

**MM2MS2 Mechanics of Solids 2**  
**Exercise Sheet 6 – Fatigue & Fracture**

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1. A circular connecting rod is made from steel having the following properties:

Ultimate Tensile Stress,  $\sigma_u$ : 950MPa

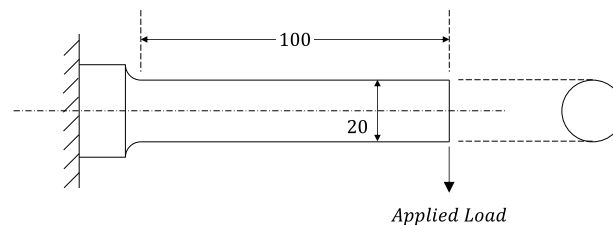
Yield Stress,  $\sigma_y$ : 800MPa

Fatigue Endurance Limit,  $\sigma_e$ : 500MPa

The rod is subjected to a fully reversed axial load of 180kN. Determine the minimum rod diameter, allowing a Factor of Safety of 2, if the rod end produces a Fatigue Strength Reduction Factor of 2.1, where the Stress Concentration Factor is 2.5.

**[Ans: 43.88mm]**

2. A Mild Steel cantilever beam of circular cross-section is subjected to a load at its free end (as shown in Fig Q2 which varies cyclically from  $P$  to  $-3P$ ). Determine the maximum value of  $P$  if the Fatigue Strength Reduction Factor for the fillet is 1.85 and a Safety Factor of 2 is assumed. (Hint: use the Goodman diagram and apply the Fatigue Strength Reduction Factor to the stress amplitude only).



All dimensions in mm

**Fig Q2**

**[Ans: 188.57N]**

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3. A circular steel shaft having a transverse oil hole is subjected to a torsional load which varies from  $-100Nm$  to  $400Nm$  (i.e. values in opposite senses). Determine the necessary shaft diameter assuming that the hole causes a Fatigue Strength Reduction Factor of 1.75 and making use of a Factor of Safety of 1.5. Assume the following properties for steel:

Ultimate Shear Stress,  $\tau_u$       400MPa

Fatigue Endurance Limit,  $\tau_e$       260MPa

**[Ans: 25.02mm]**

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- 4.
- (a) Determine the maximum size of a surface crack or flaw that may exist in the Aluminium panel shown in Fig Q4 without affecting unstable crack propagation. The applied tensile load,  $\sigma_{nom}$ , is equal to half of the material yield stress,  $\sigma_y$ . Assume linear elastic material and that Fracture Toughness is given as:

$$K_I = \sigma_{nom} \sqrt{\pi a}$$

Assume:

$$\sigma_y = 280 \text{ MPa}$$

$$K_{Ic} = 32 \text{ MPa}\sqrt{\text{m}}$$

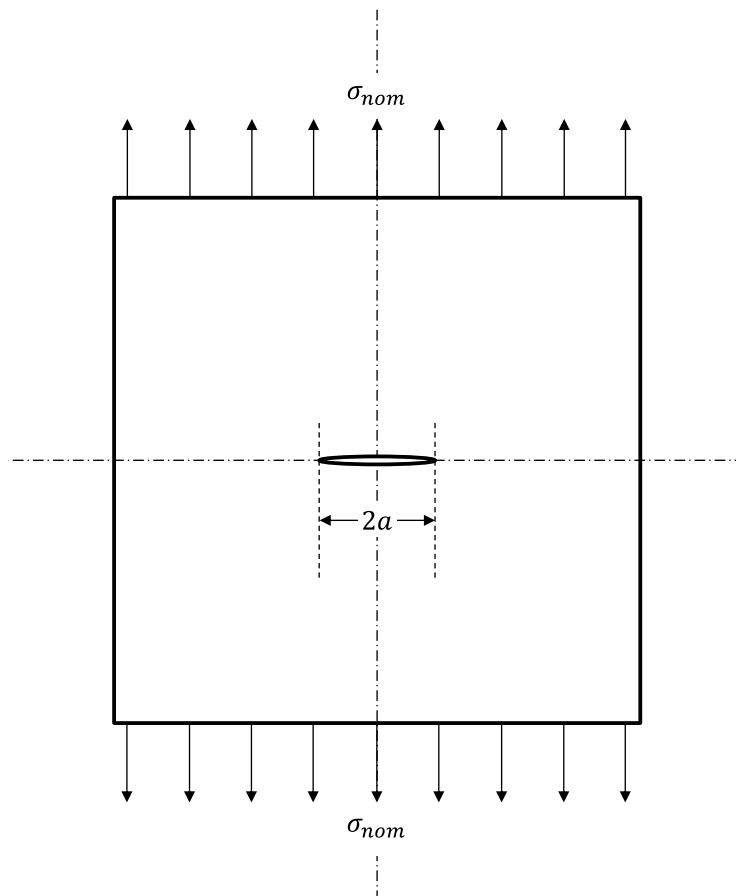


Fig Q4

- (b) It is found during inspection that there is a crack contained in the panel, size  $a = 1 \text{ mm}$ . If the external load is fluctuated such that the maximum nominal stress is half of the yield point and the minimum stress is compressive of 30 MPa, estimate the life cycle of the structure.

[Ans: a) 16.6mm, b) 4,623,661 cycles]

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5. A component is made of a steel for which  $K_{Ic} = 54 \frac{MPa}{\sqrt{m}}$ . Non-destructive testing by ultrasound methods shows that the component contains cracks up to  $2a = 0.2mm$  in length. Laboratory tests show that the crack growth rate under cyclic loading is given by:

$$\frac{da}{dN} = C \Delta K^m$$

where  $C = 4 \times 10^{-13} \frac{m/cycle}{MPa\sqrt{m}}$  and  $m = 4$ . The component is subjected to an alternating stress range,  $\Delta\sigma$ , of  $180MPa$  about a mean tensile stress,  $\sigma_m$ , of  $\frac{\Delta\sigma}{2}$  (i.e.  $R = 0$ ). Given that  $\Delta K = \Delta\sigma\sqrt{\pi a}$ , calculate the number of cycles to failure.

**[Ans: 2,404,524 cycles]**