MM2DYN Dynamics: Control Lecture 3

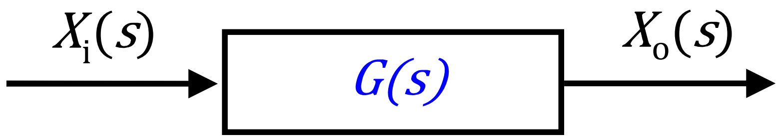
Block Diagram Representation of control systems and their manipulation
Block Diagram treatment for hydraulic position control

Lecture Objectives:

- Demonstrate Block diagram manipulation
- Demonstrate System modelling for a first order system

Block Diagrams Revisited

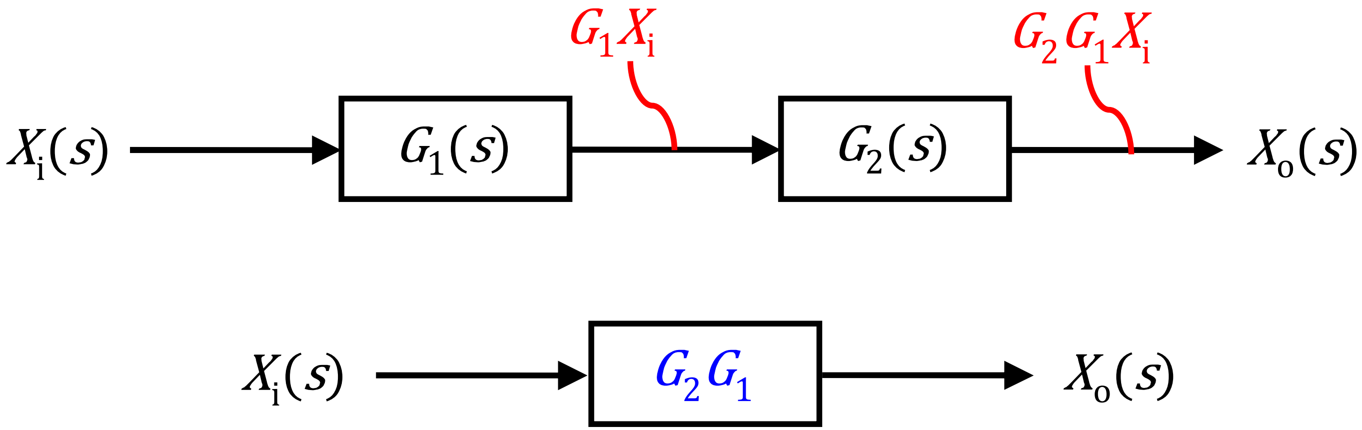
- Systems engineers represent the components of the system as a series of blocks:
- Recap:



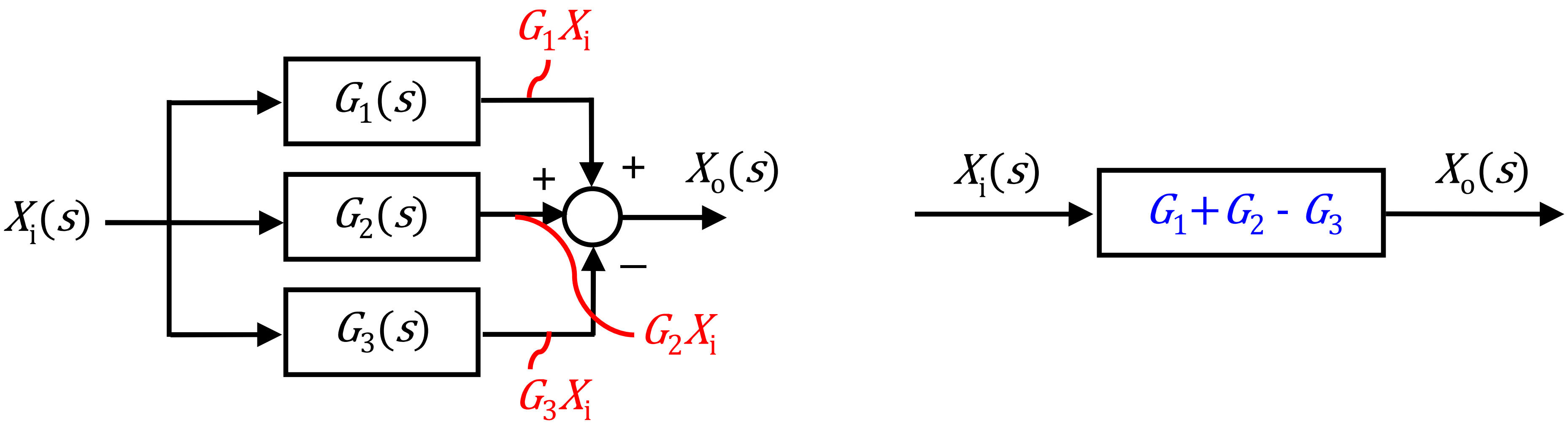
- The transfer function G(s) transforms the input X_i into the output X_0
- $X_0(s) = G(s)X_i(s)$

Block Diagram Manipulation: The rules

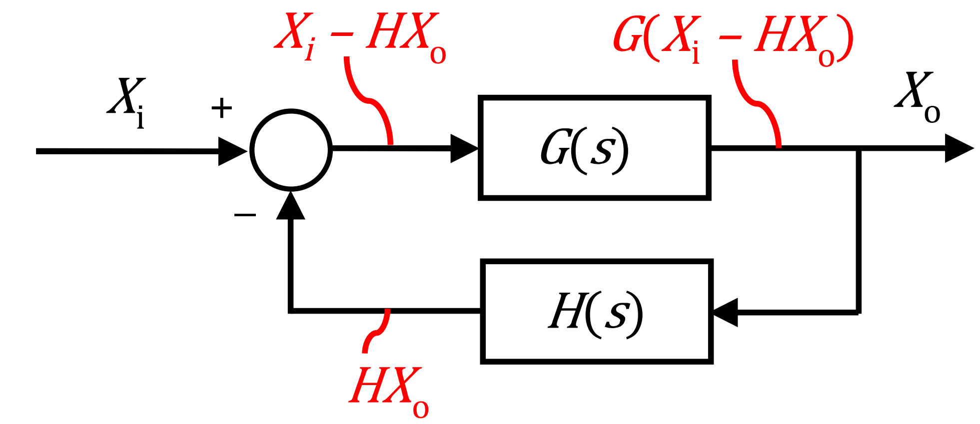
a) Elements in Series

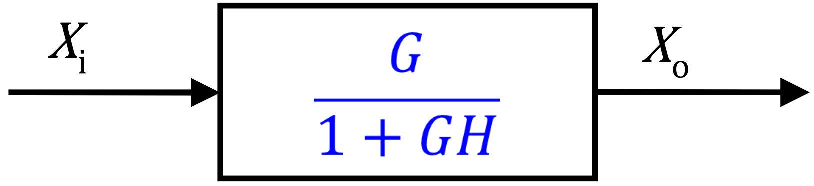


b) Elements in Parallel

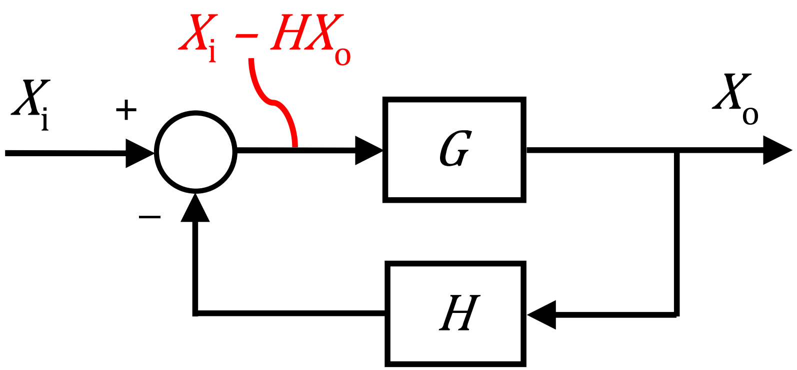


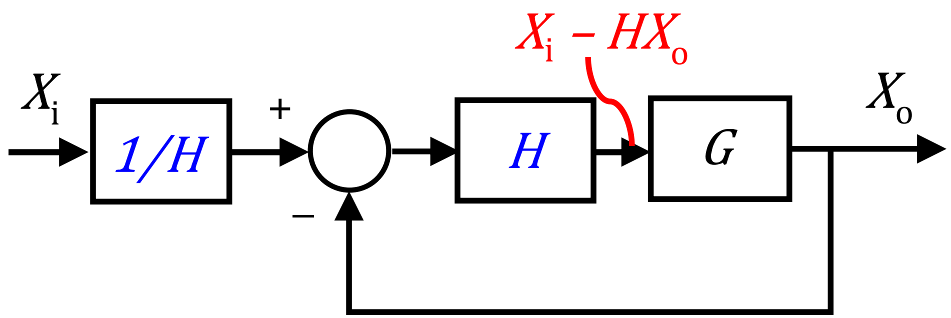
c) Feedback (Closed loop) Transfer Function

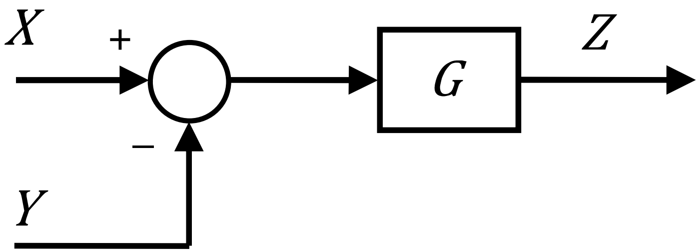


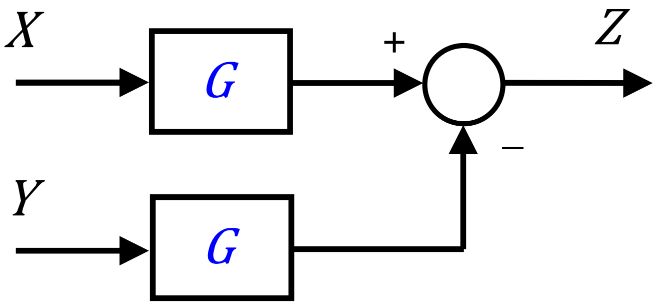


d) Changing block positions:

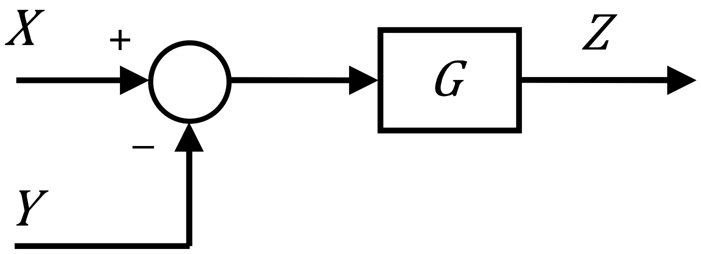


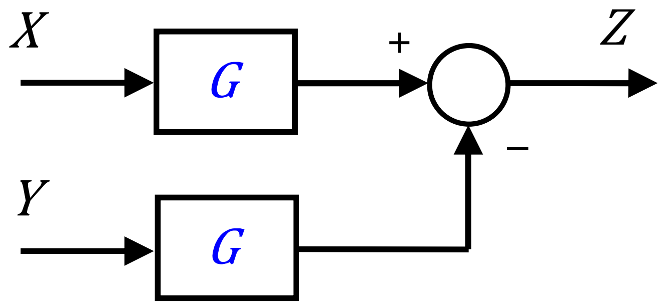






d) Changing block positions

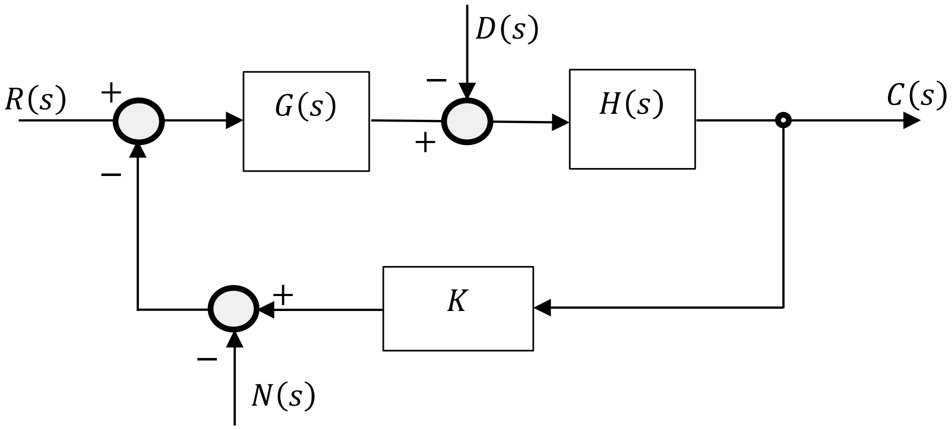




$$Z(s) = G(s)\big(X(s) - Y(s)\big)$$

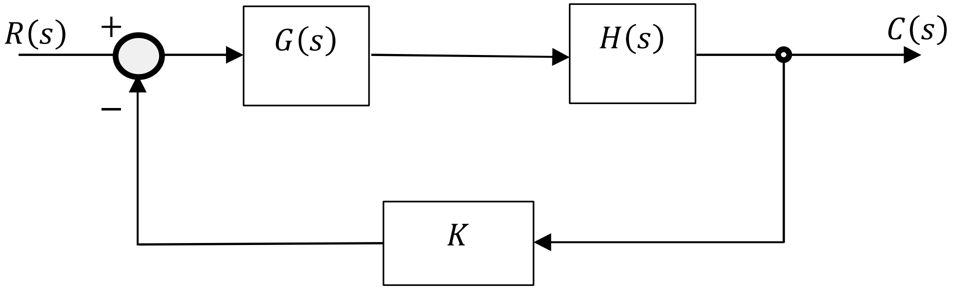
Check: Does this give the same result?

e) Dealing with disturbances



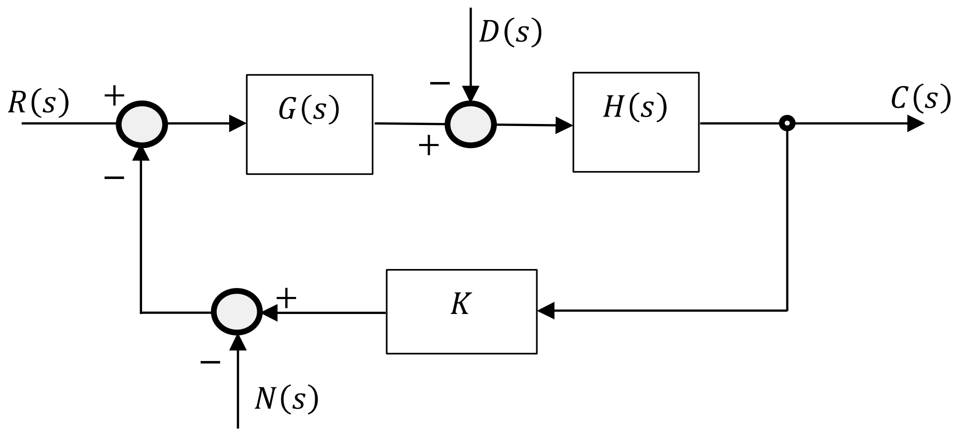
- R(s) is the input
- C(s) is the output
- D(s) is a disturbance
- N(s) is a (user controlled) compensation

- Procedure:
  - Each of the inputs has its own independent transfer function. For R(s):



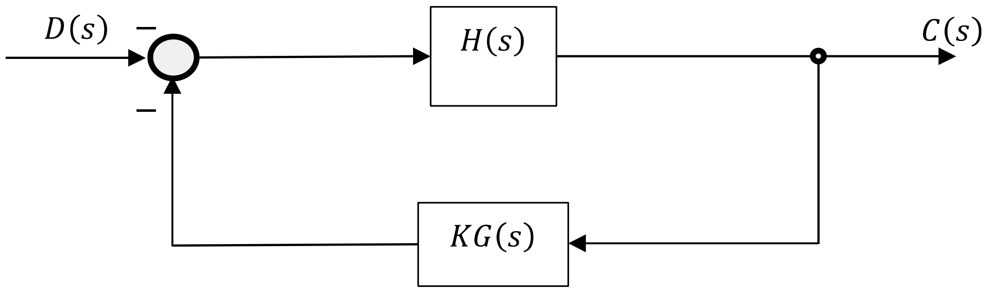
$$C(s) = \big(R(s) - KC(s)\big)G(s)H(s)$$
$$C(s)\big(1 + KG(s)H(s)\big) = R(s)G(s)H(s)$$
$$\frac{C(s)}{R(s)} = \frac{G(s)H(s)}{1 + KG(s)H(s)}$$

e) Dealing with disturbances



- R(s) is the input
- C(s) is the output
- D(s) is a disturbance
- N(s) is a (user controlled) compensation

- Procedure:
  - For D(s):

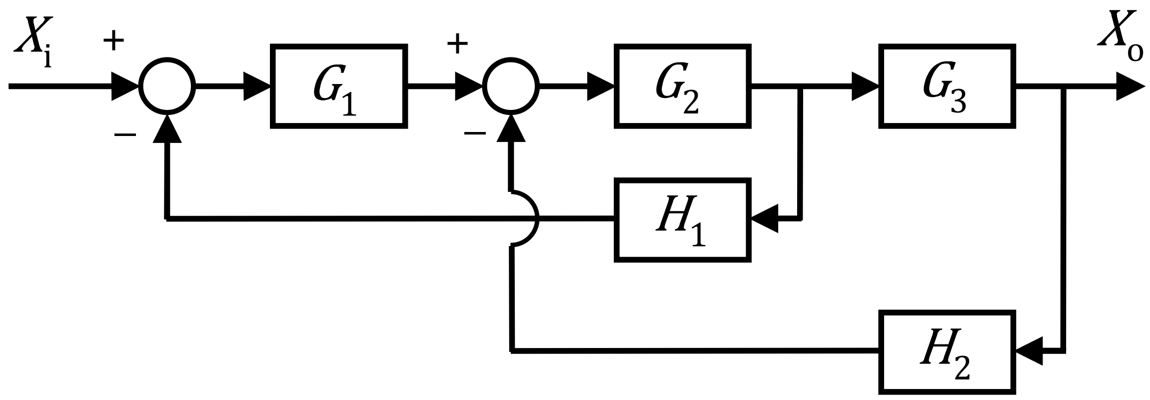


$$C(s) = -\big(D(s) + KG(s)C(s)\big)H(s)$$

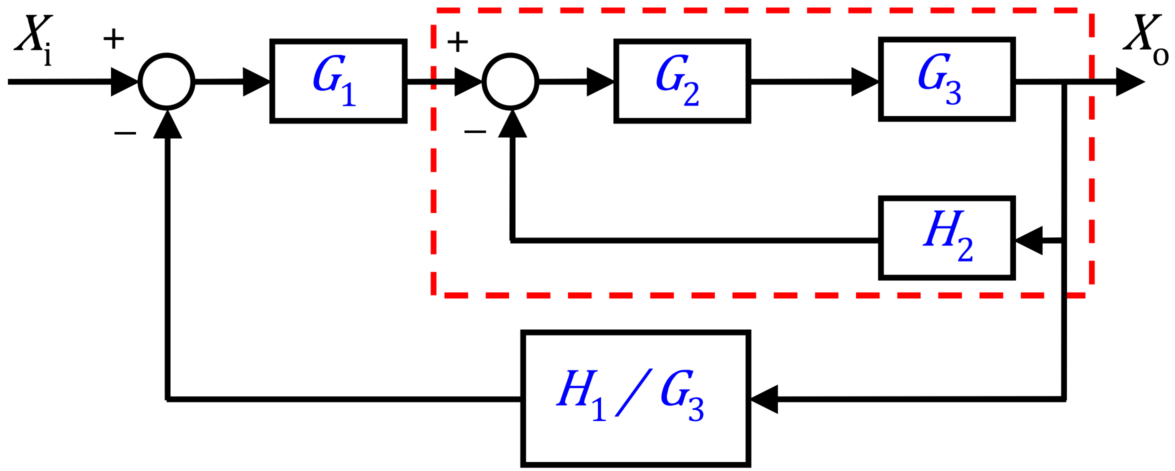
$$C(s)\big(1 + KG(s)H(s)\big) = -D(s)H(s)$$

$$\frac{C(s)}{D(s)} = \frac{-H(s)}{1 + KG(s)H(s)}$$

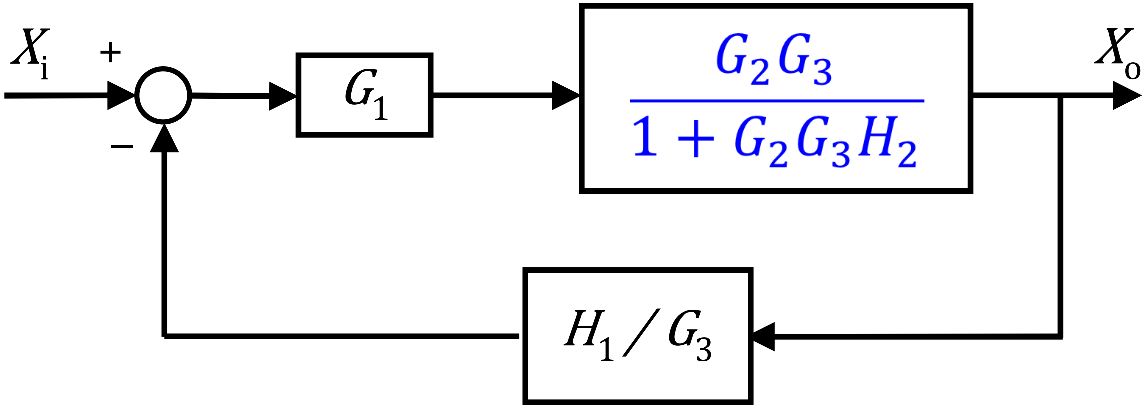
Example: Reduce the block diagram and find the overall T.F.



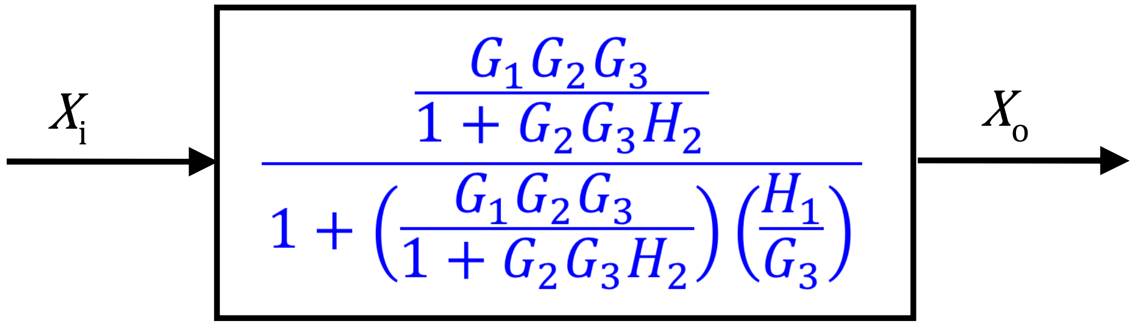
i) Rearrange to avoid interlinking loops

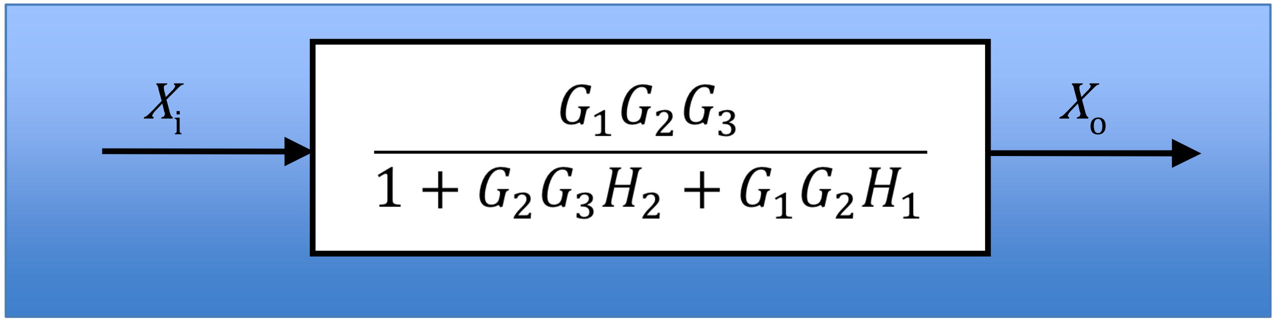


ii) Eliminate the inner loop



iii) Reduce to a single block and simplify





MM2DYN Dynamics: Control Lecture 5

Position Control Systems

(case studies in 1^{st} & 2^{nd} order systems)

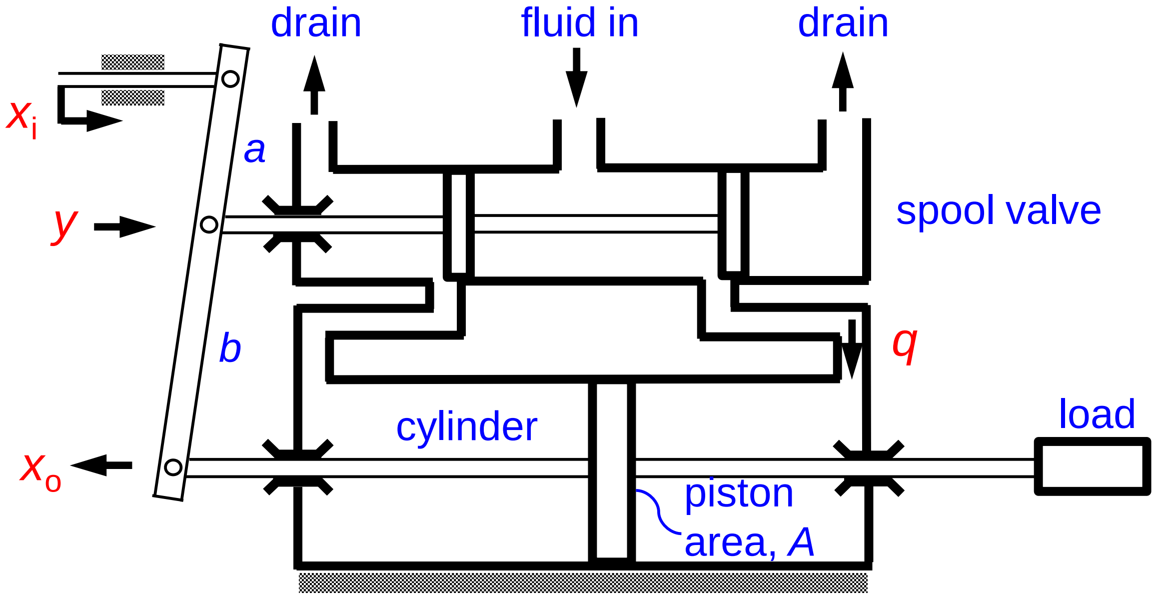
Lecture Objectives:

- Introduce the differences between 1^{st} and 2^{nd} order systems
- Analyse steady-state responses under step and ramp inputs
- Analyse transient behaviour through the roots of the characteristic equations

Video Interlude

- Motherboard Assembly
  - https://www.youtube.com/watch?v=ym64NFCWORY
- Before we watch the video:
  - Is the robot quicker than the employee?
  - Was this robot customised for the application?
  - What would happen if the robot overshoots?

Recap: Hydraulic Position Control System

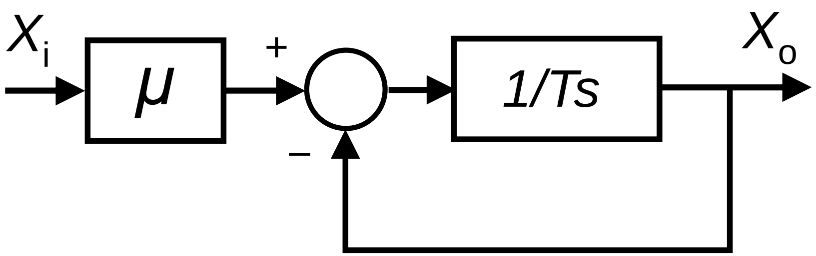


It was shown that the **transfer function** is given by

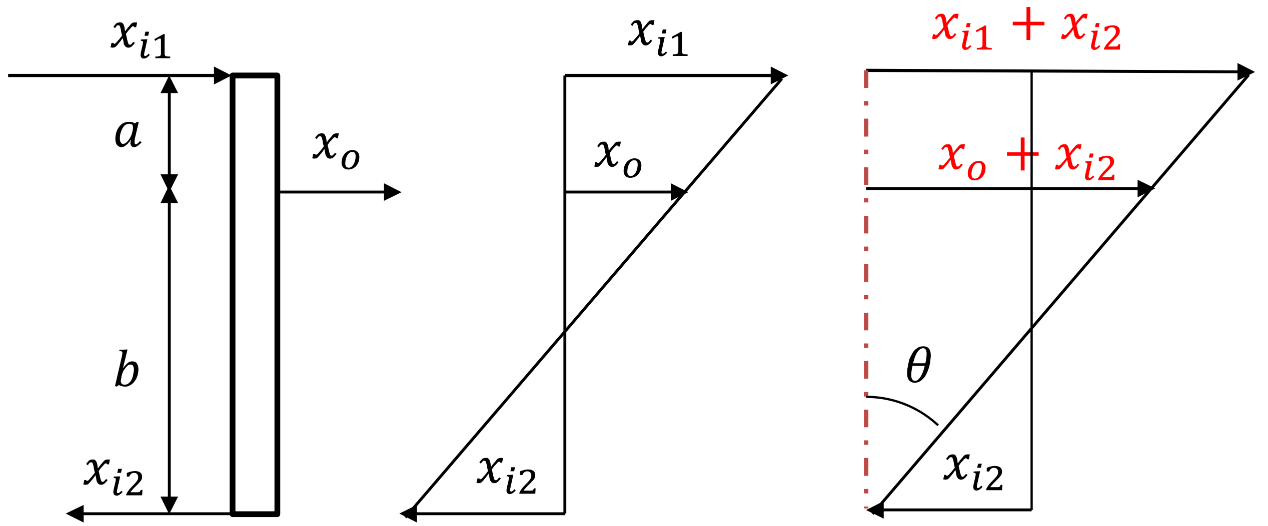
$$G(s) = \frac{X_o(s)}{X_i(s)} = \frac{\mu}{1 + Ts}$$

1st order system

with the **block diagram** (to be shown in this lecture)



Feedback link:



$$\tan\theta = \frac{x_{i1}+x_{i2}}{a+b} = \frac{x_o+x_{i2}}{b}$$

$$x_o = \frac{b}{a+b}x_{i1} - \frac{a}{a+b}x_{i2}$$

Hydraulic Position Control System: Equations for the Model

Spool Valve

in the time domain $$q = Ky$$

transfer function $$\frac{Q(s)}{Y(s)} = K \qquad (1)$$

Ram Piston

in the time domain $$A\frac{dx_\mathrm{o}}{\mathrm{d}t} = q$$

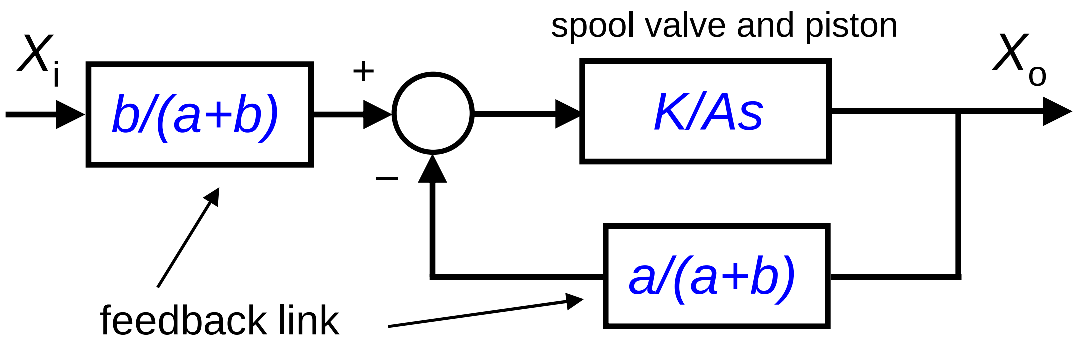
transfer function $$\frac{X_\mathrm{o}(s)}{Q(s)} = \frac{1}{As} \qquad (2)$$

Feedback Link

in the time domain $$y = \frac{b}{a+b}x_\mathrm{i} - \frac{a}{a+b}x_\mathrm{o}$$

transfer function $$Y(s) = \frac{b}{a+b}X_\mathrm{i}(s) - \frac{a}{a+b}X_\mathrm{o}(s) \qquad (3)$$

Hydraulic Position Control System: Overall Transfer Function



From the block diagram

$$X_o(s) = \left[X_i(s)\frac{b}{a+b} - X_o(s)\frac{a}{a+b} \right] \frac{K}{As}$$

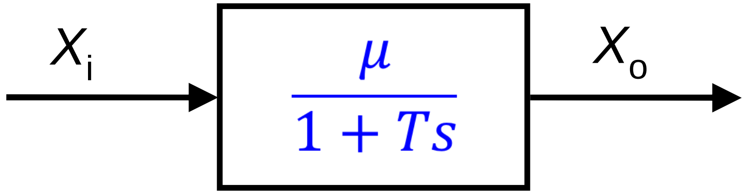
rearranging

$$\left[1 + \frac{A(a+b)s}{Ka} \right] X_o(s) = \frac{b}{a} X_i(s)$$

$$\frac{X_o(s)}{X_i(s)} = \frac{\mu}{1+Ts} \qquad (4)$$

First order system with time constant *T* and gain *μ*

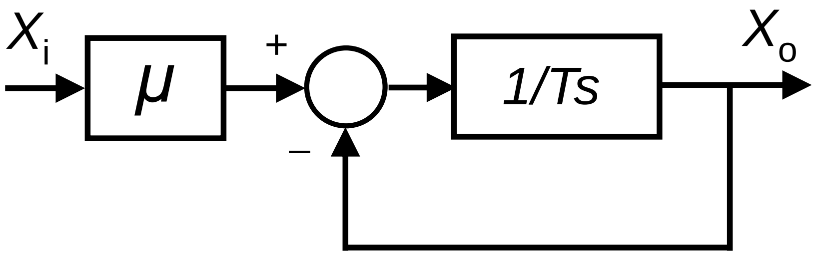
Hydraulic Position Control System: Control System Model



$$T = \frac{A(a+b)}{Ka}$$ **time constant**

$$\mu = \frac{b}{a}$$ **steady-state gain**

Exercise: Show that the block diagram for a system governed by (4) can be drawn as

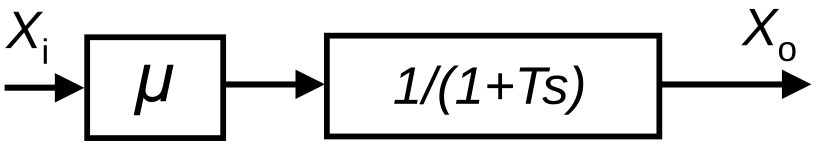


Exercise

- From Equation 4:

$$\frac{X_o(s)}{X_i(s)} = \frac{\mu}{1 + Ts}$$

- First of all: recognise that gain is μ:

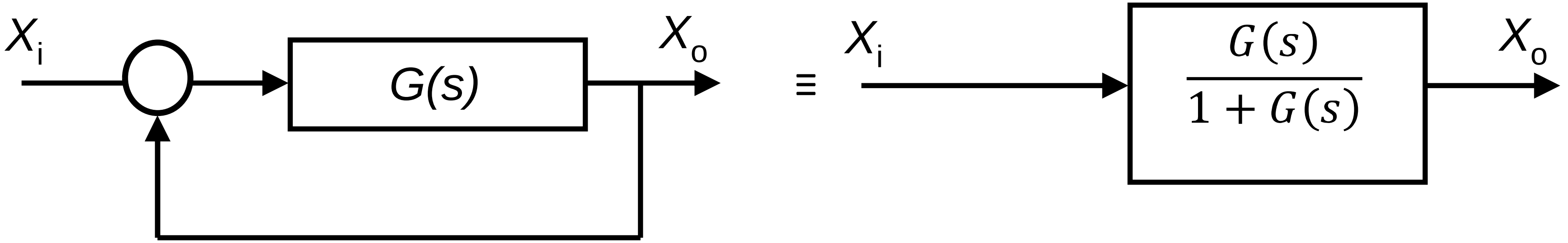


Exercise

- From Equation 4:

$$\frac{X_o(s)}{X_i(s)} = \frac{\mu}{1 + Ts}$$

- Second: Unity feedback loop

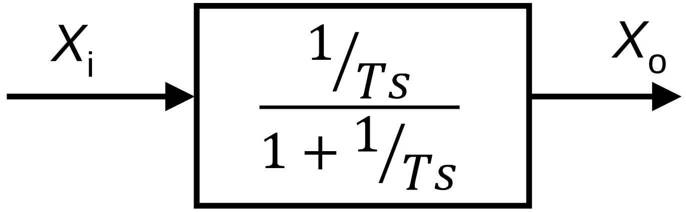


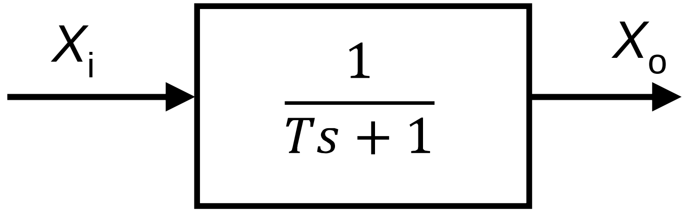
Exercise

- From Equation 4:

$$\frac{X_o(s)}{X_i(s)} = \frac{\mu}{1 + Ts}$$

- Third: $G(s) = \frac{1}{Ts}$



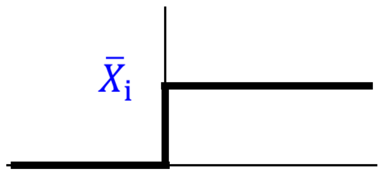


Time domain

- Input is a unit step: $X_i(s) = \frac{1}{s}$
- Output $X_o(s) = \frac{1}{s}\left(\frac{\mu}{1+Ts}\right)$
- Go to the table of Laplace transforms: No. 8
- $x_o(t) = \mu\left(1 - e^{-t/T}\right)$

Hydraulic Position Control System under Standard Inputs

i) step Input



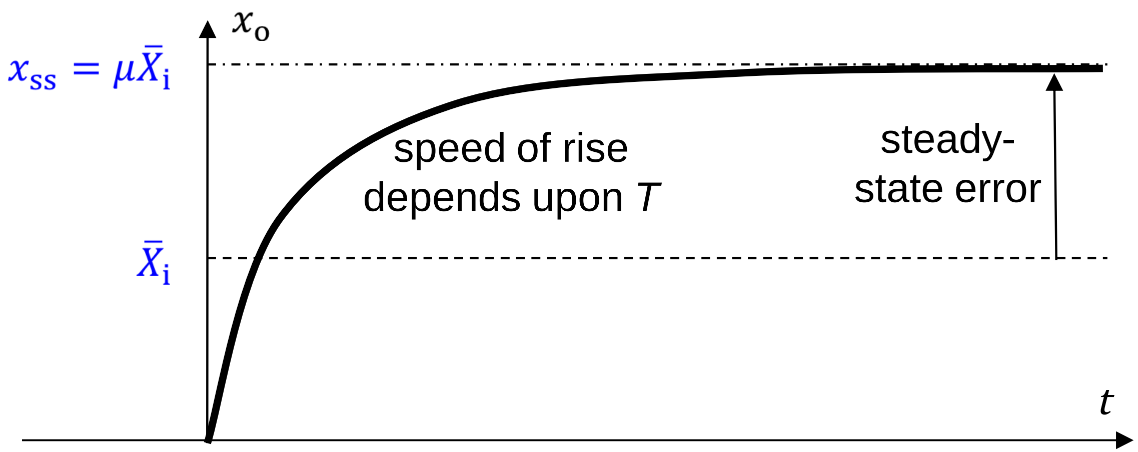
$t < 0 \qquad x_\text{i}(t) = 0$

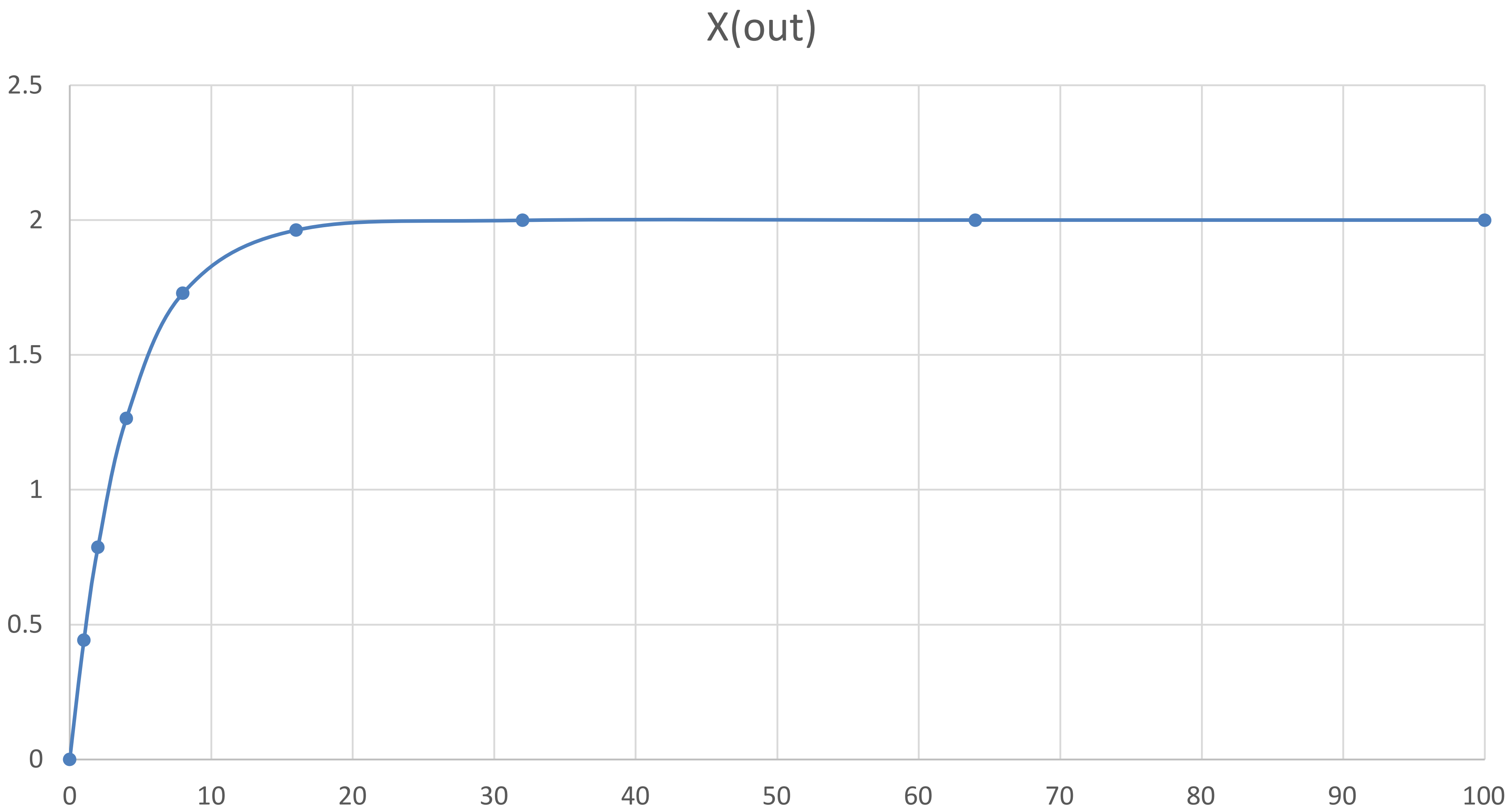
$t \geq 0 \qquad x_\text{i}(t) = \bar{X}_\text{i}$

From the table of L.T.
$$X_\text{i}(s) = \frac{\bar{X}_\text{i}}{s} \tag{5}$$

The output in s-domain
$$X_\text{o}(s) = \frac{\mu \bar{X}_\text{i}}{s(1 + Ts)} \tag{6}$$

In the time domain
$$x_\text{o}(t) = \mu \bar{X}_\text{i}\left(1 - \text{e}^{-\frac{t}{T}}\right) \tag{7}$$




X(out)
2.5
2
1.5
1
0.5
0
0
10
20
30
40
50
60
70
80
90
100

Hydraulic Position Control System under Standard Inputs

- We will return to this model in a later lecture to go through:
  - Using the Final Value Theorem to calculate the steady state response and steady state error
  - Examining the system's response to a ramp input

2^{nd} Order Control Systems

- 1^{st} order systems are
  - Reliable
  - Non-oscillatory
  - Slower than 2^{nd} order
- Hydraulics are slow and heavy
- How do they do this?
  - https://www.youtube.com/watch?v=U4y1grtRLDs

2^{nd} Order Control Systems

- Some examples – electro-mechanical position control
  - Power steering

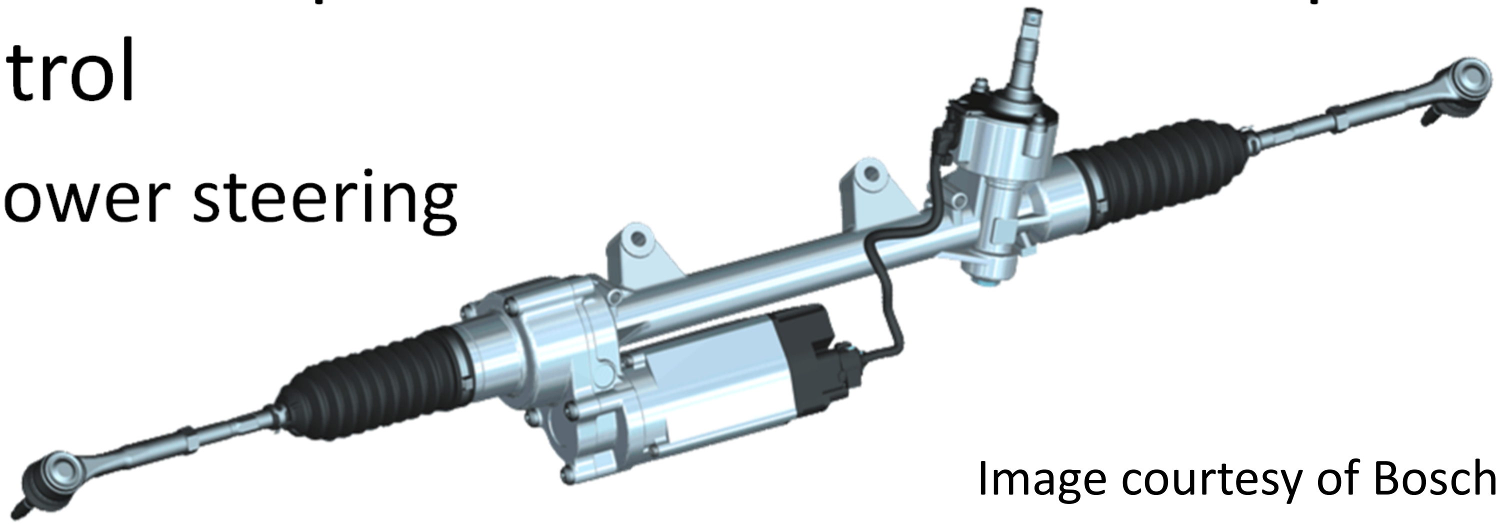


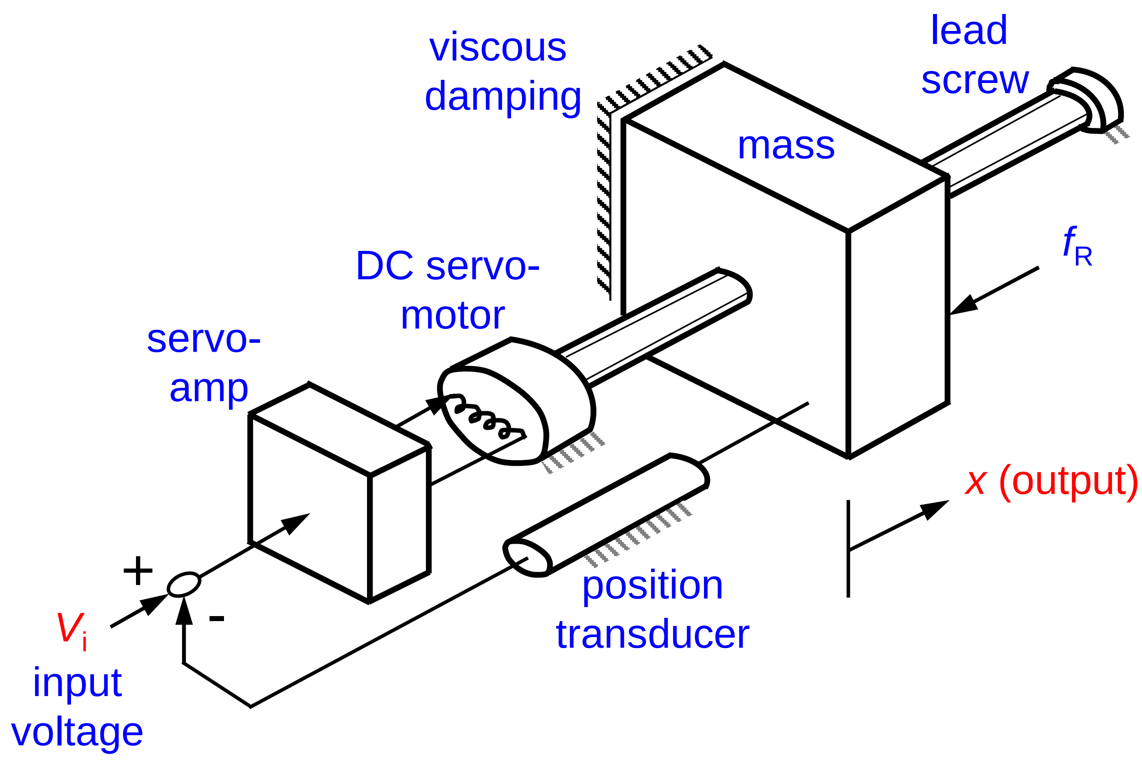
Image courtesy of Bosch AG

  - n-degree of freedom robots
  - …

2^{nd} Order Control Systems

- What is meant by "Second order systems"?
- You will be familiar with 2^{nd} order differential equations and their solutions:
  - $\frac{d^2y}{dt^2} + A\frac{dy}{dt} + B = 0$
  - 2 real roots (overdamped)
  - 1 root (critically damped)
  - 2 complex roots (underdamped)

Example: Electro-Mechanical Position Control System



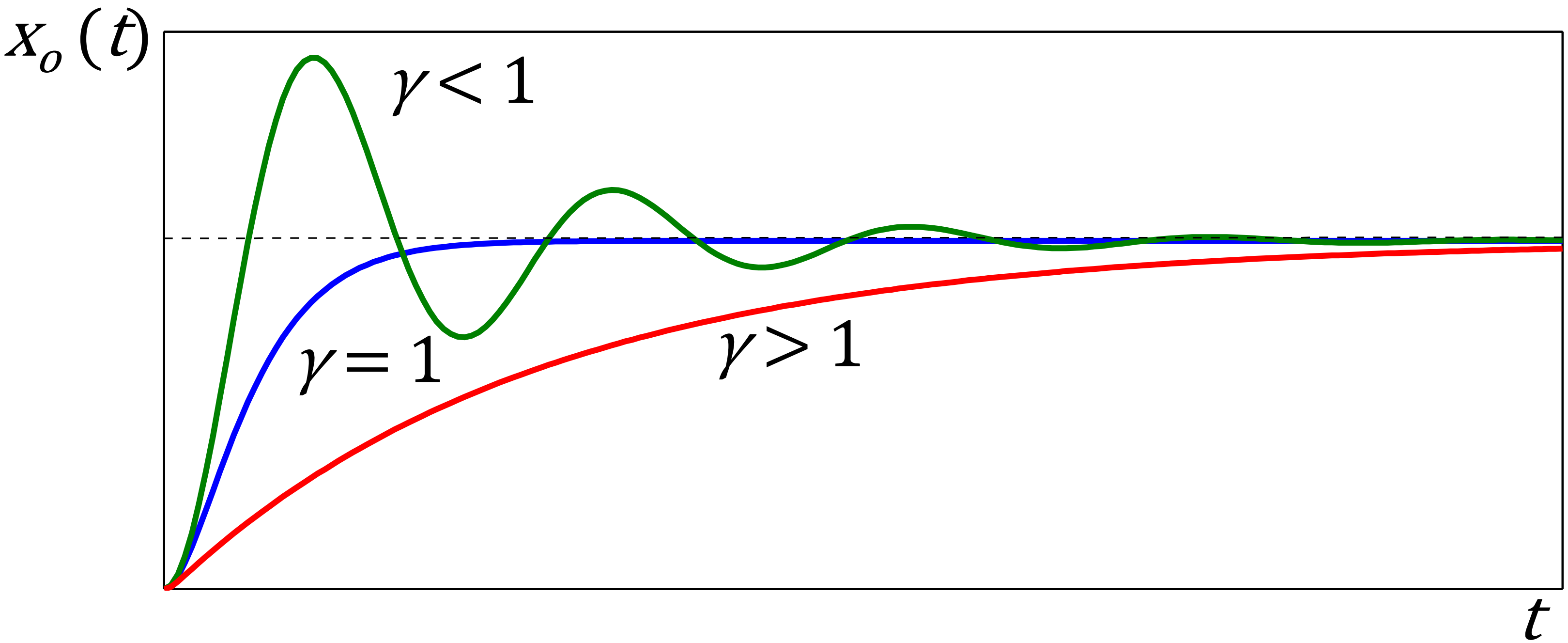
It will be shown that the **transfer function** may be written as

$$X(s) = \frac{\omega_n^2 X_i(s) - \frac{F_R(s)}{M}}{s^2 + 2\gamma\omega_n s + \omega_n^2}$$

2nd order system

2^{nd} Order System

The transient responses under a step input for all three cases can be summarised



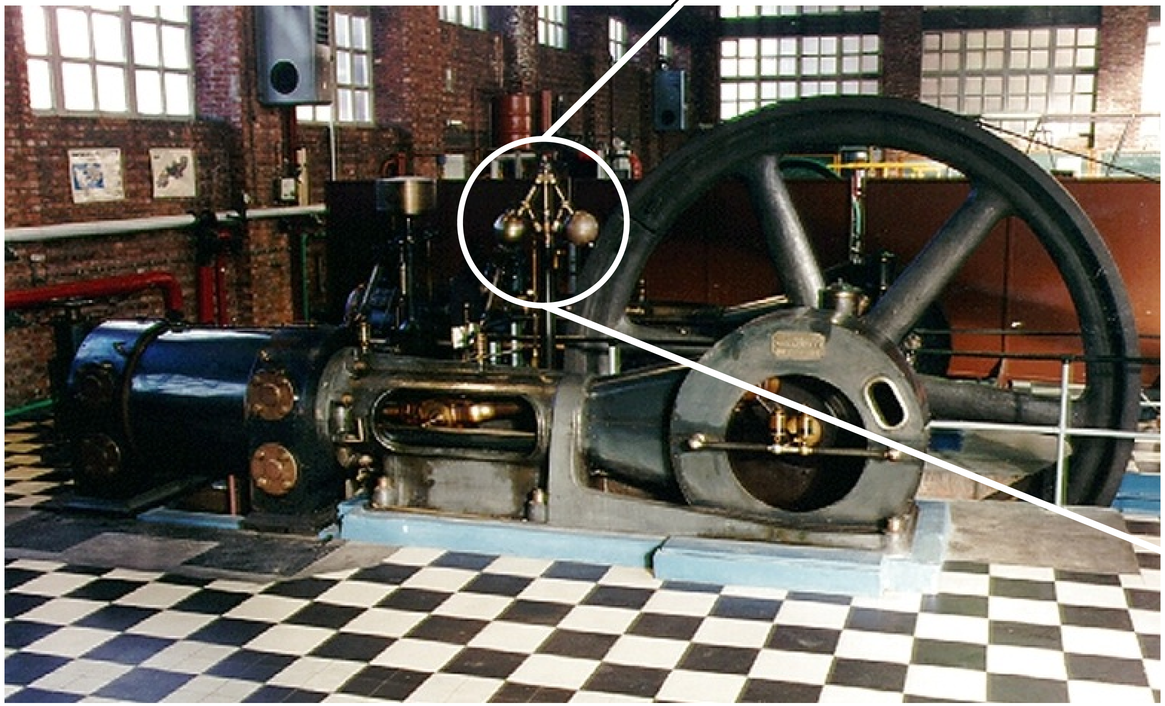
As can be seen, there is the danger of oscillation or overshooting

Video Interlude

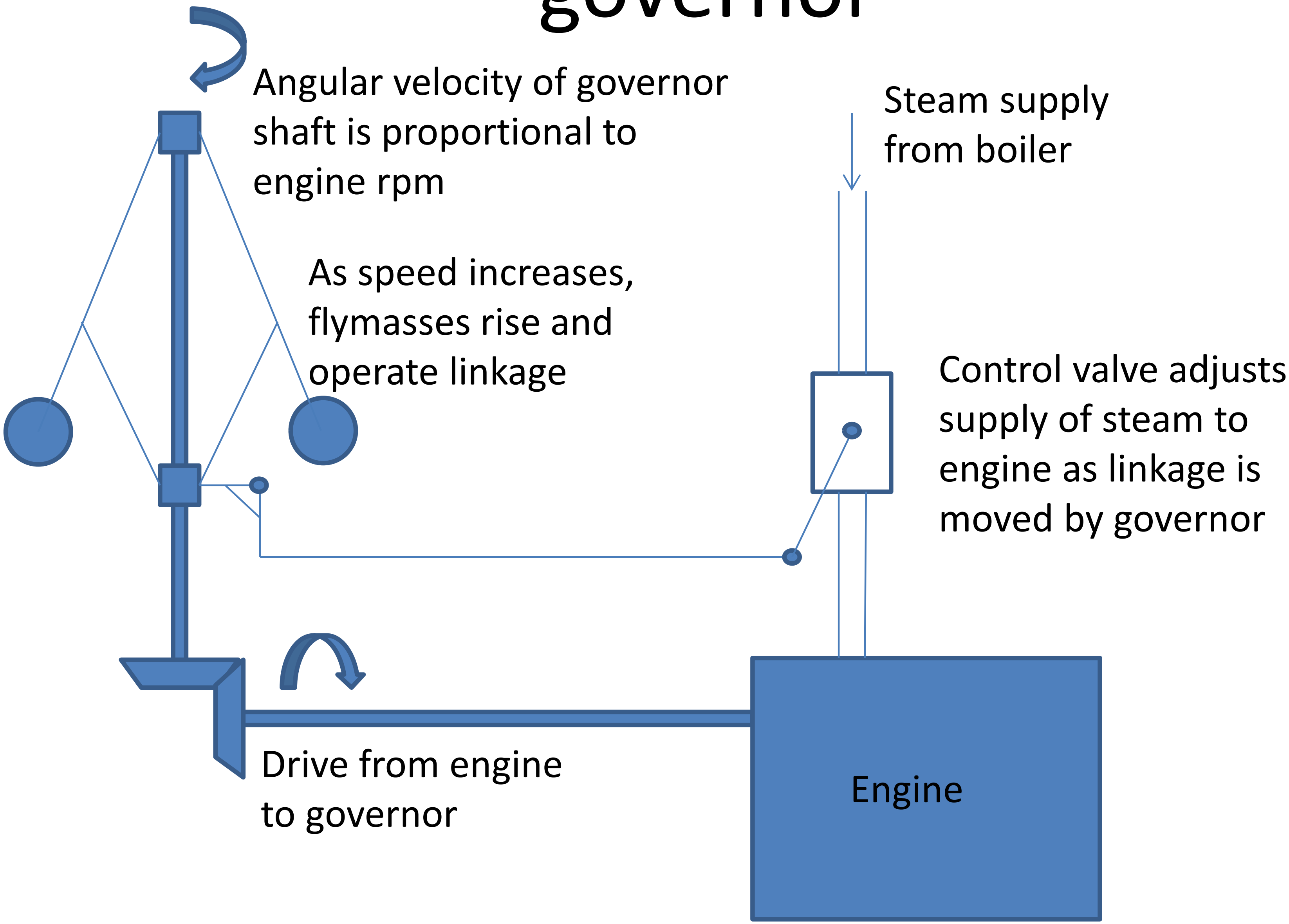
- Highly automated VW Golf production line
  - https://www.youtube.com/watch?v=3H1c_6_AxrQ
- Questions:
  - What would happen if the robot overshoots?
  - Can speed/precision be improved?
  - What controls the robots? Are they open or closed loop?

Brilliant idea no. 1 - revisited

- Centrifugal governor
  - Patented 1788 by James Watt



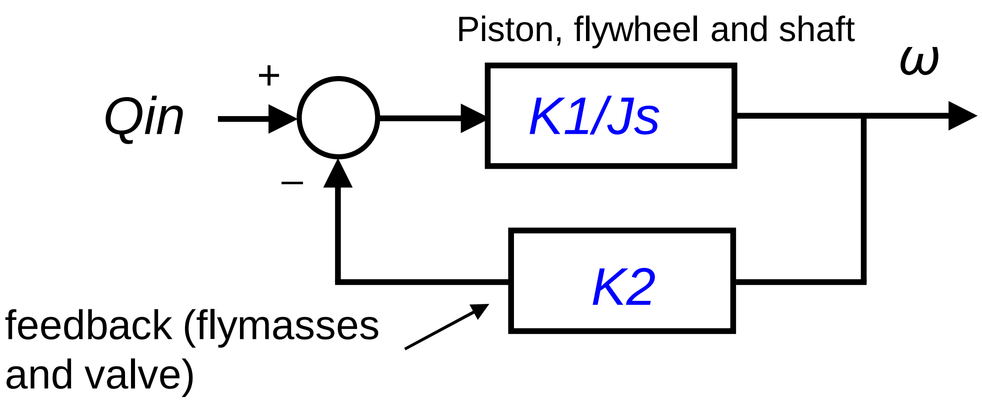
Method of operation for centrifugal governor



Video: https://www.youtube.com/watch?v=OG1AiaNTT6s

Brilliant idea no. 1 – centrifugal governor revisited

- Speed control is achieved by negative feedback loop
  - Too fast – steam input is reduced.
  - Too slow – steam input is increased.



What other information can we use?

- Position – “are we there yet?”
  - $x(t) \xrightarrow{L} X(s)$
- Velocity – “can we go any faster?”
  - $\frac{dx}{dt} \xrightarrow{L} sX(s)$
- Integral – “how far have we come?”
  - $\int x(t)dt \xrightarrow{L} \frac{X(s)}{s}$

This will be covered in the control lectures next week.

- Any questions?

- P.S. There is an overview/revision lecture towards the end of the course.