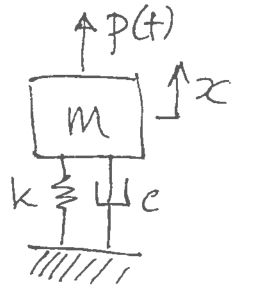
MM2DYN - DYNAMICS

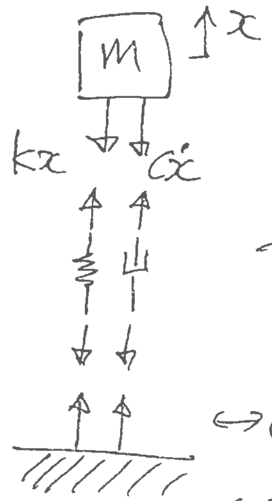
VIBRATION: SHEET 7 - VIBRATION ISOLATION PART I.

<u>SOLUTIONS</u>

Q.1



F.B.D.



$$\Sigma F_x = m\ddot{x} = -kx - c\dot{x} + p(t)$$

$$\Leftrightarrow m\ddot{x} + c\dot{x} + kx = p(t)$$

Transmitted force to the ground:

$$q(t) = kx + c\dot{x}$$

$$\Leftrightarrow Q^* \cancel{e^{i\omega t}} = (k + i\omega c) X^* \cancel{e^{i\omega t}}$$

$$\Leftrightarrow$$

Consider:

$$p(t) = P.e^{i\omega t}$$

$$q(t) = Q^*.e^{i\omega t}$$

$$x(t) = X^*.e^{i\omega t}$$

From the equation of motion $\quad (-m\omega^2 + i\omega c + k) X^* \cancel{e^{i\omega t}} = P.\cancel{e^{i\omega t}}$

$$\Leftrightarrow X^* = \frac{P}{-m\omega^2 + i\omega c + k}.$$

Force transmissibility $\quad T_F = \left|\frac{Q^*}{P}\right| = \left|\frac{k + i\omega c}{(k - m\omega^2) + i\omega c}\right| = \sqrt{\frac{k^2 + c^2\omega^2}{(k - m\omega^2)^2 + (c\omega)^2}}$

(a) Assuming zero damping: for $\omega > \omega_n$

$$T_F = \frac{k}{|k - m\omega^2|} = \frac{k/m}{|k/m - \omega^2|} = \frac{\omega_n^2}{|\omega_n^2 - \omega^2|}$$

$$= \frac{1}{\left|1 - \frac{\omega^2}{\omega_n^2}\right|}. \quad \text{since } \omega > \omega_n$$

$$T_F = \frac{1}{\frac{\omega^2}{\omega_n^2} - 1}$$

For $\frac{\omega_{min}}{\omega_n} = 5$, $\quad T_F = \frac{1}{25 - 1} = 0.0417$

Isolation efficiency $= (1 - T_F) \times 100\% = \underline{\underline{95.83\%}}$

(b). If $m_2 = \frac{1}{2}m$, $\omega_{n_2}^2 = \frac{k}{\frac{1}{2}m} = 2k/m = 2\omega_n^2$

where ω_{n2} is the natural frequency of the new system, with mass M_2.

The force transmissibility for $\omega > \omega_n$, $T_{F_2} = \frac{1}{\frac{\omega^2}{\omega_{n_2}^2} - 1}$.

$$\Leftrightarrow T_{F_2} = \frac{1}{\frac{\omega^2}{2\omega_n^2} - 1}.$$

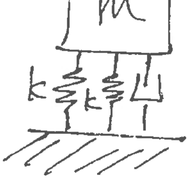
At a given ω, $T_{F_2} > T_F$, so the isolation efficiency will be lower than the first system.

(c) To achieve the same vibration isolation characteristics, in terms of T_F, $\omega_{n_2}^2 = \omega_n^2$.

$$\Leftrightarrow \frac{k_2}{\frac{1}{2}m} = \frac{k}{m}$$

$$\Leftrightarrow k_2 = \frac{1}{2}k.$$

This can be achieved by adding a parallel spring of stiffness, k, as follows:

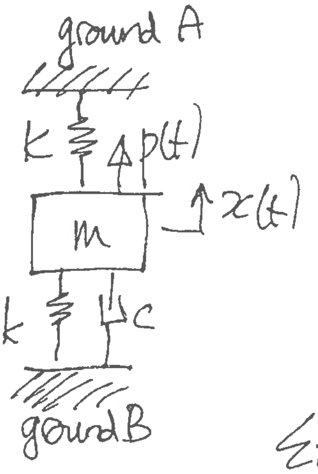


$k_2 = k_{parallel}$

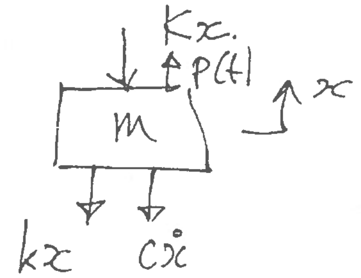
where

$$K_{parallel} = k_1 + k_2 = k + k = 2k$$

Q(2)



FBD:



$$\Sigma F_x = m\ddot{x} = p - Kx - kx - c\dot{x}$$

(a) Equation of motion $\Leftrightarrow m\ddot{x} + (K+k)x + c\dot{x} = p(t)$

(b) Transmitted force to ground A:

$$q_A(t) = Kx$$

Consider: $p(t) = P.e^{i\omega t}$

$$q_A(t) = Q_A^* e^{i\omega t}$$

$$x(t) = X^*.e^{i\omega t}$$

Using the equation of motion $\left(-m\omega^2 + i\omega c + (K+k)\right)X^* = P$

and $Q_A^* = K.X^*$

(Force transmissibility to ground A) $T_{FA} = \left|\frac{Q_A^*}{P}\right| = \left|\frac{K}{(K+k-m\omega^2) + i\omega c}\right| = \frac{K}{\sqrt{(K+k-m\omega^2)^2 + c^2\omega^2}}$

(c) Transmitted force to ground B:

$$q_B(t) = kx + c\dot{x}$$

Let: $q_B(t) = Q_B^*.e^{i\omega t}$ $\Big\}$ $Q_B^* \cancel{.e^{i\omega t}} = (k + i\omega c).X^* \cancel{e^{i\omega t}}$

So: $T_{FB} = \left|\frac{Q_B^*}{P}\right| = \left|\frac{k + i\omega c}{(K+k-m\omega^2) + i\omega c}\right| = \sqrt{\frac{k^2 + c^2\omega^2}{(K+k-m\omega^2)^2 + c^2\omega^2}}$

(force transmissibility to ground B)