Solutions to sheet 6 Vibrations

Q1

Rayleigh's method

$$T_{max} = \frac{1}{2}\omega^2\left[I\theta_1^2 + 2I\theta_2^2 + I\theta_3^2\right]$$

$$U_{max} = \frac{1}{2}\left[2k\theta_1^2 + k(\theta_1-\theta_2)^2 + k(\theta_2-\theta_3)^2\right]$$

mode shape guesses:

A) for mode 1, $\theta_3 > \theta_2 > \theta_1$ and all deflections are in-phase

$\therefore$ try $\theta_1 : \theta_2 : \theta_3 = 1 : 2 : 3$

$$\omega_n^2 = \frac{2}{9}\frac{k}{I} \Rightarrow \omega_n = 0.471\sqrt{\frac{k}{I}}$$

B) "static" deflection obtained by applying a static torque to each disk proportional to it's inertia.

$\therefore$ $\theta_1 : \theta_2 : \theta_3 = 2 : 5 : 6$

$$\omega_n^2 = \frac{1}{5}\frac{k}{I} \Rightarrow \omega_n = 0.447\sqrt{\frac{k}{I}}$$

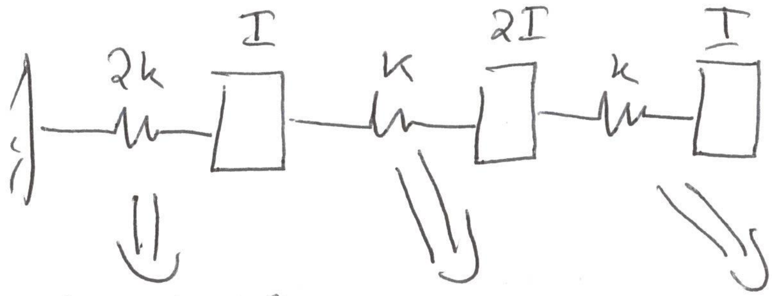
c) Exact value from matlab would be

$$\omega_n = 0.445\sqrt{\frac{k}{I}}$$

Q1 B) supplemental

to find $\theta_1 : \theta_2 : \theta_3$ using static deflection

think of it as:



deflection θ_1 deflection θ_2 deflection θ_3

$$\Delta\theta_1 = \frac{I+2I+I}{2k} = \frac{4}{2}\frac{I}{k} = 2\frac{I}{k}$$

$$\Delta\theta_2 = \frac{2I+I}{k} = 3\frac{I}{k}$$

$$\Delta\theta_3 = \frac{I}{k} = 1\frac{I}{k}$$

the $\frac{I}{k}$ will drop out

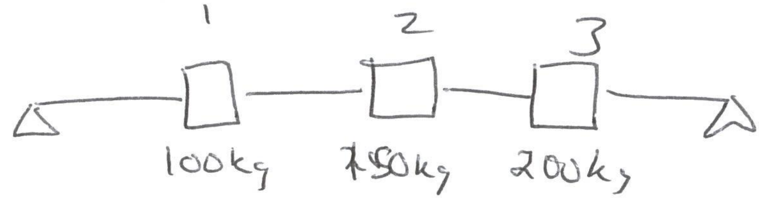
for θ_1 consider $\Delta\theta_1$ only $\Rightarrow 2$

~~$\Delta\theta_1$~~

for θ_2 conside $\Delta\theta_1 + \Delta\theta_2 \Rightarrow 5$

for θ_3 consider $\Delta\theta_1 + \Delta\theta_2 + \Delta\theta_3 \Rightarrow 6$

Q2



$$k_i = \frac{3EIL}{x_i^2 (L - x_i)^2}$$

(i) One choice is $Y(x) = \sin \frac{\pi x}{L}$ as this is the beam's mode shape w/out masses

$$U_m = \frac{EI \pi^4}{4L^3} = 5.8 \times 10^6$$

$$T_m = \frac{1}{2} \omega^2 \sum_{i=1}^{3} m_i [Y(x_i)]^2 = 126.8\, \omega^2$$

$\therefore$ $\omega_n = 34.14$ Hz $= 2048 \frac{\text{rev}}{\text{min}}$

(ii) $T_{mT} = T_m + T_{beam} = \frac{1}{2} \omega^2 \sum_{i=1}^{3} m_i [Y(x_i)]^2 + \frac{1}{4} \rho A L \omega^2$

$T_{mT} = 239.32\, \omega^2$

U_{max} is unchanged

$\therefore$ $\omega_n = 24.9$ Hz $= 1491 \frac{\text{rev}}{\text{min}}$

Q3

$$T_{max} = \frac{1}{2} 3m\omega^2 X^2 + \frac{1}{2} m\omega^2 Y^2 = \frac{1}{2} m^+ \omega^2 X^2$$

$$\therefore \quad m^+ = m\left(\frac{3X^2 + Y^2}{X^2}\right)$$

$$U_{max} = \frac{1}{2} k X^2 + \frac{1}{2} k (X-Y)^2 + \frac{1}{2} k Y^2 = \frac{1}{2} k^+ X^2$$

$$\therefore \quad k^+ = k\left(\frac{X^2 + (X-Y)^2 + 2Y^2}{X^2}\right)$$

$$\omega_n = \sqrt{\frac{k^+}{m^+}}$$

Guess

A) $X : Y = 2 : 1$

$$m^+ = 3.25\,m$$

$$k^+ = 1.75\,k$$

$$\therefore \quad \omega_n = 0.734\sqrt{\frac{k}{m}}$$

B) $X : Y = 3 : 1$

$$m^+ = 3.11\,m$$

$$k^+ = 1.67\,k$$

$$\therefore \quad \omega_n = 0.732\sqrt{\frac{k}{m}}$$