The University of Nottingham
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MMME2046 Dynamics and Control: Lecture 1

Introduction to Machine Dynamics

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Handouts Chapter I

Objectives

- Sum up the main **prerequisites** for Machine Dynamics part of MMME2046
- **Introduction** to Machine Dynamics
- **Revision** case studies

For your information

- Handouts are available online
- Slides will be made available on Moodle
- Four exercise sheets
- Solutions to exercise sheets will be on Moodle a week after the seminar
- Recommended textbook: Hibbeler R.C. “Engineering mechanics: Dynamics”, chapters 16 and 17
- Coursework assignment (25%)

  Out on 27th October 2022

  Submission: 1st December 2022

Prerequisites

Starting Point

Basic concepts of mechanics:

- Secondary school,
- MMME1028: Statics and Dynamics

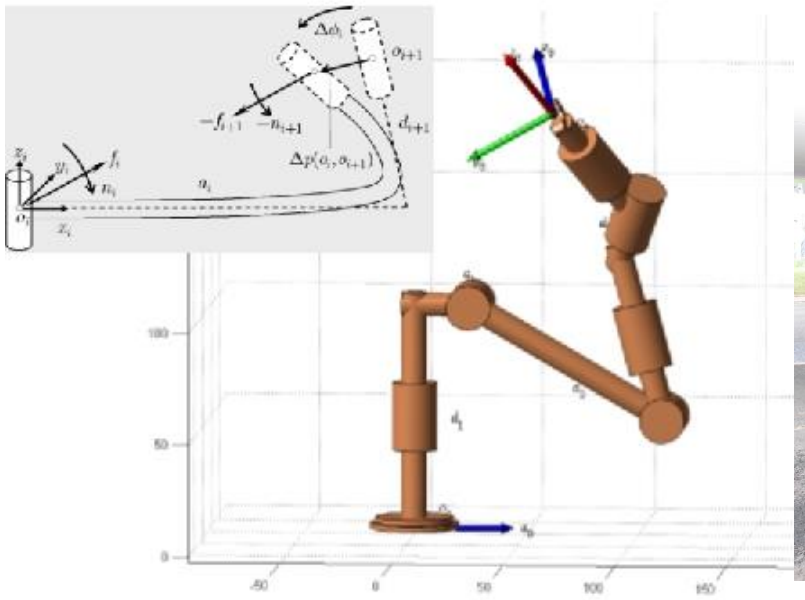
Essential tools

- Newton's Laws,
- Free body diagrams,
- Particle kinematics and dynamics,
- Inertia properties of rigid bodies,
- Vectors and vector algebra

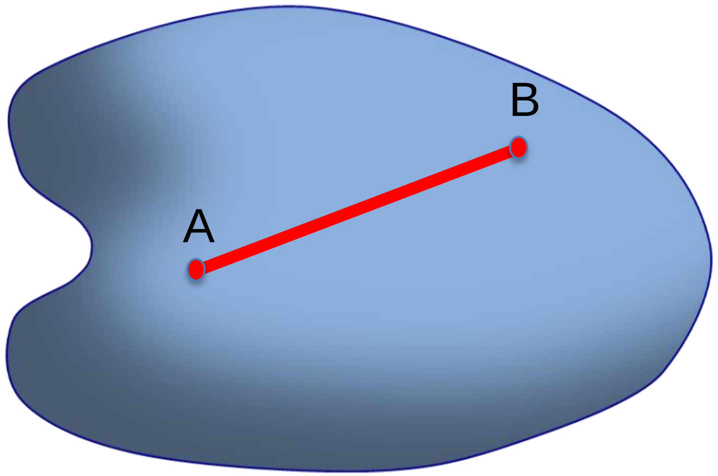
What is Machine Dynamics

Analysis and design of **rigid mechanisms** and structures in motion

Examples: industrial robots, landing gears, wind turbines



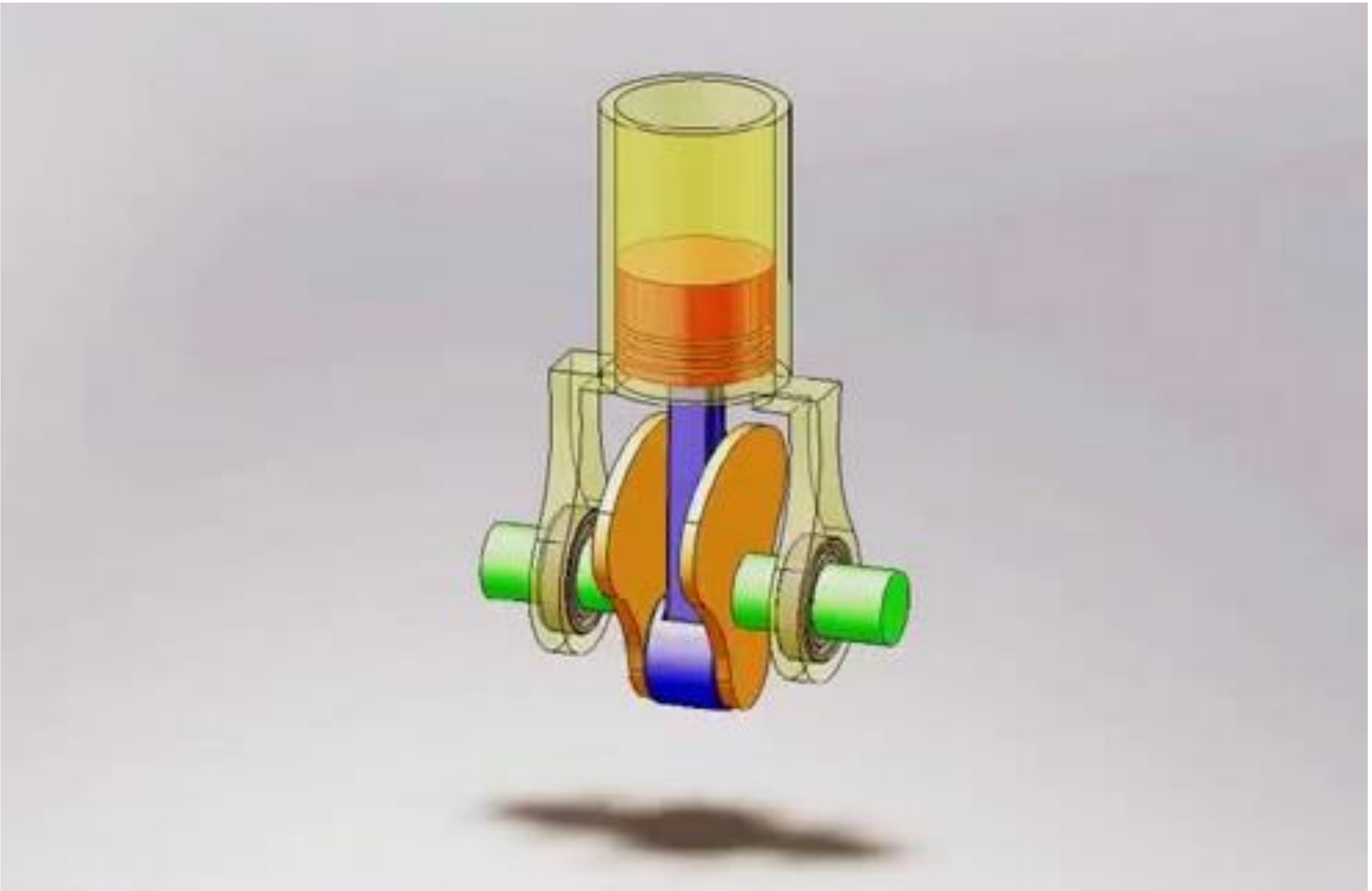
Rigid Body definition



- System of particles
- Distances between particles remain unchanged
- Deformations are neglected

Particle − Rigid body − System of rigid bodies

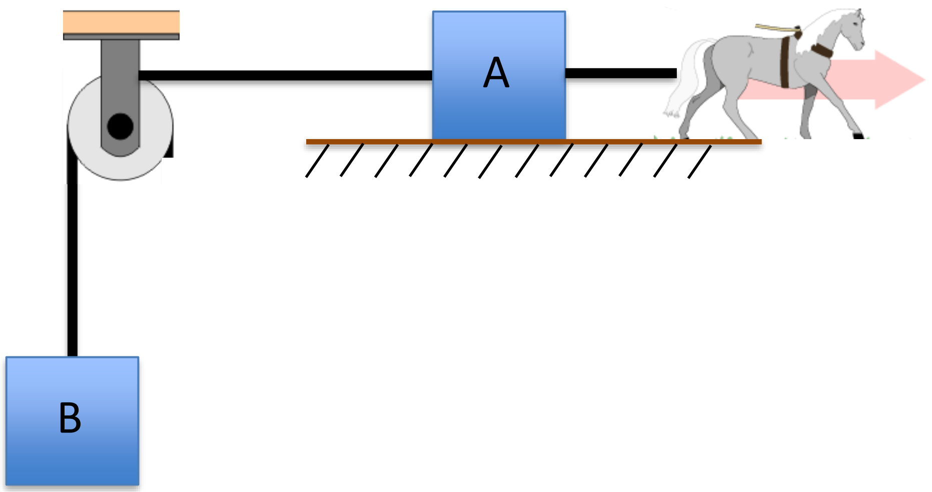
System of rigid bodies - mechanism



Revision case studies

- Newton's 2nd Law / Free body diagrams,
- Vectors and vector algebra,
- Particle kinematics in circular motion

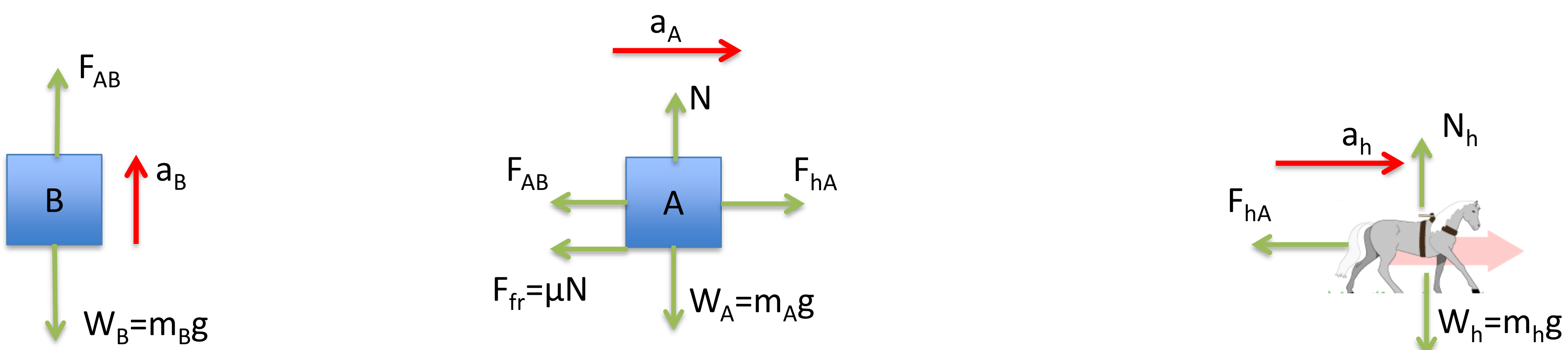
Case study 1: FBDs, Newton's 2nd Law



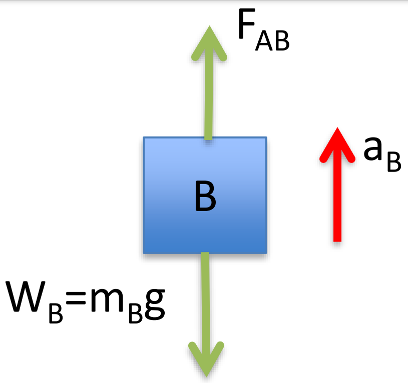
Draw FBDs for A, B and the horse. Calculate the force exercised by the horse as a function of the acceleration of mass B, a_B.

Assume that the ropes are under tension and massless pulley. Coefficient of friction is equal to μ.

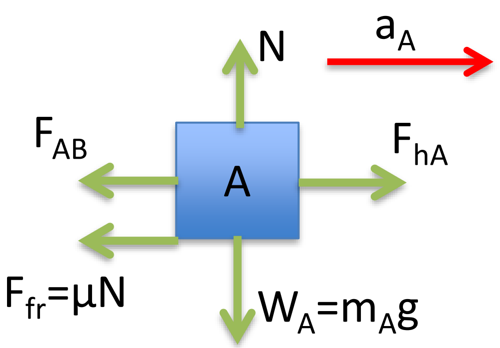
Free Body Diagrams (FBDs)



Newton's 2nd Law, Equations of motion



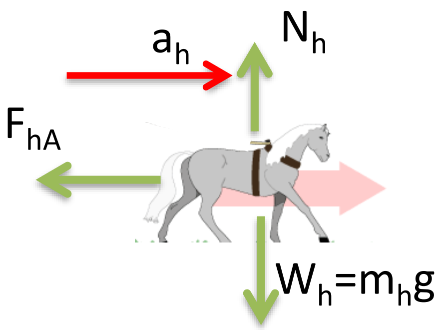
$\overset{+\uparrow}{\Sigma F_y}=m_Ba_B \qquad F_{AB}-W_B=m_Ba_B \Rightarrow F_{AB}=m_Ba_B+m_Bg \qquad (1)$



$\overset{+\rightarrow}{\Sigma F_x}= m_Aa_A \qquad F_{hA}-F_{AB}-\mu N = m_Aa_A \qquad (2)$

$\overset{+\uparrow}{\Sigma F_y}= m_Aa_{Ay} \qquad N-m_Ag = 0 \Rightarrow N=m_Ag \qquad (3)$

$F_{hA}=\underbrace{m_Ba_B+m_Bg}_{F_{AB}}+\mu m_Ag+m_Aa_A \qquad (4)$



However we assumed that $a_B=a_A=a_h=a$ (5)

Therefore: $F_{Ah}=(m_B+m_A)a+m_Bg+\mu m_Ag$

Notes on FBD and EOM

- FBD: Applied Forces + accelerations
- Action = –Reaction
- A particle can give 2 e.o.m towards any two selected directions.

Case study 2: Vector resolution

$a//CD =$

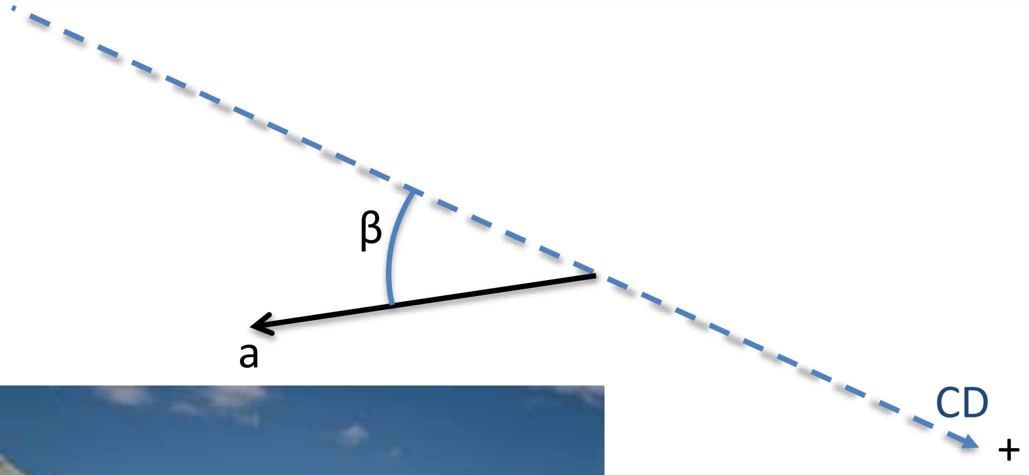
$a \times \cos\beta$ ✗

$a \times sin\beta$ ✗

$a \times \cos(180 - \beta)$ ✓

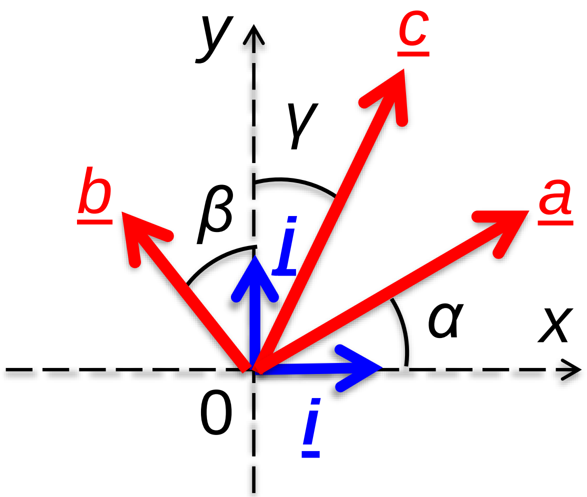
$-a \times \cos\beta$ ✓

$a \times \cos(\beta + 180)$ ✓



A 2D vector carried two pieces of information!!

Method 1: Geometrical



$$\underline{\boldsymbol{c}} = \underline{\boldsymbol{a}} + \underline{\boldsymbol{b}} \qquad (1)$$

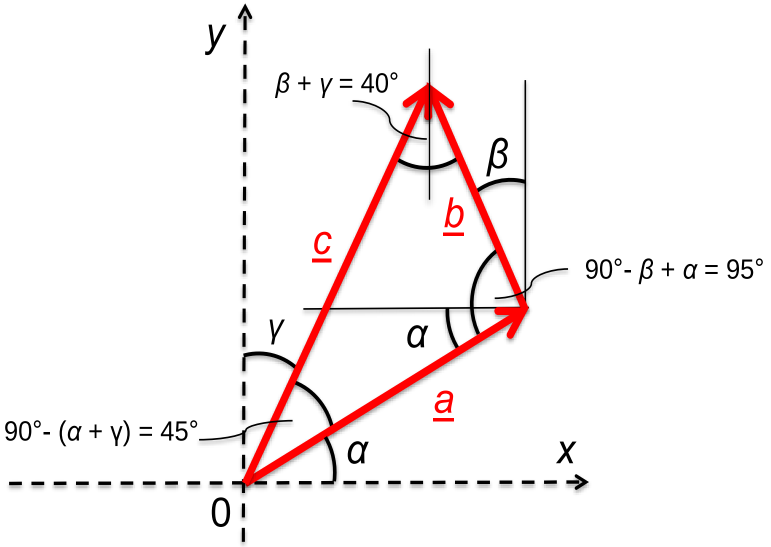
See Ch. I, pg.6

Given: $a = 5 \quad \alpha = 20^o$

$b = ? \quad \beta = 15^o$

$c = ? \quad \gamma = 25^o$

Method 1: Geometric calculations



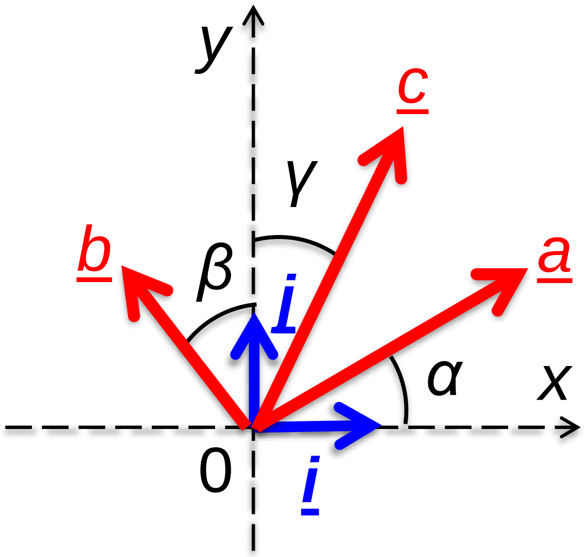
$$\frac{\sin 40^o}{a} = \frac{\sin 45^o}{b} = \frac{\sin 95^o}{c}$$

$$b = 5\,\frac{\sin 45^o}{\sin 40^o} = 5.500$$

$$c = 5\,\frac{\sin 95^o}{\sin 40^o} = 7.749$$

Check: $c^2 = a^2 + b^2 - 2ab\cos 95^o$

Method 2: X and Y projections



$$\underline{c} = \underline{a} + \underline{b} \qquad (1)$$

Given: $a = 5$ $\quad \alpha = 20^o$

$b = ?$ $\quad \beta = 15^o$

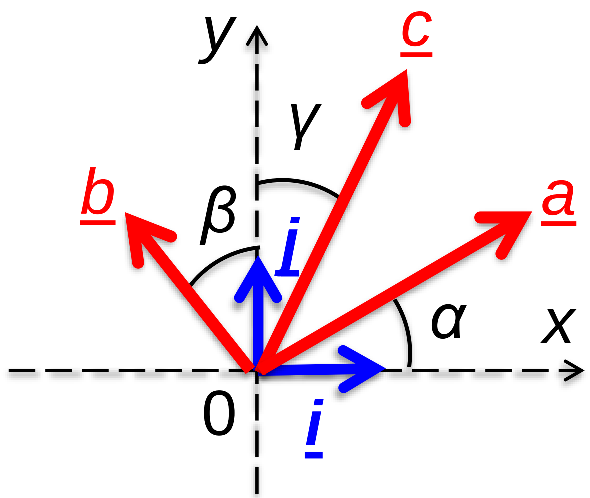
$c = ?$ $\quad \gamma = 25^o$

$$\xrightarrow{+}\ \Sigma X:\ c_x = a_x + b_x \rightarrow c\sin\gamma = a\cos\alpha - b\sin\beta$$

$$\uparrow^{+}\ \Sigma Y:\ c_y = a_y + b_y \rightarrow c\cos\gamma = a\sin\alpha + b\cos\beta$$

$$\begin{bmatrix} 0.2588 & 0.4226 \\ -0.9659 & 0.9063 \end{bmatrix} \begin{Bmatrix} b \\ c \end{Bmatrix} = \begin{Bmatrix} 4.698 \\ 1.710 \end{Bmatrix}$$

Method 3: Arbitrary axis projections

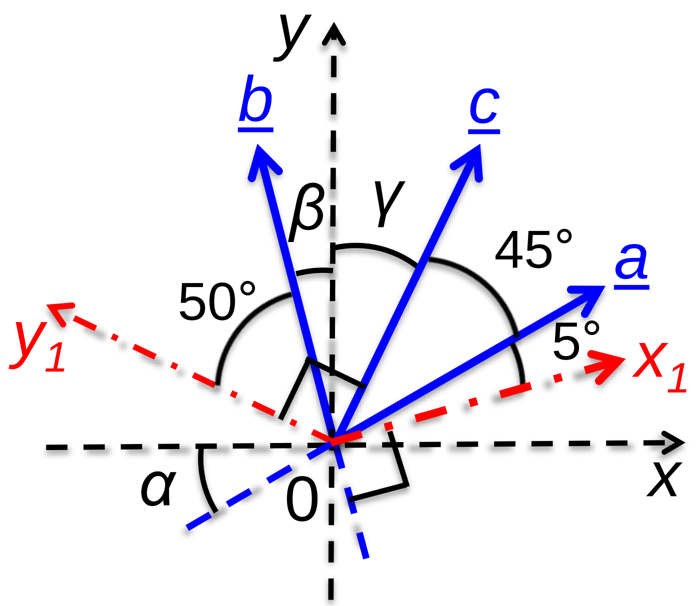


$$\underline{c} = \underline{a} + \underline{b} \qquad (1)$$

Given: $a = 5$ $\quad \alpha = 20^o$

$b = ?$ $\quad \beta = 15^o$

$c = ?$ $\quad \gamma = 25^o$



$\nearrow^+ \Sigma X_1$: $\quad c \cos 50^o = a \cos 5^o + 0$

$$c = 5 \frac{\cos 5^o}{\cos 50^o} = 7.749$$

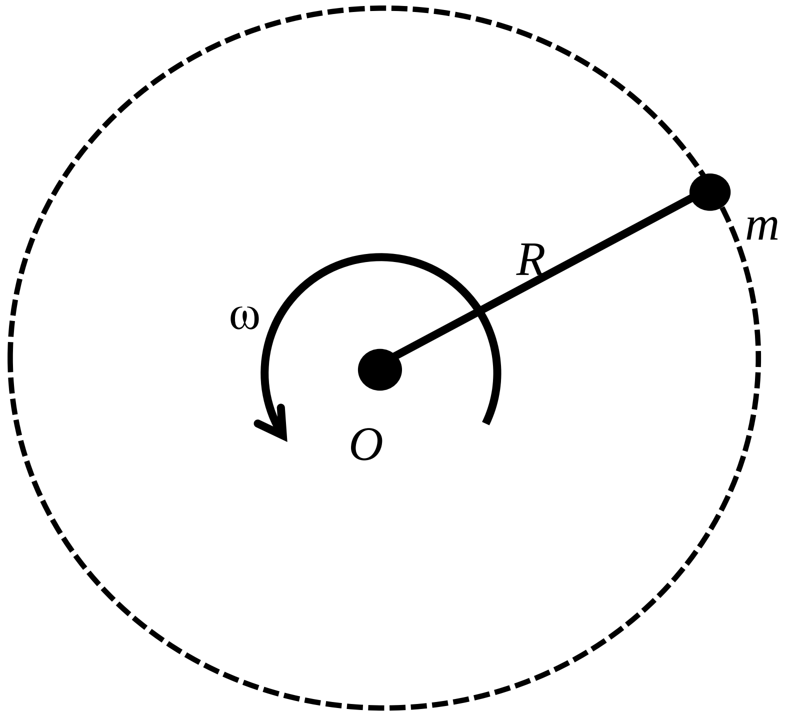
$\nwarrow^+ \Sigma Y_1$: $\quad 0 = -a \cos 45^o + b \cos 50^o$

$$b = 5 \frac{\cos 45^o}{\cos 50^o} = 5.500$$

Notes on resolving vectors

- There are several ways to solve a system of vectors
  - You should always go for the fastest one. Wise selection comes with experience so practise!

Case study 3: Circular motion



- A particle m attached to one end of a rigid light rod rotates in a circle of constant radius R about O at angular speed ω.
- Determine the velocity and acceleration of the particle in polar coordinates.

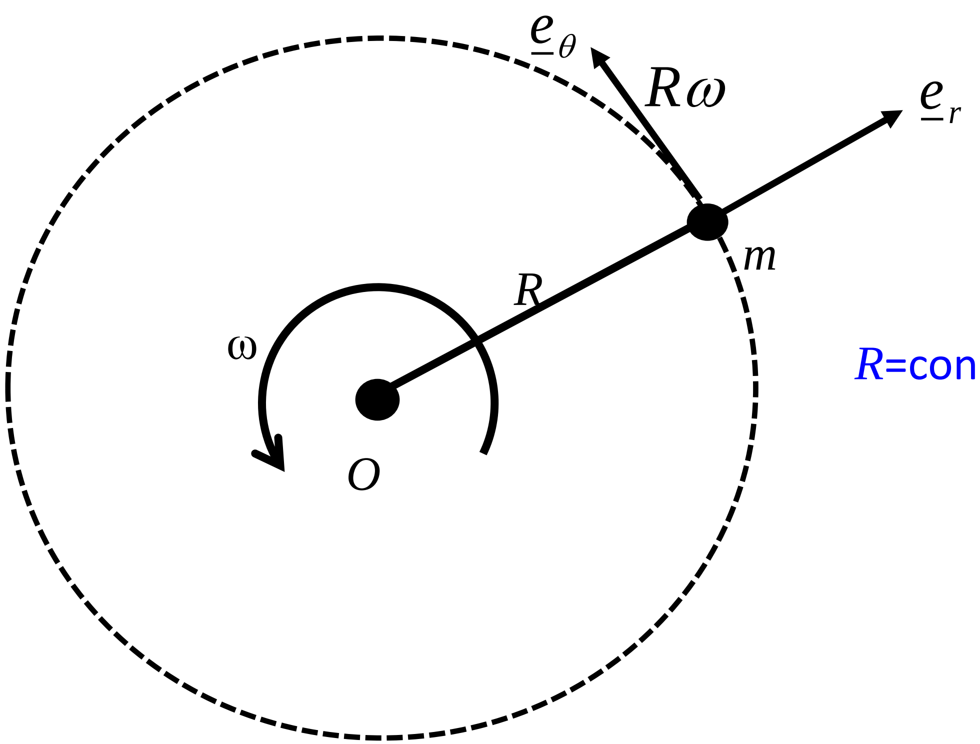
Circular motion - Velocity

Position $\underline{r} = R\underline{e}_r$

Velocity $\underline{v} = R\omega\underline{e}_\theta$

Interpretation:

m has tangential velocity Rω along $\underline{e}_\theta$



R=constant

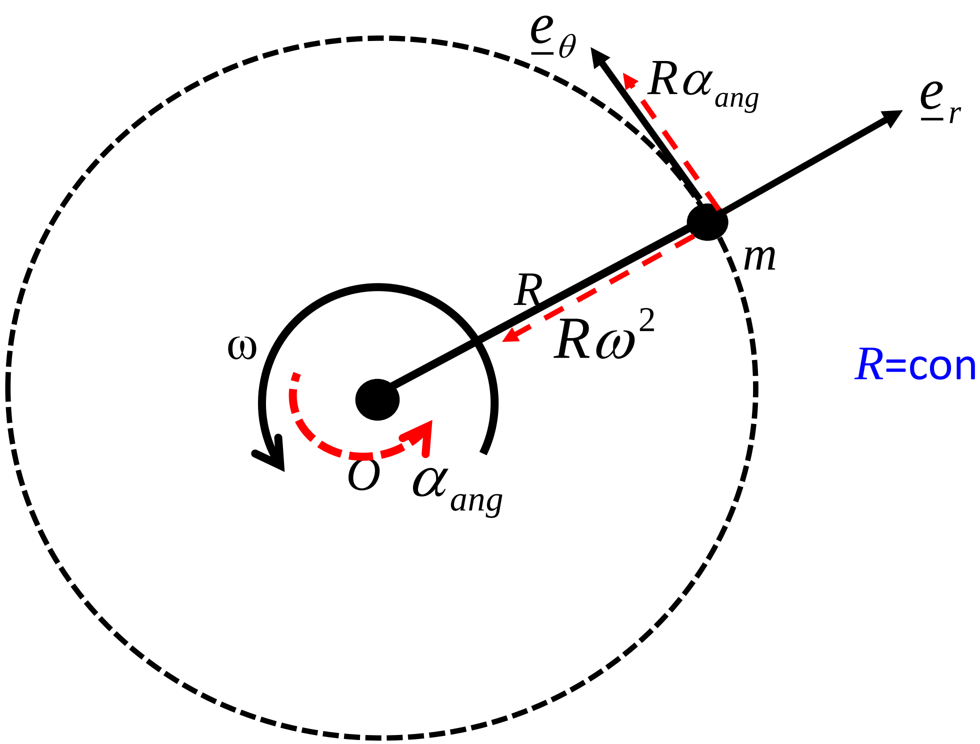
Circular motion - Acceleration

Position $\underline{r} = R\underline{e}_r$

Velocity $\underline{v} = R\omega\underline{e}_\theta$

Interpretation:

m has tangential velocity Rω along $\underline{e}_\theta$



R=constant

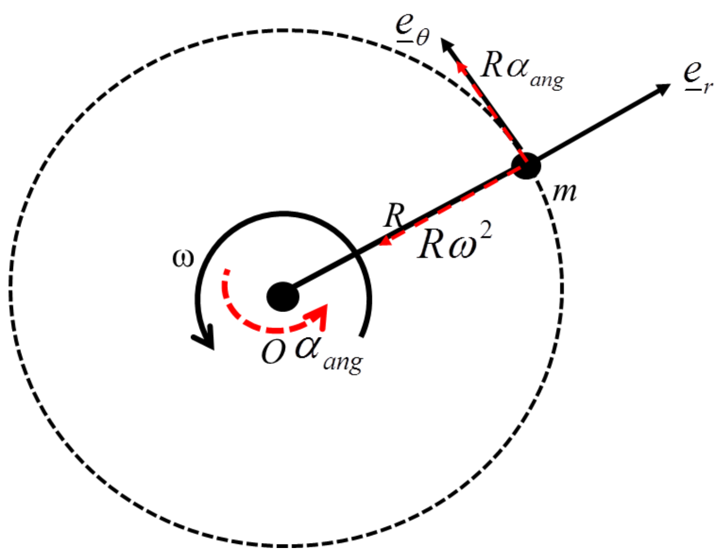
Acceleration $\underline{a} = (-R\omega^2)\underline{e}_r + (R\alpha_{ang})\underline{e}_\theta$

Interpretation:

m has radial acceleration Rω^2 towards O along $\underline{e}_r$ and tangential acceleration along $\underline{e}_\theta$

Notes on circular motion

- When a point conducts circular motion:
  - The velocity is tangential to the circle with direction defined by ω.
  - There are two acceleration components (tangential + normal)
    - Normal component always has direction towards the centre of rotation.
    - Tangential component is tangential to the circle with direction defined by α.

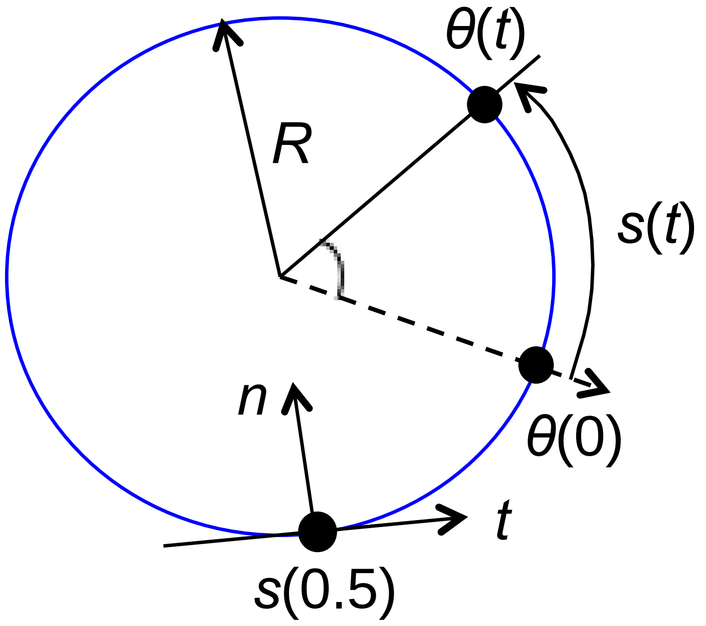


Case study 3: Circular motion example

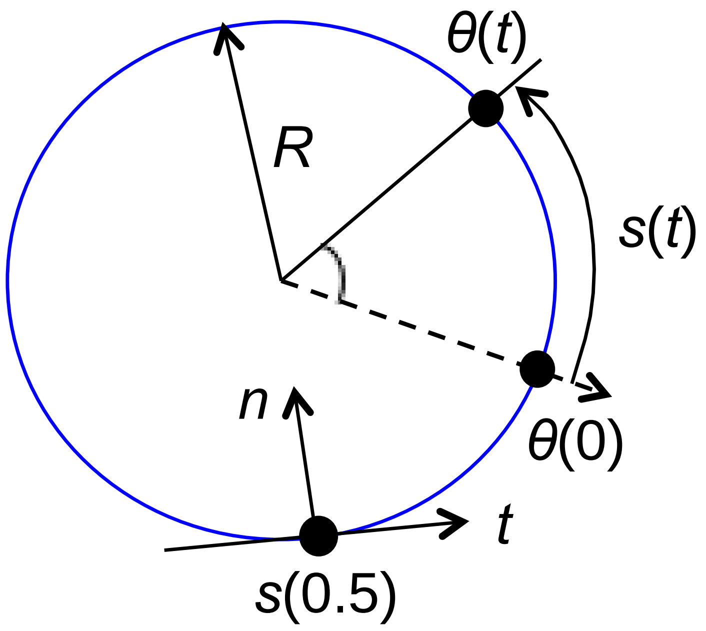
A particle is moving in a plane on a circular orbit with radius $R = 2\ \mathrm{m}$. Its motion is described by the angle between the particle's initial and current positions on the circular trajectory

$$\theta(t) = t^2 - 3t\ \mathrm{rad}.$$

Calculate and describe geometrically the kinematic variables of the particle at $t = 0.5\ \mathrm{s}$.



Case study 3: Circular motion example



$$\underline{r} = R\underline{e}_r$$

$$\underline{v} = R\omega\underline{e}_\theta \qquad \omega = \dot{\theta}$$

$$\underline{a} = (-R\omega^2)\underline{e}_r + (R\alpha)\underline{e}_\theta \qquad \alpha = \ddot{\theta}$$

The particle position can be given by the arc length, a scalar function, in the following form

$$s(t) = R\theta(t) = 2(t^2 - 3t)\ \mathrm{m}.$$

The velocity magnitude and the tangent acceleration are given by

$$v(t) = \dot{s}(t) = R\dot{\theta}(t) = 2(2t - 3) = 4t - 6\ \mathrm{m/s}$$

$$a_\mathrm{t}(t) = \ddot{s}(t) = R\ddot{\theta}(t) = 4\ \mathrm{m/s^2}.$$

The normal (centripetal) acceleration is

$$a_\mathrm{n}(t) = R\omega^2 = R(2t - 3)^2 = 8t^2 - 24t + 18\ \mathrm{m/s^2}$$

In the next lecture…

Kinematics of machines:

- Classify various type of rigid body motion
- Perform velocity and acceleration analysis on simple mechanisms