Lecture Notes: Design with light alloys

1. Introduction

Light alloys are of great interest in automotive and aerospace industries. For example, the Jaguar XJ series and Audi A8 series have used aluminium alloys to in many automotive components. This saves at least 15% in weight and largely reduces fuel consumptions. In the aerospace industries, aluminium alloys are used in aircraft frames and titanium alloys employed in engine structural components, such as fan blades. Usually, metals or alloys with densities smaller than 4.5 gcm^{-3} and typically less than 3 gcm^{-3} are called light alloys. Ideally we also want light alloys to exhibit high strength. Thus, high strength and low density are often associated with light alloys. The ratio of strength/density is defined as specific strength. Aluminium and Magnesium alloys are characterised with high specific strength whilst Fe and Ni alloys are not due to their higher density.

Light alloys are commonly available in two conditions, namely cast products and wrought products.

Cast alloys. By controlling the cooling rates and using the grain refiners then the strength and toughness of cast alloys can be improved. Eutectic compositions such as Al-12wt% Si are good candidates for casting casted because of their lower melting point. Upon cooling from molten liquid to solid, alternating layers of two-phase structure is formed. Such structure is termed an eutectic structure and this specific composition is termed an eutectic composition. Hypo and hyper eutectic compositions are characterised with primary phase as well as eutectic phase structures as you have discovered in the laboratory. Another example Al-Zn alloy that is commoly used in die casting (As you know from first year die casting is a manufacturing process for producing metal parts by forcing molten metal under high pressure into a die cavity. The die or mould cavities are often made from hardened tool steel that have been previously machined to the net shape or near net shape of the die cast parts).

Wrought alloys mean that they have undergone deformation processes to achieve the required shapes and properties. The deformation processes can be rolling, forging, extrusion, etc. The final properties depend on the heat treatment and the amount of working (deformation). They can be strengthened through age (precipitate) hardening by heat treatment and the properties depend on the distribution, size and coherency of precipitates. They can also be strengthened by work hardening which is governed by the final grain size and dislocation density.

2. Case study – engine piston

Figure 1 shows an illustration of an engine piston. The piston with the piston rings form a combustion chamber with the cylinder head, sealing the combustion gases. The combustion pressure is transferred to the crankshaft via the piston pin and connecting rod. In two stroke engines the piston act as a valve, exchanging gas. The piston is subjected to an aggressive environment. For example a 56 mm piston needs to withstand a 20 kN load reaching speeds of 15 ms^{-1}. Assuming an average rpm of 15000 then after only 11 h it has been subjected to 10^7 fatigue cycles! The head of the piston can reach 350 °C while the skirt is 150 °C. To generate high power output the piston needs to be as light as possible yet still function. Here we will consider the materials selection for a **petrol piston**. (Note Diesel engines have lower rotation but much higher pressures, 18 MPa).

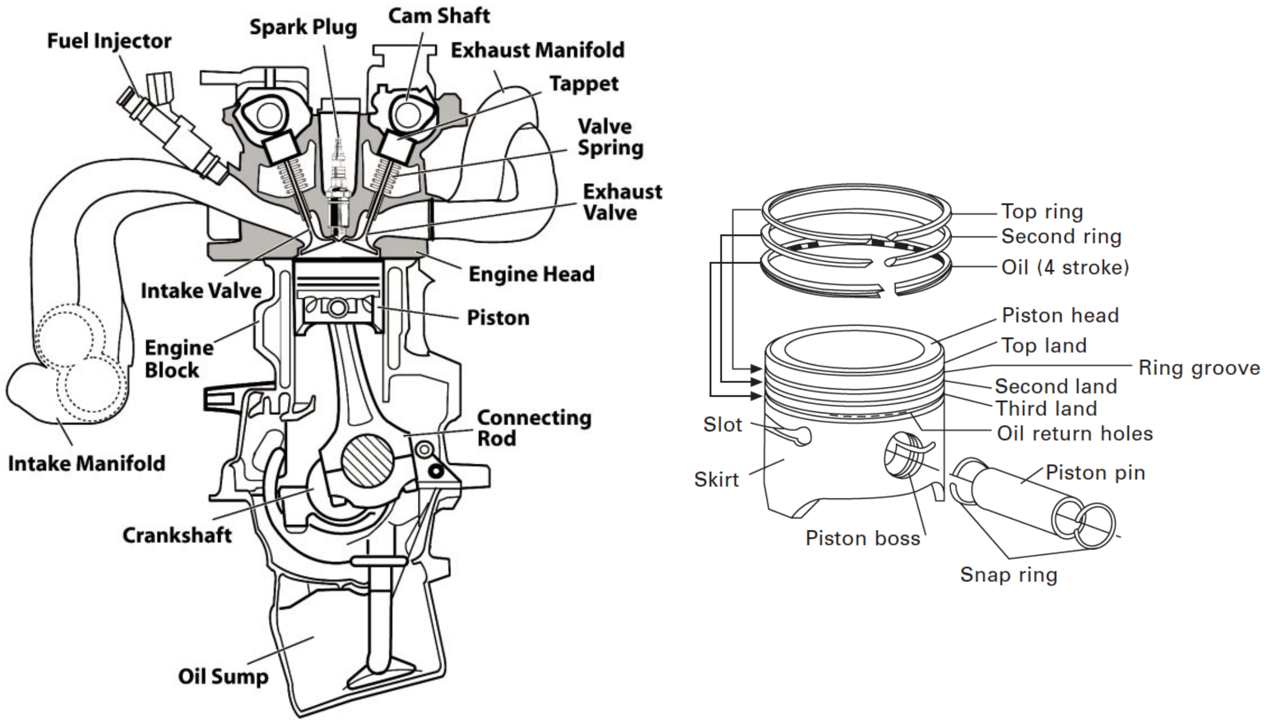


Figure 1 Illustration of the engine piston.

We do not want the piston to have a high thermal expansion otherwise it will contact with the casing as it heats up. Thus a low thermal expansion material is preferred. Wear may happen at the ring grooves during each engine cycle and it may also occur around the piston pin hole, hence good wear resistance is also preferred. Considering the piston is working at high temperature and loading environment, then good creep resistance and fatigue strength are also desirable. To reduce the engine weight and maximise the fuel efficiency, then we also require a high specific strength. Other properties that are related to the ease of manufacturing are of further interest, such as good machinability, low cost and capable of near net shape processing. Combining the above constraints, we want to select a right material for the piston.

Having considered all the requirements, we are now in a position to formulate the selection process for a piston. Therefore we need to define its engineering function, the design constraints, the design objectives, and the free variable which we can change freely to meet design constraints and objectives.

Function: Forms a combustion chamber by sealing hot gases

Objectives: Transmitting combustion power efficiently, allowing gas exchange (two strokes), must transmit heat from piston head to cylinder wall,

Constraints: Must operate at elevated temperatures defined by engine type, low thermal expansion, must not seize, high fatigue strength at temperature, must be mass produced and low cost

Free variable: Choice of material

Usually there are multiple constraints in any problem however we can initially explore the key ones initially. In this case we will investigate high strength to weight ratio, high thermal conductivity and low coefficient of expansion initially. Then we can consider low friction, strength at operating temperature and low cost to select the optimal material. We can look at two possible simple performance indexes, M_1 and M_2 to help us rank and select different materials.

(1) High strength to weight Performance index: $M_1 = \sigma_f/\rho$

(2) High thermal conductivity and low thermal expansion Performance Index: $M_2 = \lambda/\alpha$

To achieve the design objectives, the material with the highest value of M is desirable. The material selection process can be done by going through the table of material data to generate the performance index, or we can plot the performance indices graphically which is more useful and practical to compare different materials. Due to the wide range of material properties, the material selection graph is plotted in log-log scale in Figure 2A. It helps us to identify the materials selection using the slope.

Given the performance index: $M_1 = \sigma_f/\rho$

Taking logs, then $\log M_1 = \log \sigma_f - \log \rho$

Thus, $\log \sigma_f = 1 \times \log \rho + \log M_1$, which is the form $y = mx + c$, $m = 1$ in this case.

Therefore if we plot any line on a plot of log σ versus log ρ and lines with **slope 1** they will have the same performance index. The desired higher performance index will have higher y intercepts. By moving line of slope 1 towards top left of graph identification of material, the material with higher performance index can be found.

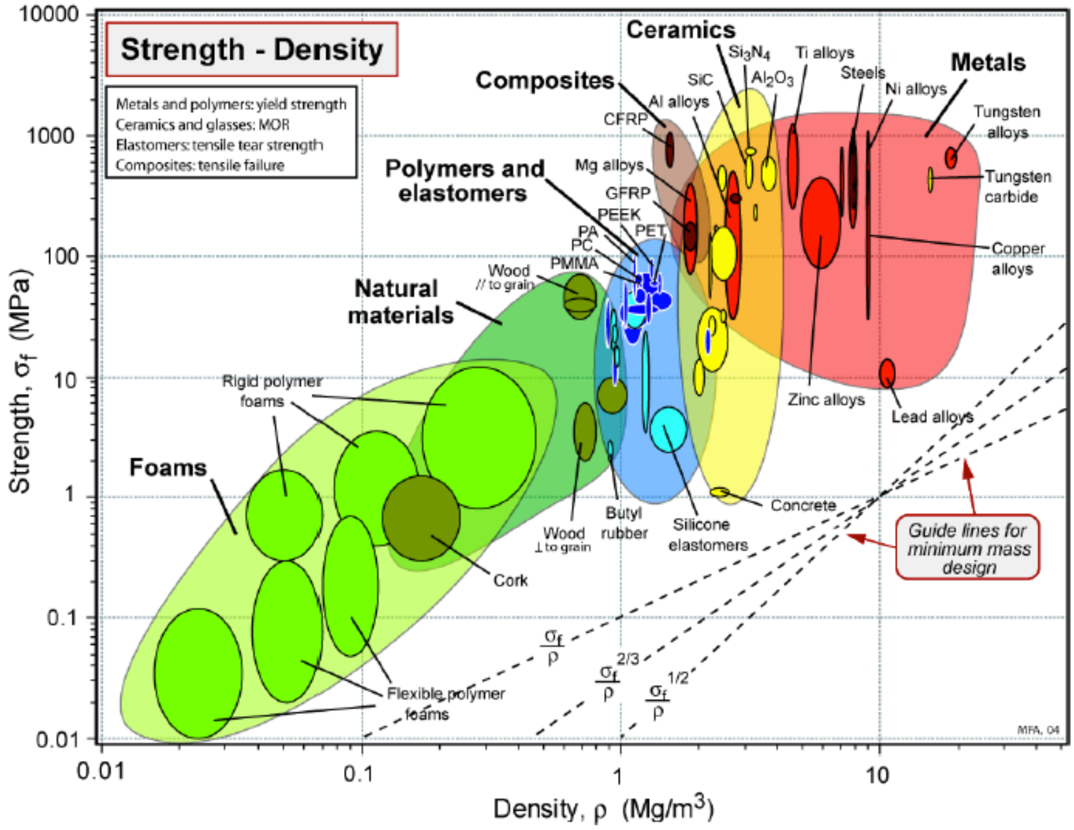


Figure 2A Material selection chart of strength over density.

A similar procedure can be used for the merit index: $M_2 = \lambda/\alpha$

Again taking logs, then $\log M_2 = \log \lambda - \log \alpha$

Thus, $\log \lambda = 1 \times \log \alpha + \log M_2$, which is the form $y = mx + c$

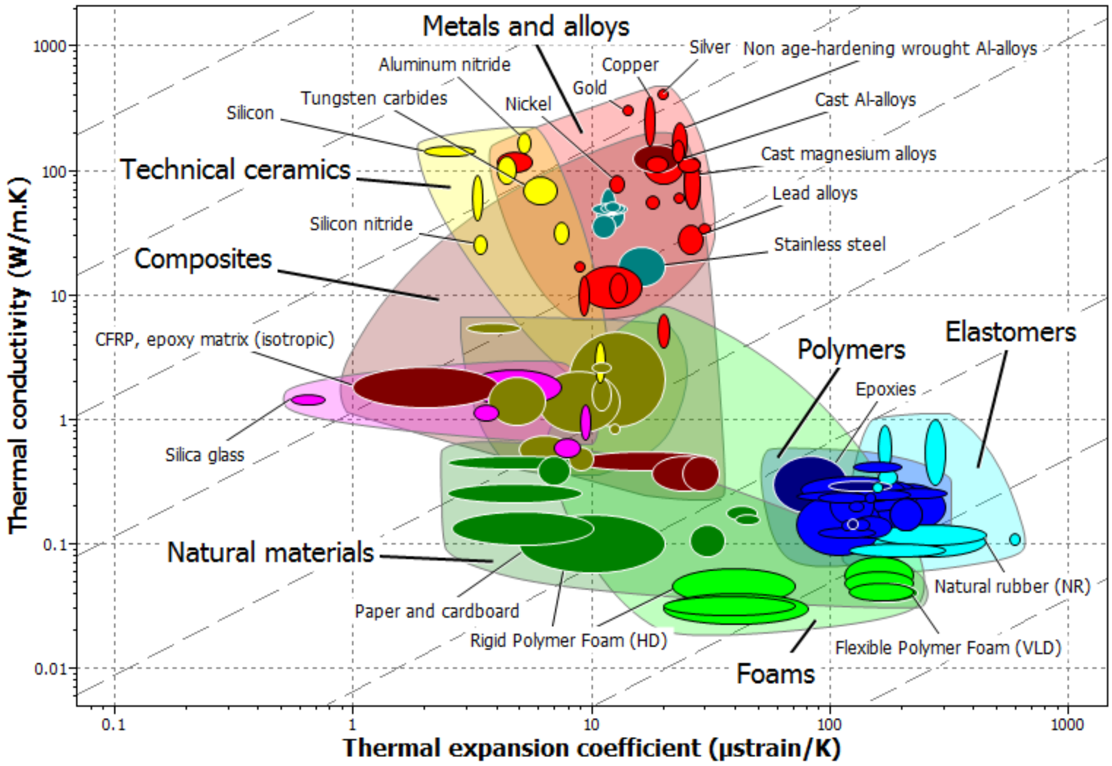


Figure 2B Material selection chart of thermal conductivity versus thermal expansion

Therefore, when we plot log λ versus log α and lines with **slope 1** we will again have the same performance index along that line, see figure 2B. The desired higher merit index will have a higher y intercept thus by moving line of slope 1 towards top left of graph identification of material which maximises that particular performance index and the optimal material that meet this performance index can be found.

Based on the design requirement, we can explore further on other constraints such as maximum service temperature, coefficient friction, cost, etc. This allows us to consider various materials and identify a suitable material to explore for a petrol piston. Based on the selection plots and other factors then the following conclusions can be made towards the selection:

- Many classes of materials have the required strength to weight ratio
- However, the constraint of service T limits materials to ceramics and metals

- The cost of ceramics not just raw cost but processing (high pressure, high temperatures) means metals are favourable
- Examining the high thermal conductivity and low thermal expansion Merit Index ($M = \lambda/\alpha$) for the metal then alloys of Al, Mg, Cu and Pb are of interest
- Discount lead, Pb, due to its poor inservice temperature, low Tm and low strength to weight ratio
- Cu alloys significantly more expensive and its higher Tm makes it more expensive to cast
- Mg and Al alloys only just make the inservice temperature conditions
- Al is cheaper than Mg and cheaper to process. Both alloys are on limit of inservice temperature
- Al is easier to machine in terms of safety that Mg alloys
- Al has lower thermal expansion coefficient

The above indicates the Al alloys are a good candidate material for the petrol piston. High Strength to weight ratio is important for this application so selecting Al with light alloying element important as is improving the strength by alloying. However its low T_m means that Al alloys are susceptible to creep at the service temperatures, thus alloy selection is key to minimise this. We also have identified low thermal expansion is important. Howver there are a wide range of Al alloys available and several series of Al alloys by alloying with other elements have been developed. So we need to now look at aluminium series as shown below combined with the processing route to make a final selection.

Aluminium Alloy Series

1xxx: Minimally alloyed and generally used in the annealed condition, low yield stress at 10 MPa and UTS 70 - 180 MPa often considered to be 'pure' with purity no less than 99%. While these alloys can be hardened they are not usually used where hardness is a requirement and are not suited to machining (unless hardened). They anodise well. Applications: electrical conductors, chemical equipment, food, foil and architecturally.

2xxx: This series is alloyed primarily with Copper to give the alloys in this series a high strength (UTS 180 – 430 MPa) and very good machining characteristics. Therefore these Al-Cu precipitation hardened alloys, have high strength to weight ratio thus aerospace potential but not weldable, and aircraft are riveted. $CuAl_2$ precipitates. Li in aluminium alloys very effective: each 1 wt.% of lithium reduces the density of the alloy by 3% and increases stiffness by ~5%. These alloys tend to have poor corrosion resistance and so they are typically coated in one form or another and anodisation is least effective on this series.

3xxx: Al–Mn or Al–Mn–Mg alloys: This series is alloyed primarily with Manganese. This gives excellent corrosion resistance that makes them suitable for architectural/outdoor applications as well in areas where foods or corrosive chemicals are used. This Aluminium alloy series has excellent welding characteristics. 3000 series alloys can be found in a wide range of applications from drink cans to roofing to heat exchangers. The moderate strength, UTS 110 – 280 MPa, comes from dispersoids which form in the early stages of solidification and work hardening. The Mn limited to 1.25 wt% to avoid primary Al_6Mn particles. Mg (0.5 wt%) gives solid solution strengthening and the Al–Mn–Mg alloy is used in the H or O conditions. For example beverage cans Al–0.7Mn–0.5Mg wt%.

4xxx: This series is alloyed primarily with Silicon, which gives good flow characteristics when molten. Two variants that are most commonly used in this series are 4032 and 4043. 4042 is primarily used in forging processes where the high silicon content alloys the alloy to flow easily into even very complex dies. The alloy can be heat treated and offers good overall strength (UTS 170 – 380 MPa). Al–Si binary eutectic with eutectic composition ~11 wt.% Si or hyper euctectic composition of ~18 wt.% Si, used for castings, Si Needles, refined with Na, see case study, or for brazing other aluminium alloys (eutectic composition).

5xxx: This series is alloyed primarily with Magnesium, which results in a variety of useful properties. They are strain harden-able and have a moderately strength, yield strength **σy ≈** 40 – 160 MPa with UTS 120 – 350 MPa with work hardening. The higher the

Magnesium content the higher base strength. Corrosion resistance is very high, even in salt water applications making it useful for a wide range of applications. These alloys also retain a high degree of strength at very low temperatures. They are a very suitable group of alloys for welding with a wide variety of techniques - shells of boats/vehicles and are readily welded.. The magnesium concentration is less than 3–4 wt% in order to avoid Mg_5Al_8. Work hardened aluminium alloys tend to soften with age because the microstructure is not stable even at room temperature. Therefore, overwork harden then anneal.

6xxx: This series is alloyed primarily with Magnesium and Silicon and utilises Mg_2 Si precipitation hardening. . They have good strength (UTS 120 - 400 MPa) but are heat treatable. Their corrosion resistance is reasonable but not as good as the 5000 series. One characteristic that makes this series very useful is its ability to be easily extruded into a wide variety of shapes and lengths. Two common varients of the 6000 series of Aluminium alloy are 6061 and 6063. The 6061 offers greater strength and machinability that 6063 but the latter has better extrudability. The 6000 series found wide uses from window frames to bridges to sports equipment and structures because of their extrudability, corrosion resistance and tensile strength. In addition these alloys are readily welded and all tend to anodise well, hence decorative applications. 6063 anodises better than 6061 particularly where bright applications are required.

7xxx: This series is alloyed primarily with Zinc (Zinc additions ranging from 0.8 to 12.0%). High strength alloy, UTS 220 – 600 MPa, based on $MgZn_2$ precipitation hardening. Suffers from stress corrosion if Zn > 6 wt.%. These alloys are often used in high performance applications such as aircraft, aerospace, and competitive sporting equipment. Like the 2xxx series of alloys, this series incorporates alloys, which are considered unsuitable candidates for arc welding, and others, which are. The commonly welded alloys in this series, such as 7005, are predominantly welded with the 5xxx series filler alloys. Not as corrosion resistant as the 5000 and 6000 series of alloys, they are typically coated, although not the best alloys for anodising compared to the other alloys. In aircraft applications they are normally found in the “Alclad” form. Alclad is a tradename of a composite-wrought product comprised of an aluminum alloy core having on one or both surfaces a metallurgically bonded aluminum or aluminum alloy coating that is anodic to the core and thus electrochemically protects the core against corrosion. This also is applied to the 2000 series for aircraft to achieved better corrosion resistance such as cladding 2024-T3.

8xxx: Various other alloys including Li –based with δ Li_3Al precipitates. Low density, increased stiffness -aircraft applications, low toughness.

Dispersion strengthened alloys: Molten aluminium droplets oxidised on their surfaces. Solid then compacted, the surface oxide breaks up into highly stable dispersoids in an Al matrix. Up to 20 wt% of alumina or SiC or other ceramic can also be added– "metal matrix composites"

(To distinguish each alloy with different manufacturing processes, **Suffixes** are used after the alloy designation:
-Tx for precipitation hardened alloys where x is a number corresponding to a specific treatment, for example -T4 solution treated and then naturally aged -T6 solution treated and artificially aged.
-F as fabricated; **-O** annealed; **-Hx** strain hardened where the value of x denotes extent of hardening: H4 – half hard, H8 – fully hard).

By comparing different Al alloy series, the 4xxx series, i.e. Al-Si alloys exhibit promising properties including cast ability. Some properties of Al and Si are listed below.

Material	**ρ/gcm^{-3}**	**α/K^{-1}**	**Vickers Hardness Hv**
Aluminium	2.67	23.5×10^{-6}	870-1350
Silicon	2.33	9.6×10^{-6}	170

- Lower density Si means alloy has a lower density, helps with strength to weight.
- Si has a large solubility range in Al so potential for solid solution strengthening and precipitation hardening, improving strength and creep resistance.

- Si has lower thermal expansion coefficient means the alloy has a lower thermal expansion coefficient than Al.
- Improved hardness of alloy reduces piston seizure

Having selected a material or materials as with the case study, we need to examine in detail the materials properties not only meet the design but also how the material may be processed and how the processing influences the material properties and the design.

3. Al alloys: structure – properties - processing

A common process to manufacturing the engine piston is by casting, as shown in Figure 4.

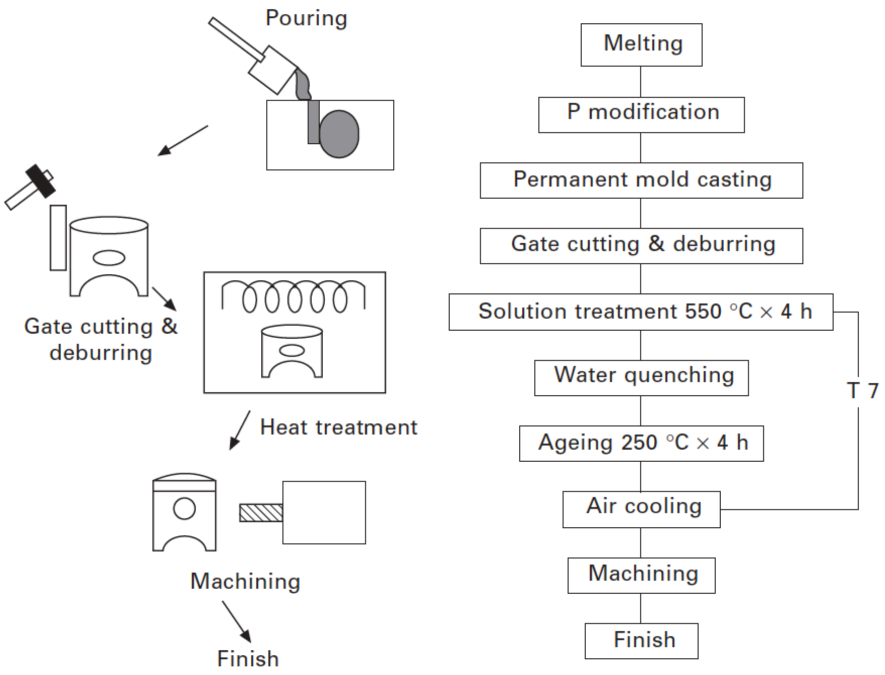


Figure 3 The casting processing of an Al alloy piston

When considering the casting process, it is useful to consult the associated alloy phase diagram. The phase diagram of the Al-Si alloy system is shown in Figure 4.

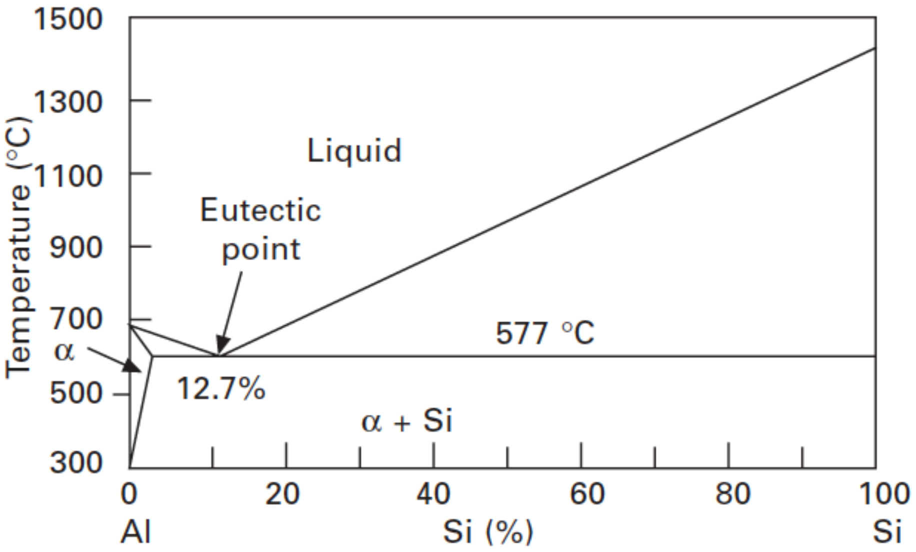


Figure 4 Al-Si phase diagram

We can identify the eutectic composition (12.7 wt% Si) and the eutectic point which has the lowest melting point. We can also identify the hypoeutectic region (<12.7 wt% Si) and hypereutectic region (>12.7 wt% Si), which exhibits higher melting points compared to the eutectic point. During solidification of the casting process, it generally follows a few stages from molten liquid to crystallised solid, which is shown in Figure 5.

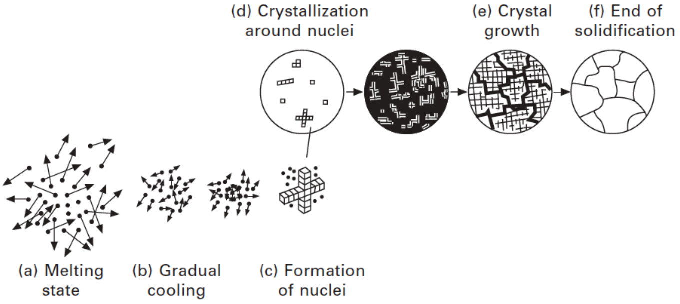


Figure 5 The typical phase formation during the solidification process

Two typical Al-Si alloys with 12 and 19 wt% Si are used for pistons.

With 12wt% Si – close to eutectic composition so get eutectic Si (grey areas) formed in between the eutectic α (Al rich phase – white areas), as shown in Figure 6.

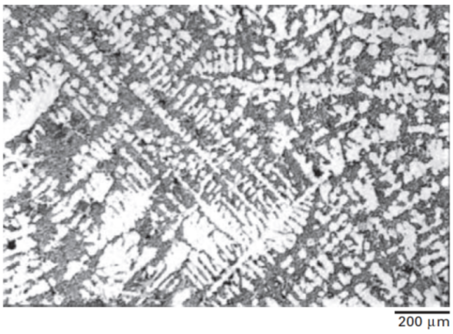
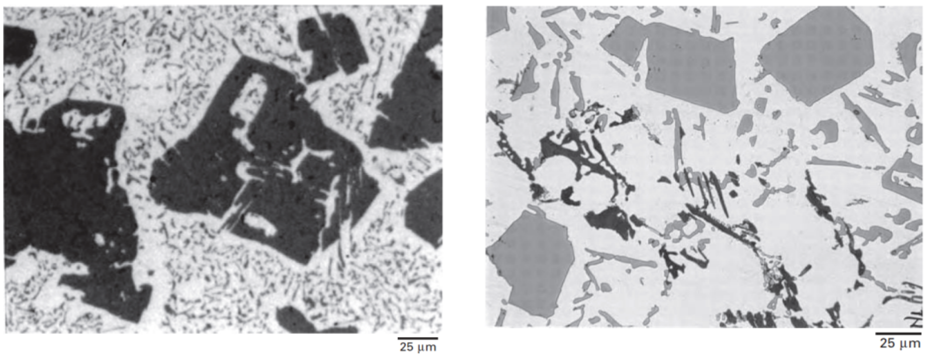


Figure 6 Eutectic structure of Al-12wt% Si alloy

With 19wt% Si – the alloy has even lower density, improving strength to weight ratio. Also the **primary silicon** (the large grain in Figure 7) that forms helps strength the alloy and improves its hardness. Ideal for the piston grooves for the rings. But it becomes susceptible to brittle failure and has to be casted at a higher temperature than a eutectic composition, see phase diagram in figure 4.

The strength and the fracture toughness are improved by reducing the grain of primary Si formed thus need to encourage nucleation of the primary silicon. To do this small amounts of Phosphorus, P, are added into the melt.



Al-19wt%Si alloy without P Al-19wt%Si with P

Figure 7 The microstructure of Al-19wt Si without and with P

Only a small amount of P is added to the melt, which is effective in reducing the primary Si size (figure 7) and improving the tensile strength, as shown in Figure 8. Also Strontium or Sodium is added to produce a rod like eutectic structure rather than a plate like structure improving fracture toughness more.

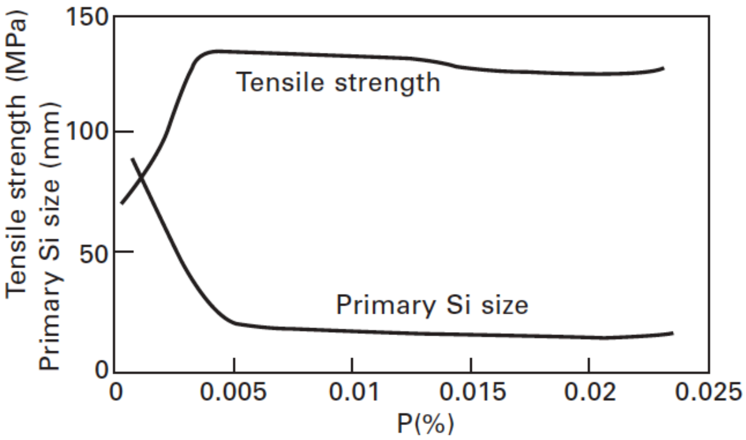


Figure 8 The effects of P on the primary Si size and tensile strength

Small amounts of intermetallic elements such as Fe, Cr or Ni can form intermetallic compounds which strengthen the alloy at higher temperatures. However, too much intermetallic compounds causes a coarse brittle structure, also heavier elements add to weight of alloy. For this case study example 1 wt% Ni has been added which then forms Al_3Ni in the structure, resulting in a strengthening effect at higher T.

3. Age hardening mechanisms

Age Hardening - Al alloy systems are often suitable to be age hardened, in which:

- A monophase forms solid solution at high temperatures,
- Rejection of a finely dispersed precipitate during aging, i.e. the phase diagram must show a declining solvus line.

The age (precipitate) hardening in Al alloys is illustrated in Figure 9 through solution treatment in the single phase region, then quenched to the two-phase region, followed by aging to allow the fine dispersion of precipitates to occur.

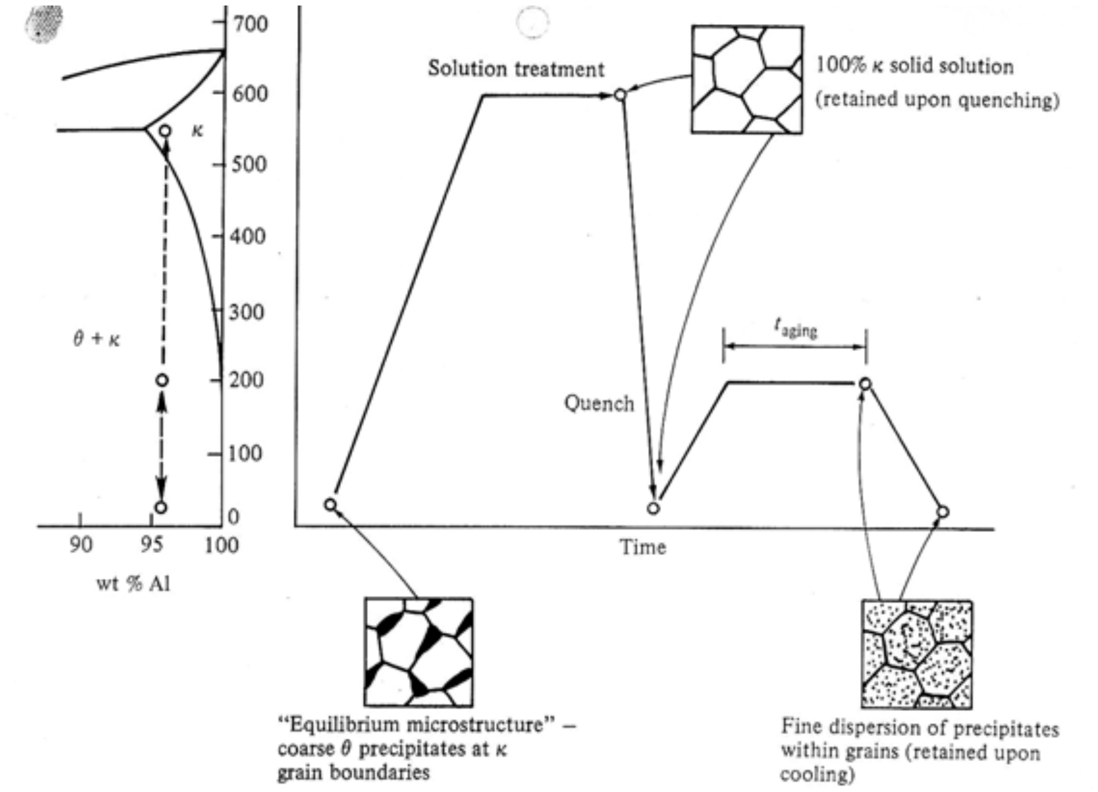


Figure 9 The age hardening process of Al-Cu alloy system.

Since the increase of strength, Δσ, is related to the spacing of precipitates, S, in that Δσ proportional to 1/S. Thus the optimum size and distribution of precipitate will results in the maximum strengthening effect, while overaged Al alloys will have coarsened precipitates which cause a reduction in strength. This is shown in Figure 10.

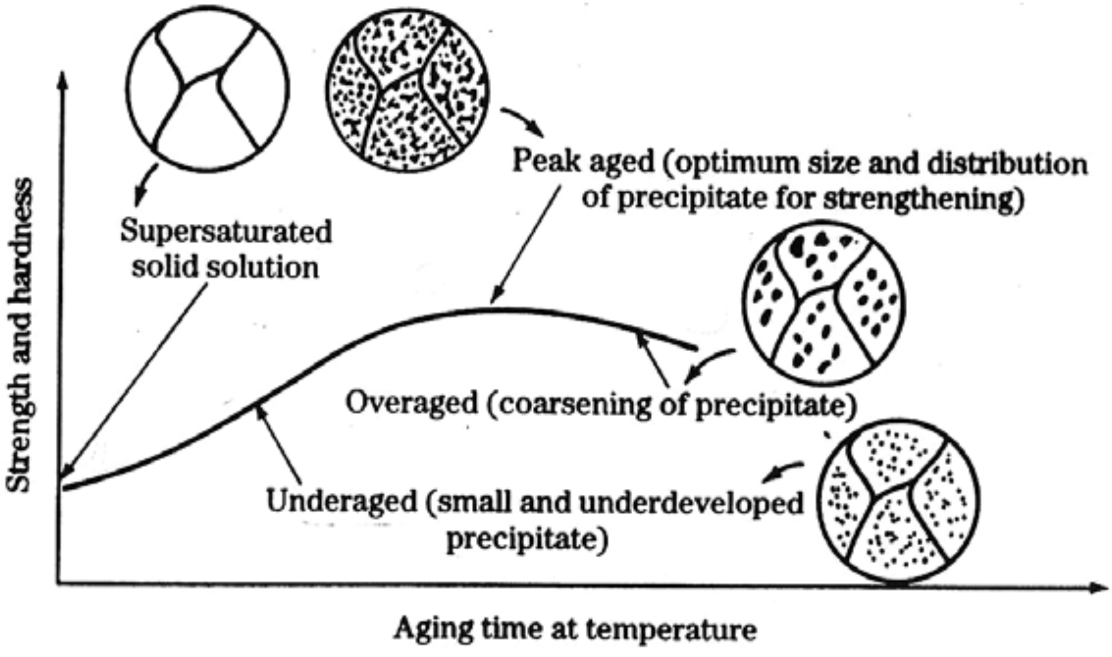


Figure 10 The strength and hardness against ageing time at ageing temperature

To explain how the precipitates are formed, we firstly need to understand the evolution of the interface between the matrix and the secondary phase. Different crystallographic relationships between **matrix** and **second** phase are: (a) Complete coherency, (b) Coherency with strained, but continuous, lattice planes across the boundary, (c) Semicoherent, partial continuity of lattice planes across the interface, (d) Incoherent equilibrium precipitate, no continuity of lattice planes across the interface. This is illustrated in Figure 11.

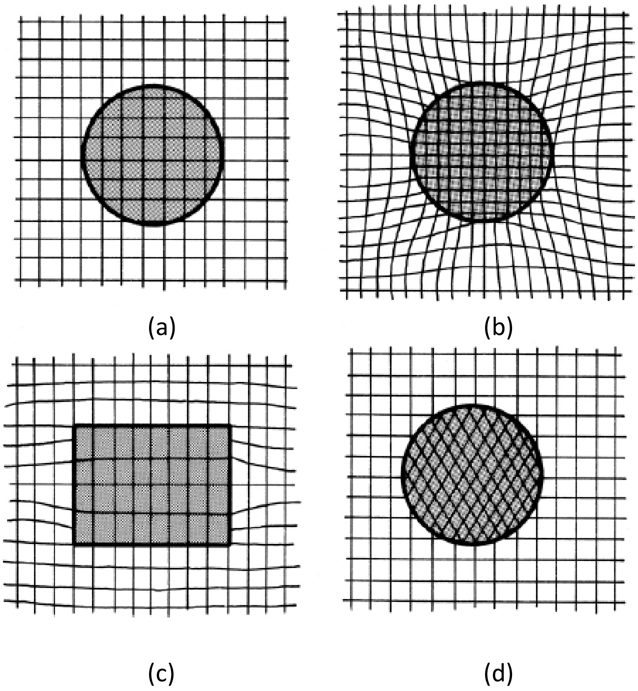


Figure 11 The coherent, coherent with strained, semicoherent and inchoherent interfaces.

The hardening mechanism is complex during the aging process. For example in the Al-Cu system the following happens over time:

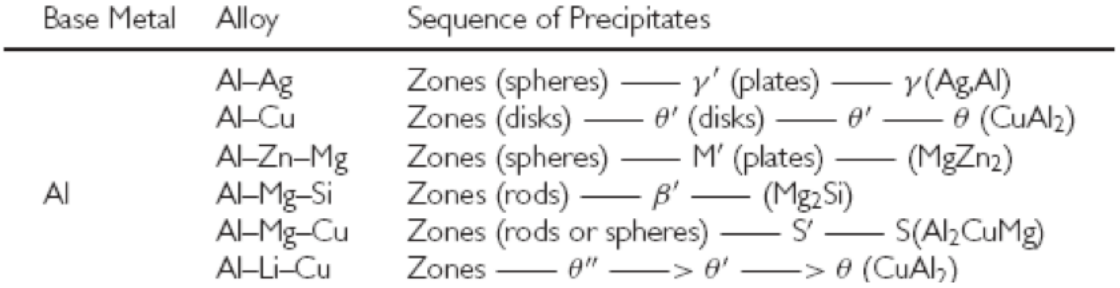
- Initially solid solution hardening occurs. The Cu trapped in solid solution in the supersaturated α phase.
- Then all the solute disappears from solution and disk shaped Guinier –Preston (GP) zones form. These GP disk faces (~ 10 nm across) are coherent with the matrix but the edges have a large coherency strain.

- Some of the GP zones extend by diffusion and form θ″ precipitates. Disc faces (now ~ 100 nm across) are coherent but again the edges have high coherency strain due to mismatch between the θ″ phase and the matrix. (**This coherency strain due to GP zones and θ″ cause the largest precipitation hardening effect by restricting dislocation movement**)
- Continuing with time the θ″ precipitates dissolves with the Cu diffusing further and forming a θ′ phase. The discs (now ~ 1 μm across) have edges that are **incoherent**.
- Finally at the edges of the θ′ – matrix and grain boundaries equilibrium θ (Cu_2Al) nucleates and grow as the θ′ precipitates dissolve . These more rounded precipitates are completely incoherent with matrix. (Their effectiveness in in stopping dislocations limited because the dislocations can either cut through them or bow between them)

Further notes in terms of the GP zone formation: in the initial stages of the aging treatment, zones that are coherent with the matrix appear. These zones are nothing but clusters of solute atoms on certain crystallographic planes of the matrix. In the case of Al-Cu, the zones are a clustering of copper atoms on the [100] planes of aluminium. The zones are referred to Guinier-Preston zones, or GP zones, named after the two researchers who discovered them. They are called zones rather than precipitates in order to emphasize the fact that the zones represent a small clustering of solute atoms that has not yet taken the form of precipitate particles. The GP zones are very small and have a very small lattice mismatch with the aluminium matrix. Thus they are coherent with the matrix i.e. the lattice planes cross the interface in a continuous manner. Such coherent interfaces have very low energies but there are small elastic coherency strains in the matrix. As these coherency strains grow, the elastic energy associated with them is reduced by the formation of semicoherent zones where dislocations form at the interface to take up the misfit strain. Further growth of the of the semicoherent zones, or precipitates, results in a complete loss of coherency: an incoherent interface forms between the precipitate and the matrix.

To summarise, the nature of precipitate/matrix interface produced during the aging treatment can be coherent, semicoherent, or incoherent. Coherency signifies that there exists a one-to-one correspondence between the precipitate lattice and that of the matrix, as shown in Figure 11 (a) and (b). A semicoherent precipitate signifies only partial correspondance between the two sets of lattice planes (Figure 11c) and an incoherent precipitate indicates no correspondence between the lattice planes and, as a result, a completely different crystal structure. The intermediate metastable phases are very important for effective age hardening.

This mechanism is also true in other combinations although the preceiptates that from are different in crystal structure and shape. Common precipitation hardened alloys include Al-Zn-Mg, Al-Mg-Cu, Al-Mg-Si and Al-Li-Cu. So the additions of Mg and Cu in Al piston alloys allow effective age hardening with Zones, S’ and θ'' precipitates forming as summarised in the following table



Base Metal	Alloy	Sequence of Precipitates
	Al–Ag	Zones (spheres) —— γ' (plates) —— γ(Ag,Al)
	Al–Cu	Zones (disks) —— θ' (disks) —— θ' —— θ ($CuAl_2$)
	Al–Zn–Mg	Zones (spheres) —— M′ (plates) —— ($MgZn_2$)
Al	Al–Mg–Si	Zones (rods) —— β' —— (Mg_2Si)
	Al–Mg–Cu	Zones (rods or spheres) —— S′ —— S(Al_2CuMg)
	Al–Li–Cu	Zones —— θ'' ——> θ' ——> θ ($CuAl_2$)

Alternative processes

Cast Al-Si is cost effective and ideal for mass production. However, casting can produce defects which limit the strength and the thick sections of the casting cool slower with a coarser and therefore weaker microstructure. Alternative processes can be: forging and powder metallurgy.

Forging of Al-Si piston is limited by the ductility (elongation) of the alloy thus forged components are rarely near net shape but require a finishing step. A high temperature needs to be selected to allow sufficient ductility for deformation (450 °C in this case). For **powder metallurgy**, molten alloy is sprayed into powders with a high cooling rate typically 10^3 K s^{-1}. The rapidly cooled alloy contains finely dispersed intermetallics. The powder can be mixed with fine SiC powder to improve wear resistance. The powders are then cold compacted in a die to produce green cylinder compacts. These are then sintered followed by hot forging.

However more can be done to improve the wear resistance of Al-Si alloys. Anodisation can to produce a hard and wear resistance oxide layer. Or insert a wear resistant material such as:

- A cast iron with high hardness and a thermal expansion close to the Al-Si alloy (problem - heavy)
- Fibres of alumina (Al_2O_3) and silica (SiO_2) in a preform in which the Al-Li alloys is squeeze cast (light weight)

4. Other light alloy material selection examples

Case study: Mast on a yacht

Function: Carrying load imposed by the boom and the mainsail

Objectives: Carries approximately uniform loading imposed by the mainsail, must be as light as possible while maintaining mechanical performance.

Constraints: Minimise mass, must be mass produced, low production costs, elastic bending, corrosion resistant, length and radius of the mast are fixed by the design, must have toughness

Free variable: Choice of material, wall thickness

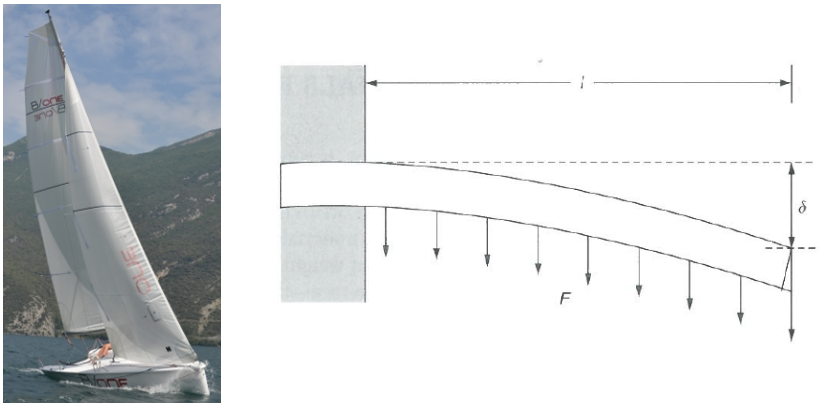


Figure 12 Mast on a yacht and its loading condition

We can treat the above case a cantilever beam subjected to uniformly distributed loading. But here we have the additional constraint that it must resist elastic bending.

The cross-section of the mast is essentially a thin walled tube of radius r and wall thickness t and length L.

The bending deflection can be given as $\delta = (1/8)\,(FL^3/EI)$

The secondary moment of area is $I = \pi r^3 t$

Combining these two equations, the bending stiffness of the tube is $F/\delta = 8\pi E r^3 t/L^3$

The mass of the beam is given by $m = 2\pi r t L \rho$

Therefore, $m = L^4 F\rho/(4r^2\delta E)$

However we do not want to maximise mass but to **minimise** mass. Thus mass of tube for a specified bending stiffness is therefore minimised by selecting a material with the maximum value of the performance index, $M = E/\rho$

Therefore plot log E versus log ρ and lines with **slope 1** will have the same merit index. To minimise mass, maximise performance index M.

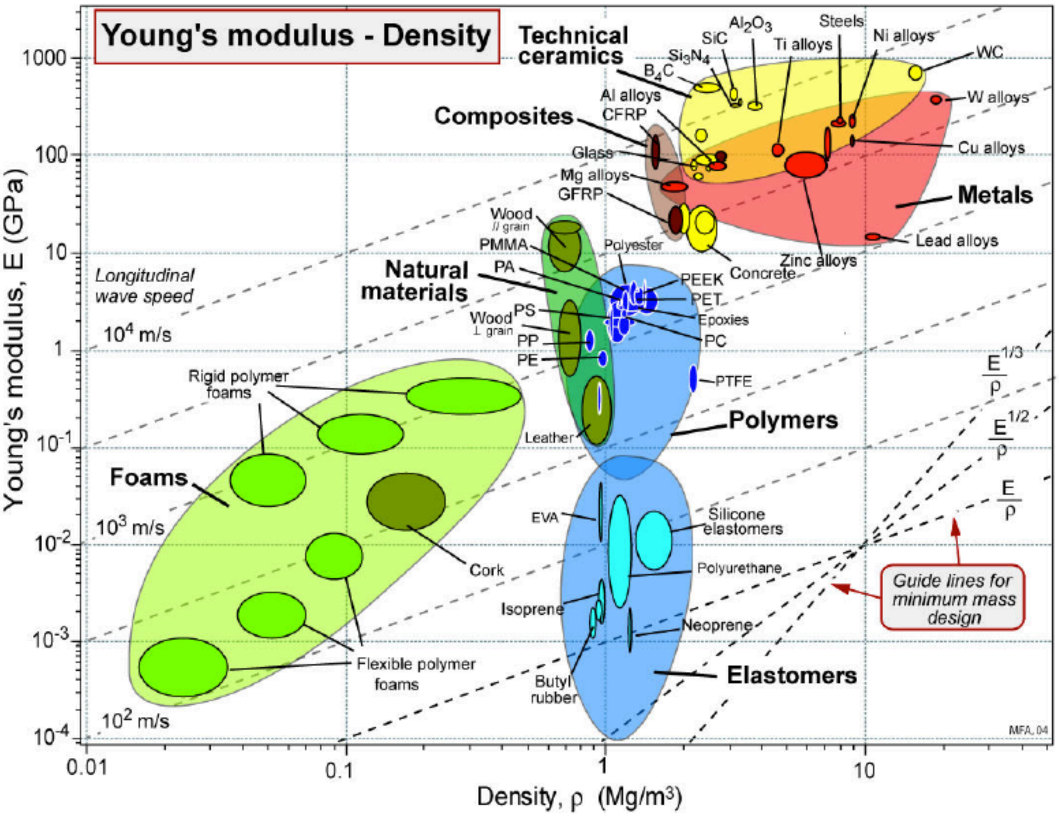
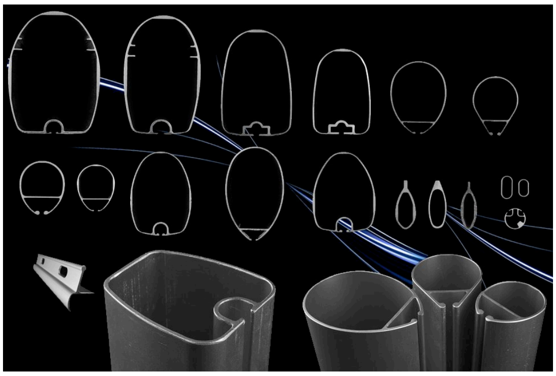


Figure 13 The material selection chart of Young’s modulus against density

We can use the merit index E/ρ plus other factors such as toughness and cost to select aluminium alloys. The specific Al series is selected based on other factors such as strength, corrosion resistance and fabrication method, in this case the ease of extrusion is preferred to manufacture the mast. Hence the ductility and ease of extrusion in the manufacturing process makes it attractive too hence 6xxx series which is also higher in strength than 5xxx, even though it is not as corrosion resistant as the 5xxx series which is also a factor. Both 5xxx and 6xxx series are considered.



Case Study: Engine connecting rod

Function: Connecting rod in a reciprocating engine

Objectives: Connects piston to crankshaft, must be as light as possible while maintaining mechanical performance in terms of strength and resisting bucking.

Constraints: Minimise mass and strength over billions of cycles (high cycle fatigue strength), must withstand elastic buckling, must be mass produced, low production costs

Free variable: Choice of material, cross section

Again strength to weight is important so again **$M_1 = \sigma_f/\rho$**

But here we have the additional constraint that it must not buckle. The cross section shape of the shaft of the con rod is essentially an I beam of width w and thickness b and length L. The compressive load ≤ the Euler bucking load, $F \leq \pi^2 EI/L^2$, where the second moment of area $I = b^3w/12$, and mass $m = V\rho$.

If we assume a shape geometry is constant such that the width of the con rod is related to its thickness such that $b = \alpha\, w$

Then $I = \alpha^3 w^4/12$ and $m = \alpha w^2 L\rho$

Rearranging, then, $w^2 = m/(\alpha L\rho)$

Therefore $I = \alpha\, m^2/(L^2\rho^2\, 12)$

Euler bucking occurs when

$F = \pi^2 EI/L^2$

$F = \pi^2 E\alpha\, m^2/(L^4\rho^2\, 12)$

$m^2 = F\, L^4\rho^2\, 12/(\pi^2 E\alpha)$

Therefore, m^2 is proportional to ρ^2/E for a set length L.

But we want to minimise mass (i.e. 1/m) so: Merit index for bucking: $\mathbf{M_2 = (E^{1/2}/\rho)}$

Note this merit index is commonly used for bucking problems such as pillars, table legs, etc.

Taking logs then $\log M_2 = \log E^{1/2} - \log \rho$

Thus $\log E = 2\log \rho + 2\log M_2$

Which is the form $y = mx + c$

Therefore if we plot log E versus log ρ and then lines with **slope 2** will have the same performance index for buckling.

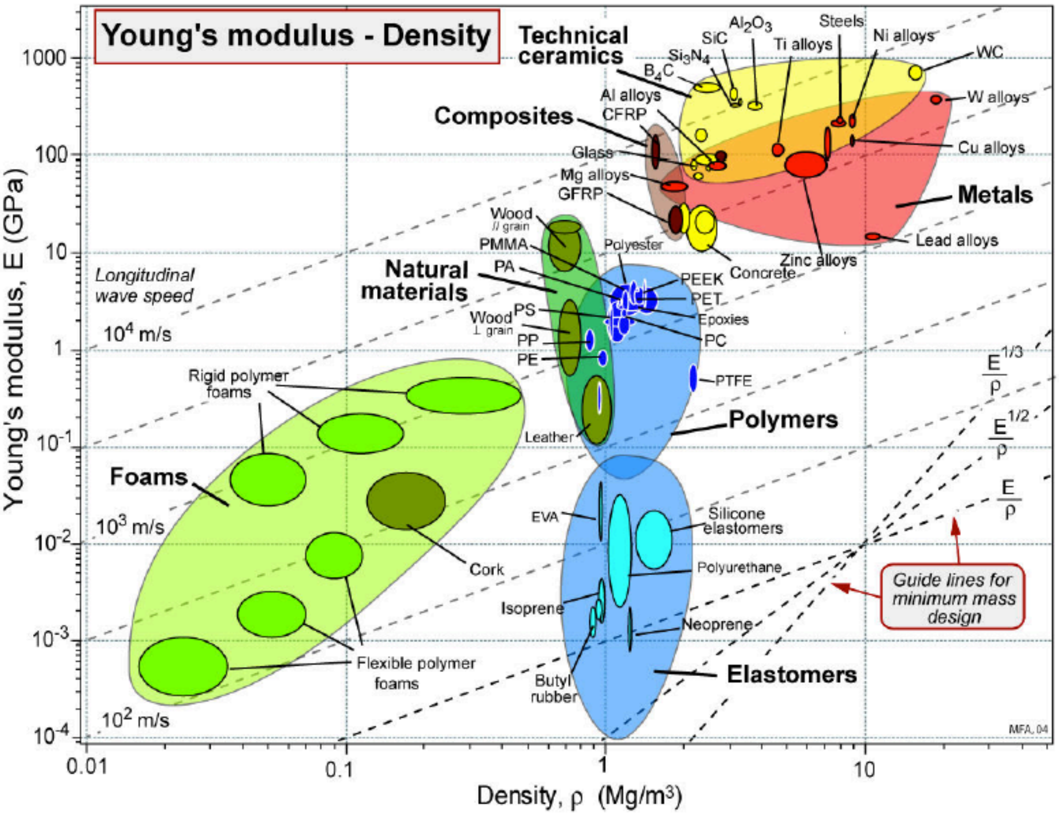


Figure 14 The material selection chart of Young's modulus against density

Selection considerations

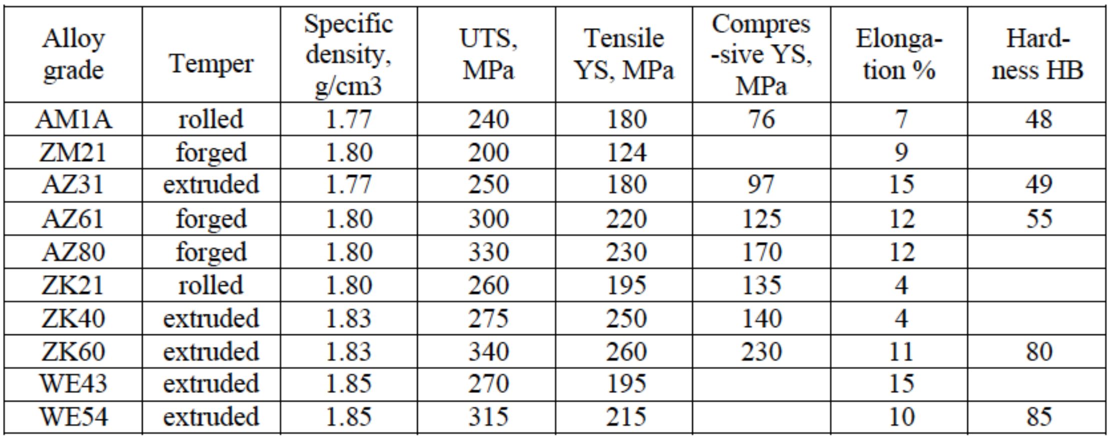
- Based on strength to weight index $M_1 = (\sigma_f/\rho)$ then Aluminium, Magnesium and titanium alloys are promising.
- Based on buckling resistance $M_2 = (E^{1/2}/\rho)$ then Magnesium best followed by aluminium, both outperform the Titanium alloys.
- Based on raw material cost: Titanium alloys are significantly more expensive to produce (and to manufacture due to high temperature processing in an inert atmosphere). Magnesium is more expensive but has better overall performance.
- Based on manufacturing: again Titanium too expensive for mass production, low cost engines, due to being processed at high temperatures Tm = 1668˚C and in an inert atmosphere. Like Al alloys, Mg alloys have limited ductility at room temperature and have to be wrought at elevated temperatures. Processing of Mg alloys is further complicated by its reactivity and corrosion in air.

4. Magnesium alloys

Magnesium, Mg, has great potential where strength to weight is required not been more utilised in Mg alloys until relatively recently. This is primarily because:

- Poor workability at room temperature
- Poor corrosion resistance
- High chemical reactivity
- Higher cost to process
- Drop in mechanical properties and creep at relatively low temperatures (150˚C)

The typical properties at room temperature of some wrought Magnesium Alloys are summarised in Figure 15 below. The wrought Mg alloys tend to be processed above 100˚C, typically in the range 175 – 350 ˚C, due to the higher ductility and ease of processing in the temperature regime.



Alloy grade	Temper	Specific density, g/cm3	UTS, MPa	Tensile YS, MPa	Compres-sive YS, MPa	Elonga-tion %	Hard-ness HB
AM1A	rolled	1.77	240	180	76	7	48
ZM21	forged	1.80	200	124		9	
AZ31	extruded	1.77	250	180	97	15	49
AZ61	forged	1.80	300	220	125	12	55
AZ80	forged	1.80	330	230	170	12	
ZK21	rolled	1.80	260	195	135	4	
ZK40	extruded	1.83	275	250	140	4	
ZK60	extruded	1.83	340	260	230	11	80
WE43	extruded	1.85	270	195		15	
WE54	extruded	1.85	315	215		10	85

Figure 15 Room temperature properties of some wrought Magnesium Alloys

The elemental additions in AZ Mg alloys are:

Aluminium (Al): improves castability, it also provides solid solution strengthening and age hardening at relatively low T (< 120°C).

Manganese (Mn): controls any Fe content by forming FeMn compounds and increases creep resistance and improves corrosion resistance.

Zinc (Zn): improves casting by decreasing viscosity allowing greater fluidicity. Zn also provides age(precipitation) hardening. Zn is a weak grain refiner but at the expense of increaseing brittleness.

Silicon(Si): increases creep resistance and is a weak grain refiner but at the cost of poorer corrosion resistance.

Strengthening in Mg alloys

Solid Solution Strengthening

Elements such as Al and Zn have significant solid solubility into the Mg HCP matrix. Unfortunately the effect on strengthening is very modest thus Mg alloys are often aged hardened.

Age hardening of Mg alloys

Improved strengthening is often achieved by age (precipitation) hardening. By a phase transformation from α-Mg to α-Mg + β structure in which the β phase is the precipitates such as β-$Mg_{17}Al_{12}$ and β-MgZn, as shown in Figure 16.

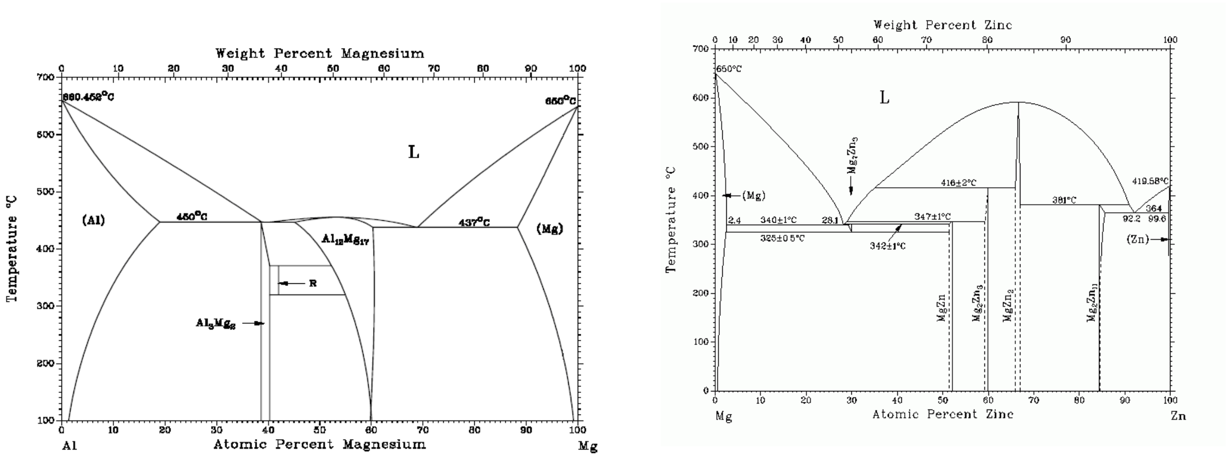


Figure 16 The phase diagram of Al-Mg and Mg-Zn

The AZ Mg alloy series (Mg-Mn-Al-Zn) such as AZ61 suffers from loss of strength and creep if operated at modest temperatures, such as above 150˚C. The WE series (Mg-Y-RE) based on Mg-RE-Zr has a greater creep resistance and good mechanical properties at both room temperature and elevated temperatures and is generally rated for use up to 300˚C. Typical rare earths (RE) used to improve yield strength and creep resistance are Dysprosium, Yttrium and Gadolinium. Other additions include Scandium, Calcium, Cerium and Manganese. Manganese, Mn, additions tend to help with fatigue resistance. Note the addition of RE and Zr combined with the higher processing temperatures make WE alloys more expensive.

5. Titanium alloys

Case study Hip Stem.

Function: provides support for hip ball and provides fixation to the femur

Objectives: Must transfer load from ball to femur, must be as light as possible while maintaining mechanical performance in terms of strength and resisting bucking. Must be biocompatible. Must transfer load evenly to bone and fit into intramedullary canal in femur

Constraints: Elastic modulus **similar** to bone, maximise strength to weight ratio, non-toxic and biocompatible, must not corrode in vivo.

Free variable: Choice of material

Three performance indices to be considered:

Strength to weight Merit Index: $\mathbf{M_1 = \sigma_f/\rho}$

The stem is essentially a cylindrical rod: The compressive load ≤ the Euler bucking load, $F \leq \pi^2EI/L^2$, where the second moment of area $I = \pi r^4/4$, and mass $m = V\rho$.

Like previous example it can be easily shown that: Buckling Merit Index $\mathbf{M_2 = (E^{1/2}/\rho)}$

Elastic Modulus similar to cortical bone: Merit Index: $\mathbf{M_3 \approx 5 - 20\ GPa}$

See figure 17 and 18

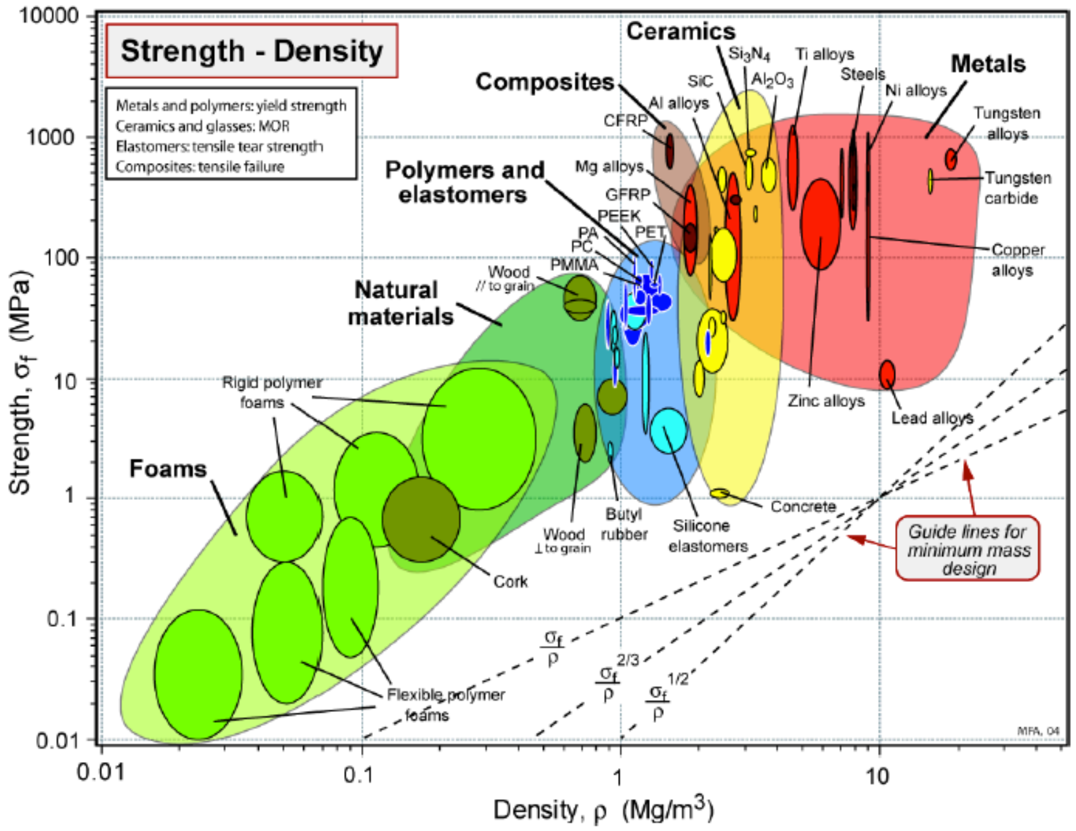


Figure 17

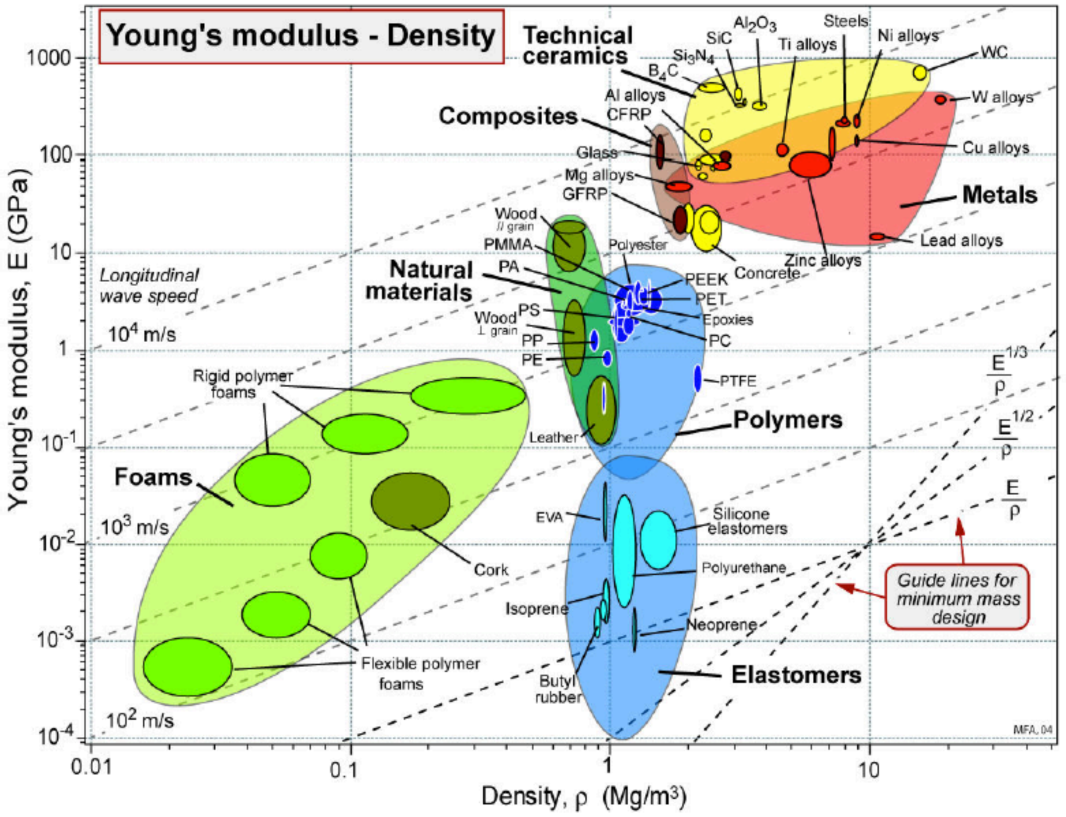
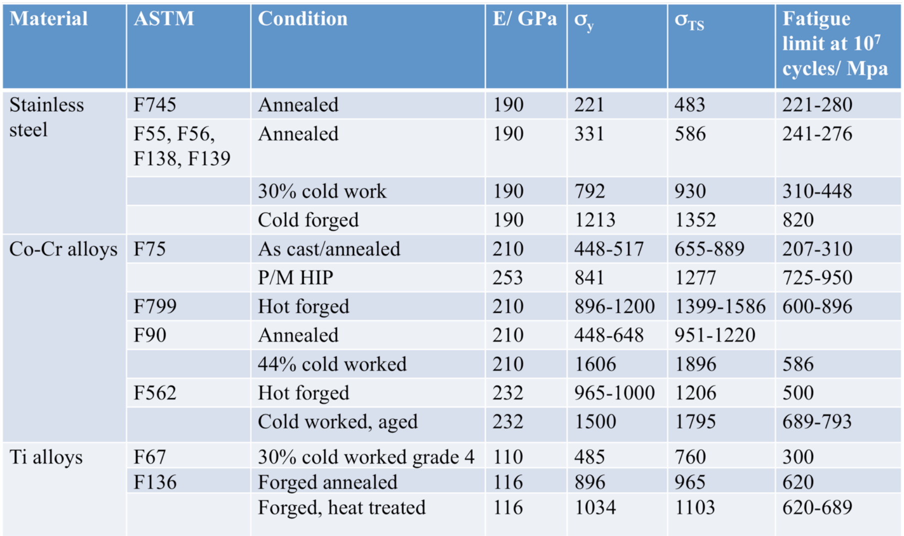


Figure 18

Selection considerations for the above work example:

- All classes of materials have the similar strength to weight merit index (M_1) as cortical bone but low density materials would not have sufficient space in intramedullary canal. Lead alloys have too low a M_1 but also toxic.
- Applying bucking Merit index M_2 then metal alloys and ceramics have sufficient high values. However in terms of fracture toughness the ceramics are an order of magnitude lower.
- For a 100 kg person running down stairs then F = ma = 10000 N. Cross section area = $\pi r^2 \approx 100$ mm. Therefore failure strength 100 MNm^{-2}. Thus selection needs to be above this value on figure 17.
- In terms of matching stiffness then all the metal alloys are too stiff. The only solution is a composite structure. For a metal implant then will have to slacken this constraint. However, this will lead to stress shielding by the implants with bone density loss and risk weakening the surrounding bone.
- Of the metal alloys the Cu, Ni and Al alloys have problems with toxicity. Mg alloys corrode.
- Therefore corrosion resistant steel alloys, such as austenitic stainless steel and Ti alloys are the most likely with Ti alloys being the favourite with their elastic modulus a factor 2 lower at ≈ 100 GPa.

In reality three different classes of materials are used for metal stems, as listed below.



Material	**ASTM**	**Condition**	**E/ GPa**	**σ_y**	**σ_{TS}**	**Fatigue limit at 10^7 cycles/ Mpa**
Stainless steel	F745	Annealed	190	221	483	221-280
	F55, F56, F138, F139	Annealed	190	331	586	241-276
		30% cold work	190	792	930	310-448
		Cold forged	190	1213	1352	820
Co-Cr alloys	F75	As cast/annealed	210	448-517	655-889	207-310
		P/M HIP	253	841	1277	725-950
	F799	Hot forged	210	896-1200	1399-1586	600-896
	F90	Annealed	210	448-648	951-1220	
		44% cold worked	210	1606	1896	586
	F562	Hot forged	232	965-1000	1206	500
		Cold worked, aged	232	1500	1795	689-793
Ti alloys	F67	30% cold worked grade 4	110	485	760	300
	F136	Forged annealed	116	896	965	620
		Forged, heat treated	116	1034	1103	620-689

Additional constraints:

- **Osteoconductivity:** for a cementless implant the metal must have a surface that is osteoconductive (that is bone will grow up to it). Ti alloys form a natural titania, TiO_2, oxide that gives it corrosion protection and also has OH groups on the surface which encourages bone cells to grow up to the surface.
- **Corrosion resistance:** we could also do a selection criteria on this and again Ti has superior corrosion performance, higher pitting potential, compared to stainless steel and CoCr alloys.

Therefore Ti alloys are a good materials selection choice. Titanium alloys are either α, α + β, or β alloys. A common alloy used for hip stem is an α + β alloy Ti-6wt%Al-4wt%V often shortened to (Ti6Al4). The alloying elements in Ti alloys exhibit different stabilising effects in the phase formation of Ti alloys.

Ti Alloy additions:

- Atoms of elements that form less than four bonds with other atoms, raise the temperature over which α phase is stable (Al, O, N, Ga).
- Atoms of elements that form four bonds with other atoms (Zr, Sn, Si) tend to be more neutral in their effect on the α an β phases.
- Atoms of elements that form more than four bonds with other atoms, stabilise β phase either continuously (Mo, Nb, V, W) or sluggishly form a eutectoid (Cu, Mn, Fe, Ni, Co, H).

α alloys have single phase microstructure therefore good weld properties. The stabilising effect of these high Al alloys also improves strength and oxidation resistance. Single phase therefore cannot use heat treatment for strengthening. High percentage of β stabilisers, such as in Ti-13V-11Cr-3Al alloy, cause a predominantly β phase structure.

In Titanium alloys α + β mixtures such as Ti6Al4V are annealed in one of two ways:

- Anneal above 1003°C in the β phase, then cool slowly to allow α phase to form by diffusion. The new grains of α have distinctive flattened shape called Widmanstatten.
- Anneal at approximately 700°C where it forms equiaxed grains of α and β phase. Then cool slowly whereupon the β phase transforms slowly to form Widmanstatten grains surrounded by α phase.

Both heat treatments are examples of strengthening by solute atoms dissolved in each phase and by the refinement of grains by the transformation of β to α phase.

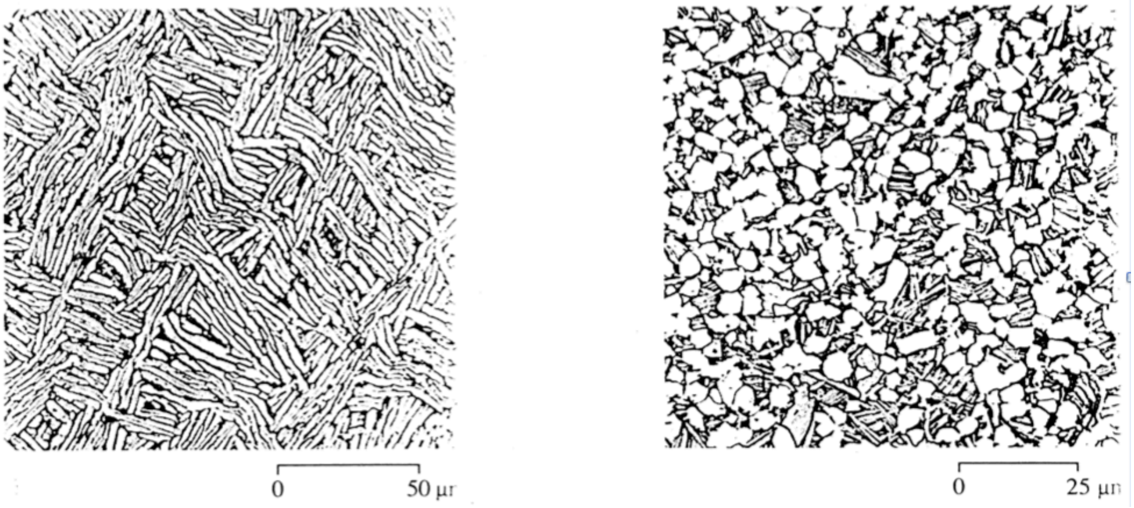


Figure 19 Left: Widmanstatten structure . Right α phase and Widmanstatten structure