theta)d\theta = \frac{SR}{\pi}[\theta - \sin\theta]_0^{2\pi} = 2SR$$

Therefore, the shear centre is 2*R* from *O* along the axis of symmetry X-X, away from the slit