Thermofluids 3: Turbomachinery and compressible flow

1. Introduction

In this section of thermofluids, the connection between fluid mechanics, which drives the thermodynamic changes, which in turn interacts with the machine dynamics, is explored. The fluid mechanics of turbomachines and the characteristics of compressible flows which relate to turbomachinery in their wider context are explored. The outcome is a summary of the engineering principle connecting the disciplines that have been learned in isolation to this point.

The complex flows within the cascade passages of turbomachines provide an elegant and contained location for analysis of the forces developed within the fluid, the work extraction from the machine, and the consequent thermodynamic property changes. Ultimately this provides a way to investigate from momentum theory how energy is extracted, which informs how the fluid has to change in response.

An important consideration in turbomachines is that due to their compact power density and high rotating speed, it is likely that the flow will encounter compressible conditions for the gases. The other part of this section therefore explores the nature of compressible flows where interactions with profile changes of the surfaces can be the cause of normal and oblique shocks.

Both of these subjects are larger than the concise summary approach presented here, and the learning outcome will be specific set point calculations within defined parameters which expose the basic concepts, together with an overview of the entire subject of each in which the key terms of reference are briefly explored.

Finally a short introduction is given on nuclear power generation to compliment the conventional power systems section.

Aim: explore the nature of turbomachines and compressible flow to gain a practical understanding of the connection of thermodynamics and fluid mechanics within mechanical power transformations.

Objectives: connection of this section with previous study in thermofluids; develop the Euler turbine equation and describe the connection with reaction; velocity diagram methodology; flow in axial turbomachines with analysis of the velocity and reaction; overview of radial flow turbomachines and the variation of the analysis from axial flow machines; effects of compressible flow within turbo compressors and turbines; normal and oblique shocks; Prandtl-Meyer waves; shock turning.

Texts used in preparation of material: two text books contain the information summarised in these notes, which are worth referring to. A text on turbomachines[1] provides a great deal more information on the wider context and the details of all calculations summarised here. A book on compressible flow[2]

[1] Dixon, S.L. and Hall, C.A. Fluid mechanics and thermodynamics of turbomachinery, 7th ed. Oxford : Butterworth-Heinemann 2014. Library location George Green TJ266DIX

[2] Oosthuizen, P.H. and Carscallen, W.E. Introduction to compressible flow, Boca Raton, Fla. ; London CRC Press c2014. Library location George Green QA911OOS

a) Compressible flow: material to be covered – 2 lectures

Since compressible flows can occur in turbomachinery, particularly in the case of supersonic flow into a gas turbine, then the compressible flow study will be expanded to address what happens in flow turning and in internal flows:

- review of compressible flow covered in thermofluids 2
- what happens when supersonic flows interact with inclined surfaces in the case of compressing and expanding flows will be examined
- the case of internal compressible flows with friction and heat addition.

b) Axial flow turbomachines

The commonly seen compressor-turbine propulsion units on aircraft are familiar, and this section allows a detailed investigation of how the power is converted out of the gas streams into shaft power, and an introduction to the concepts of how axial flow device design is commenced.

c) Radial flow turbomachines

Less commonly seen, but significant in process industry and in smaller units, radial flow turbomachinery has geometrical considerations not required in axial flow. This section shows how the considerations made for axial turbomachines are adapted into the 3-dimensional flow of radial devices.

d) Nuclear Power guest lecture by usual lecturer: Material to be covered – 1 lecture

Despite the ongoing debate on nuclear power leaning towards avoiding the pursuit of future installations in many countries, it is still considered to be a potentially useful contributor to the power mix. A brief introduction to the concept of nuclear power engineering will be given:

- the principle of operation of PWR and BWR nuclear reactors
- moderation and the importance of thermal neutrons
- overview of control strategies to maintain critical operation

2. Compressible flow introduction

Previously in Thermofluids 2, the idea of compressible flow and Mach number related to sound speed in a fluid was presented in terms of isentropic compressible flow and normal shock waves, and the discussion of sound speed in general.

In thermofluid machines using compressible gases there is significant potential for flow speeds being so high as to approach or exceed sonic velocity. In this section compressible flow is considered in interaction with flow over obstacles such as oblique objects and expanding turning as well as pipe flow with friction and with heat transfer. Expansion fans and oblique shocks lead to ideas of how common high speed components in turbomachines will interact with the flow when it is compressible. The following are presented:

- compressible fluid theory and maths
- oblique shock prediction
- expansion wave prediction
- friction and heat transfer in pipe flow of compressible gases

a) Review of compressible flow

It can be proven that the speed of sound in a perfect gas (i.e. a gas in which temperature does not affect c_p and which is far from saturation) is:

$$a = \sqrt{\gamma RT}$$

The Mach number is the dimensionless expression of how close to sound speed a flow is at a particular velocity, c:

$$M = \frac{c}{a}$$

Using the perfect gas equation and considering the change between two points which results in a small change of density, pressure and temperature:

$$\frac{p}{\rho T} = \frac{p + dp}{(\rho + d\rho)(T + dT)} \rightarrow \frac{dp}{p} + \frac{d\rho}{\rho} + \frac{dT}{T} = 0$$

The differential equation is useful for the characteristics of the gas. If we combine this with the Euler equation, which is for inviscid flow relating pressure and velocity, and expressing in terms of Mach number:

$$\frac{dp}{p} = -\frac{\rho V^2}{p}\frac{dV}{V} \rightarrow \frac{dT}{T} = -(\gamma - 1)M^2\frac{dV}{V}$$

Combining the Euler and gas law equations, an equation relating the change of density and velocity at a particular Mach number is developed:

$$\frac{d\rho/\rho}{dV/V} = -M^2$$

This equation shows the effect of the Mach number of a flow on fractional changes of density and velocity on each other. If M = 0.1, M^2 = 0.01, therefore:

$$\frac{d\rho}{\rho} = -0.01\frac{dV}{V}$$

the density change corresponding to a 1% change in velocity, will be 1/100th of the velocity change ratio, i.e. 0.01% in this case – which is negligibly small. Therefore the compressibility of the flow can be ignored for quite large velocity variation at this Mach number (provided the Mach number doesn't change by too much of course).

Similarly if M = 0.33, M^2 = 0.11, and a 1% change in velocity will lead to a 0.11% change in density – which is approaching being significant, but is still reasonably low and compressibility can be ignored in most cases. At M = 0.4, M^2 = 0.16, at M = 0.5, M^2 = 0.25, and at these levels the density changes are sizeable and compressibility must be considered. The cases considered in this section have mainly to do with Mach numbers around 1 and above, but in the later sections it becomes important again as the Mach number increases from low values in long pipes with roughness or heat addition.

The idea of isentropic compressible flow was introduced in Thermofluids 2. If the flow is considered to be isentropic, it means no heat transfer (relatively easy to achieve by quick changes in a short space with little exposed surface to external) and reversible (i.e. no friction – increasingly hard to achieve due to effects of viscosity). This assumption was safely used to consider what happens up to the point where the flow reaches the speed of sound, which is a good first approach to subsonic compressible flow. The formulae are available in the Thermofluids 2 formula sheet. In these cases, because of the isentropic condition, the stagnation temperature remains constant (due to adiabatic condition) and the stagnation pressure remains constant (due to the 'no' loss by viscosity condition).

Mach waves represent the front of the pressure waves expanding from supersonic flow interactions where there is not a strong shock wave.

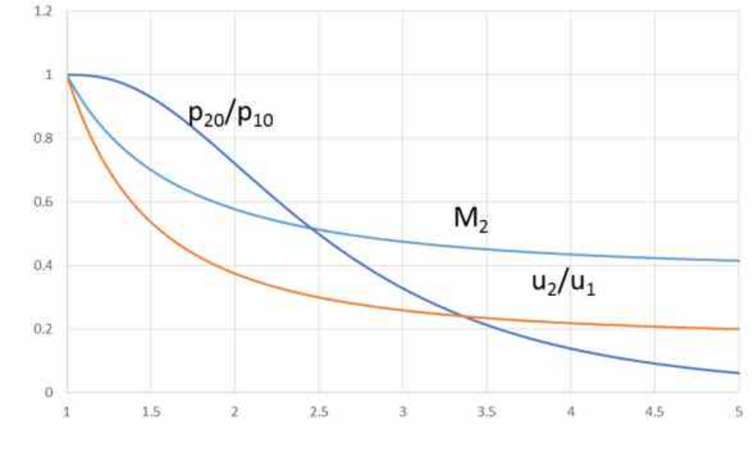
Flow can be accelerated to Mach 1 by a convergent-divergent nozzle, where M = 1 is achieved at the smallest section and M > 1 in the divergent section. If this flow interacts with bodies in the flow, the supersonic flow can produce a normal shock (by 2nd law consideration, entropy must increase). The shock abruptly drops velocity and increases the static pressure, it is called a <u>strong</u> shock. The isentropic condition is no longer valid – although adiabatic is reasonable, reversible is not reasonable due to the significant friction effects in the shock. Stagnation temperature does not change across the shock (adiabatic) but stagnation pressure changes across the shock, by friction. Change in entropy across the shock is the starting point:

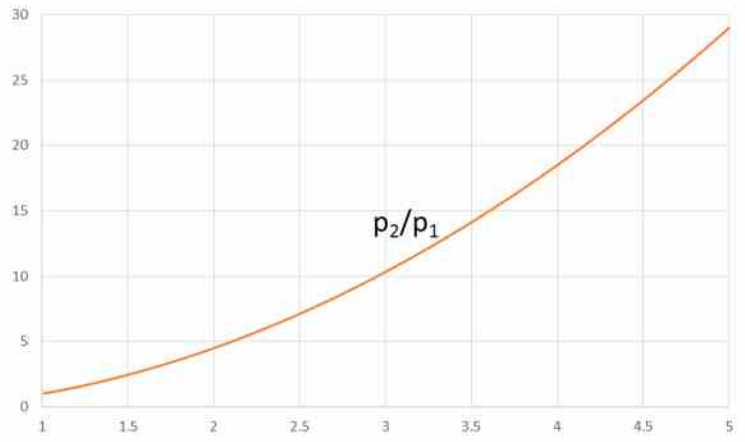
$$s_2 - s_1 = \cancel{c_p ln\frac{T_{02}}{T_{01}}} - Rln\frac{p_{02}}{p_{01}}$$

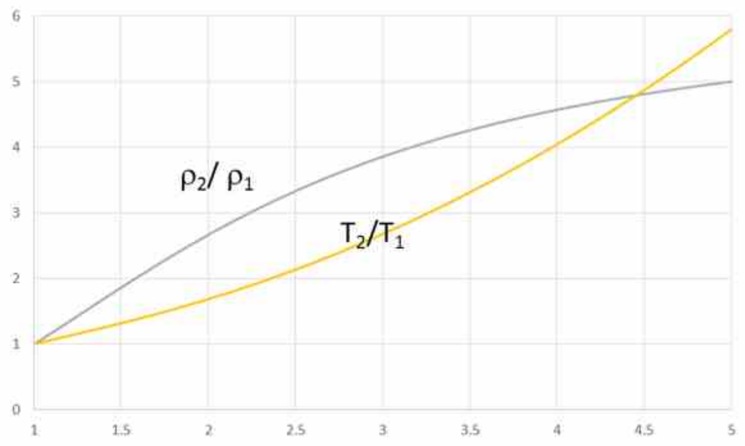
because it shows that entropy change is related only to stagnation pressure because stagnation temperature does not change; since the entropy must increase to satisfy the Second Law of Thermodynamics, the stagnation pressure must decrease (negative in front of ln pressure term inverts it, $p_{01} > p_{02}$).

There are a series of formulae for shock waves and critical flow (flow reaching sonic velocity) equations in the TF2 formula sheet. If there is supersonic flow which approaches a convergence, it will shock and produces a subsonic flow. If there is a subsonic flow which approaches a convergence it will accelerate until it reaches sonic velocity and no more, and no further reduction in downstream pressure or increase in upstream pressure can make the mass flow rate any higher. A moving normal shock (e.g. due to a sudden expansion or explosion) can be considered as a stationary normal shock by applying a relative velocity to the shock wave.

The formulae for the normal shock can be summarised in graphical form as illustrated in the following graphs. The horizontal axis is the Mach number ingoing, and the other data on the vertical axis are ratios.







As can be deduced from the graphs for normal shock:

(a) Stagnation pressure ratio decreases with M_1 and is <1

(b) M_2 decreases with increasing M_1, and is <1

(c) Velocity ratio decreases with M_1, and is <1

(d) Static pressure ratio increases with M_1, and is >1

(e) Density ratio increases with M_1 and is >1

(f) Temperature ratio increases with M_1 and is >1

b) Formulae for isentropic, weak shock, compressible flow

The formulae met had to do with comparing stagnation conditions to static conditions for isentropic conditions, for weak Mach waves where the pressure change is negligible:

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2; \ \frac{p_0}{p} = \frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma-1)}$$

where subscript '0' indicates total or stagnation condition, considering the flow being brought isentropically to rest; and combining these equations with continuity equation for flow passage normal-to-flow area the mass flow rate of the gas can be obtained:

$$\frac{\dot{m}\sqrt{c_p T_0}}{A_n p_0} = \frac{\gamma}{\sqrt{\gamma - 1}} M \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}$$

Where A_n is the area normal to the flow. The term on the left is caled dimensionless mass, since the units of the terms other than mass flow rate in it are:

$$\frac{\sqrt{c_p[m^2 s^{-2} K^{-1}] T_0[K]}}{A_n[m^2] p_0[kg m^{-1} s^{-2}]} = \frac{1}{kg s^{-1}}$$

The effect of convergent and divergent passages on subsonic flows affects the acceleration of the fluid. In the case of convergent 1D flow, subsonic flows accelerate and reduce pressure and density, but supersonic flows decelerate and increase pressure and density; the reverse is the case for a divergent flow.

For supersonic flows, as M increases, mass flow rate per unit area decreases. Plotting the mass rate in kg/s for air for a 0.1 m^2 flow area for T_0 = 346K and p_0 = 250 kPa, the graph clearly shows the maximum at M = 1. This is the reason why flow rate reaches a maximum when the flow chokes and no further increase in flow rate is possible, despite the possibility of speed increase.

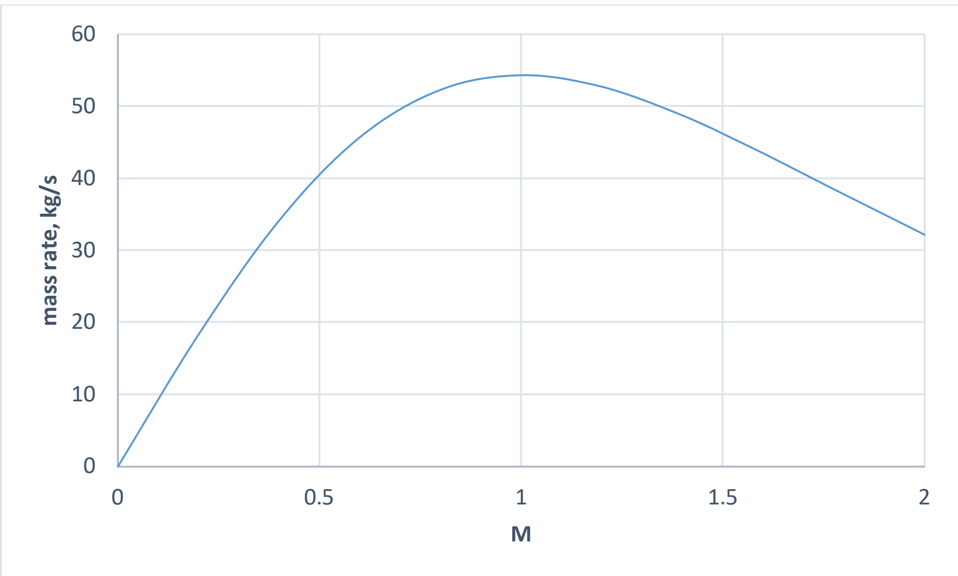


Figure 1 effect of increasing Mach number on flow rate

From this point of view we see that once M = 1 for a particular flow cross section area, nothing can be done to increase the mass flow rate of the fluid through the machine.

Example: Calculate the stagnation temperature and pressure for air at M = 0.8, p = 0.3 bar and T = 220 K. Calculate the mass flow rate given that this flow enters a compressor with annulus mean radius of 0.1 m and blade height 0.01 m

c) Formulae for strong shock waves

In the case of strong shock waves, the irreversibility caused by viscous friction has to be taken into account and a further set of equations for the general shock wave were presented:

$$\frac{T_2}{T_1} = [2 + (\gamma - 1)M_1^2]\frac{2\gamma M_1^2 - (\gamma - 1)}{(\gamma + 1)^2 M_1^2}; \frac{p_2}{p_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}$$

$$\frac{\rho_2}{\rho_1} = \frac{v_1}{v_2} = \frac{(\gamma + 1)M_1^2}{(\gamma - 1)M_1^2 + 2}; M_2^2 = \frac{(\gamma - 1)M_1^2 + 2}{2\gamma M_1^2 - (\gamma - 1)}$$

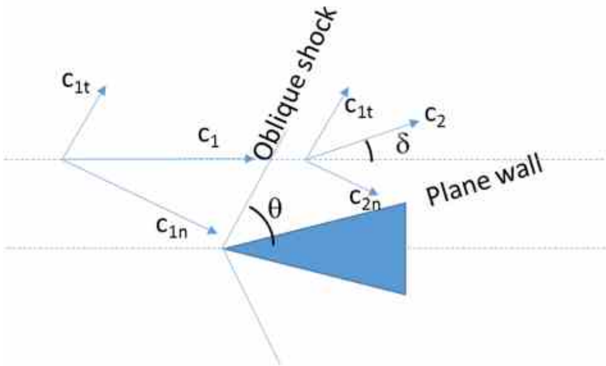
These have particular application when considering practical compressible flows where there is significant strength to the shock, as in supersonic flows, and when there is friction flow or heat addition to the flow.

Introductory section questions

1. Steam flows into the low pressure turbine of a steam power plant at 250 m/s at 20 bar and 400°C. The speed of sound in steam at these conditions is 623 m/s. Calculate the Mach number of this flow and state whether compressibility is likely to affect the flow. Derive the stagnation enthalpy for this condition if stagnation is achieved isentropically.
2. Calculate the Mach number, and stagnation conditions, for air at 315 K and 180 kPa flowing at 250 m/s. What mass rate is possible through a nozzle of 0.1 m^2 aperture? Assume no vena-contractor.
3. Calculate the static temperature for a flow at Mach 2 which experiences a shock, when the static temperature before the shock 250 K and static pressure 0.4 bar.
4. Describe why weak compressibility effects can be considered as isentropic, but flow through shock waves is adiabatic but not isentropic. Key: think about isentropic vs. adiabatic.

3. Oblique shock

When a supersonic flow is turned by a plane wall at an angle, δ, to the flow, an obliquely inclined plane shock will occur, and the flow can be considered as resolved parallel and perpendicular to the plane shock wave. The resultant velocity after the shock will proceed parallel to the wall. The oblique shock is then managed as a normal shock perpendicular to the shock wave, and the flow is unaffected parallel to the oblique shock wave, i.e. $c_{1t} = c_{2t}$ where t indicates tangential to the shock wave. The angle of turning of the flow is equal to the angle of the wall, δ, and the shock wave is at angle θ.



The formulae are derived in the same way as for normal shock but using $M_1\sin\theta$ instead of M_1. To get the relation between turning angle, δ, wave angle, θ, and incident Mach number:

$$\tan\theta = \frac{c_{1n}}{c_{1t}}; \tan(\theta - \delta) = \frac{c_{2n}}{c_{2t}}$$

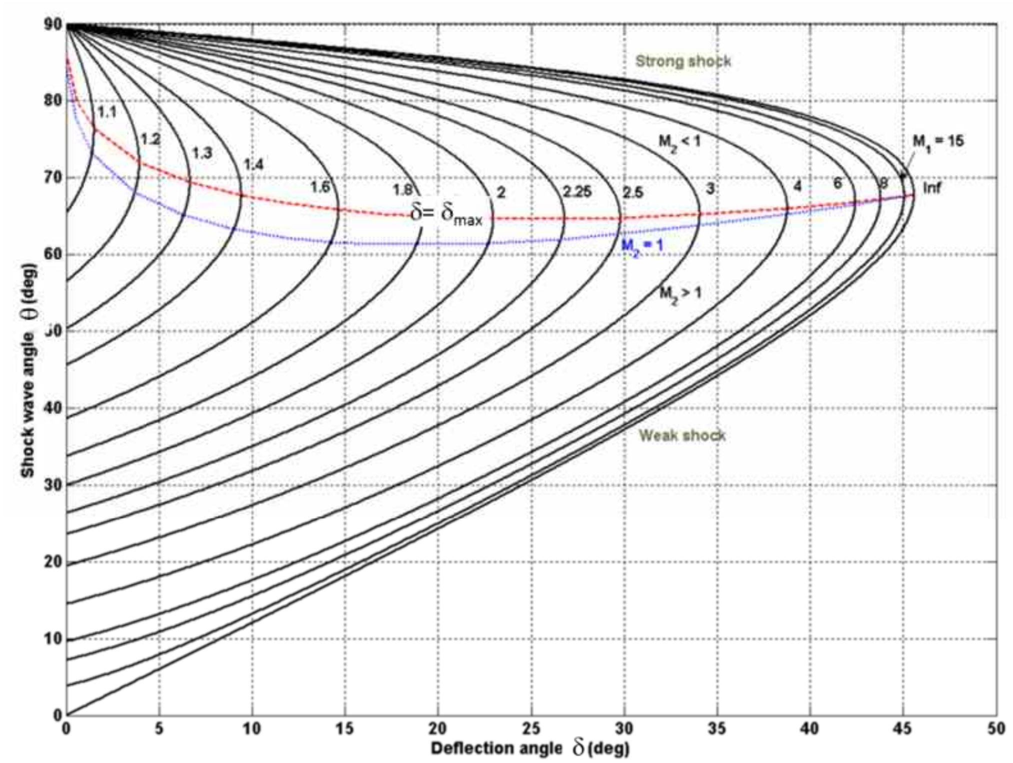
and

$$\frac{\tan(\theta - \delta)}{\tan\theta} = \frac{c_{2n}}{c_{1n}} = \frac{2 + (\gamma - 1)M_1^2 sin^2\theta}{(\gamma + 1)M_1^2 sin^2\theta}$$

Therefore

$$\frac{\tan\theta - \tan\delta}{1 + \tan\theta\tan\delta} = \frac{c_{2n}}{c_{1n}}\tan\theta \text{ and then } \tan\delta = \frac{2\cot\theta(M_1^2 sin^2\theta - 1)}{2 + M_1^2(\gamma + \cos 2\theta)}$$

This is easy to solve if M_1 and θ are given, but usually we have the wall angle, δ, not the wave angle. The formula is best represented in a chart as shown in the figure below.



Creative Commons – oblique shock chart, γ=1.4 https://en.wikipedia.org/wiki/Oblique_shock#/media/File:ObliqueShockAngleRelation.png

To use the chart:

- There are two solutions of θ given M_1 and δ
- Usually, experimental evidence shows that the 'weak shock' solution is obtained, by which the Mach number of the flow is reduced, but still supersonic
- Occasionally the 'strong shock' can occur which results in subsonic flow after the shock

It is worth noticing some of the features of the chart. At 0° deflection angle, δ, one of the two solutions for any particular entering Mach number show the 90° normal shock characterised in the formulae from Thermofluids 2 and the new graphical illustrations given here above. The other one is the weak shock solution for a small disturbance on the wall and is prescribed by the fact that for a normal shock to occur the entering Mach number must be greater than 1. In this case, the entering Mach number is perpendicular to the shock wave and is $M_1 \sin\theta >= 1$. For example, for entering Mach number of 2 at zero degree deflection:

$$M_1 \sin\theta = 1; 2\sin\theta = 1$$

Therefore θ for that case is 30°.

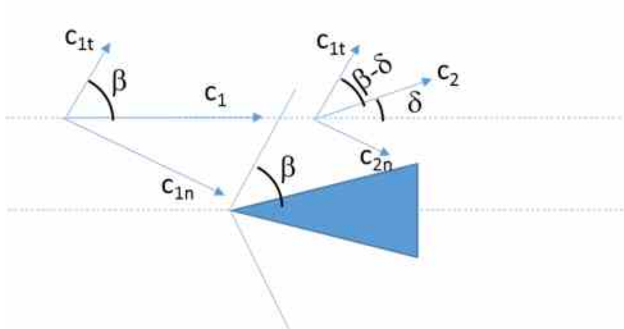
Finally, there is a maximum angle through which the flow can be converged, indicated by the apex of the curves on the right of each Mach number. A flow at Mach 2 cannot be turned more than 23°. Doing so will result in a detachment of the supersonic flow, and a normal shock will develop upstream of the point of impingement with a subsonic section in from of the point.

Example: Oblique shock in air

Flow approaches a plane wall at an angle of 20° to the incoming flow, which has a Mach number of 2. Find the angle of the weak and strong shocks and hence the resulting Mach number, and the temperature after the shock.

Ans: from the chart, the weak shock is at 53° and the strong shock is at 75°. The corresponding Mach numbers are by trigonometry. We need to know the Mach number of the speed normal to the shock wave before and after the shock wave. We use the normal shock relationship formulae (either calculate it or use a spreadsheet – provided with formlae in already in Moodle). Calculate the normal ingoing Mach number – i.e. the component normal to the shock wave. Then use the formulae to calculate the outgoing Mach number component normal to shock wave. Then use the trigonometry of the outgoing velocity triangle to convert the M_{2n} component to M_2.

$M_{n1} = M_1 \sin\beta$, therefore $M_{n1,weak} = 1.6$, $M_{n1,strong} = 1.92$



From calculation (or quicker a spreadsheet calculation):

$M_{2n,weak} = 0.668$; $M_{2n,strong} = 0.592$

And $M_{2n} = M_2 \sin(\beta\text{-}\delta)$ therefore $M_2 = M_{2n} / \sin(\beta\text{-}\delta)$

$M_{2,weak} = 0.668/\sin(53\text{-}20) = 1.84$,

$M_{2,strong} = 0.592/\sin(74\text{-}20) = 0.73$

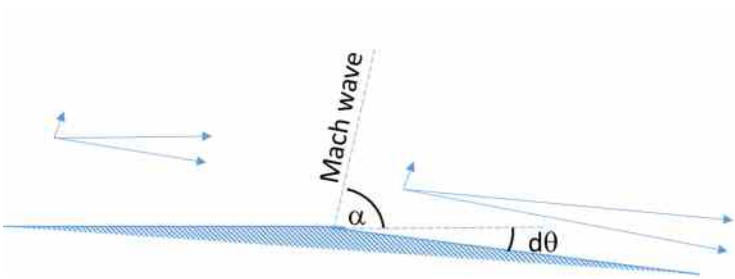
Outgoing temperature is now easy.

Temperature ratio: 1.388 weak, 1.624 strong

Weak, $T_2 = 333$ K, strong, $T_2 = 390$ K

d) Supersonic flow expansion – Prandtl-Meyer fan

Prandtl-Meyer flow occurs when supersonic flow turns to expand the flow area, i.e. flow around a convex corner, rather than the concave corner just considered for oblique shocks. By previous understanding that a shock wave must not produce an increase in entropy, the previous analysis for normal shock showed that an expansive shock is not possible since it would result in a decrease of entropy.



The flow on the expanded side is parallel to the expanded wall. The flow is isentropic, occurring in infinitesimal (sound strength) waves. These can be integrated together, and a straightforward derivation (not necessary to show here) produces:

$$\frac{dM}{M} = \left[1 + \frac{\gamma - 1}{2} M^2\right] \frac{-d\theta}{\sqrt{M^2 - 1}}$$

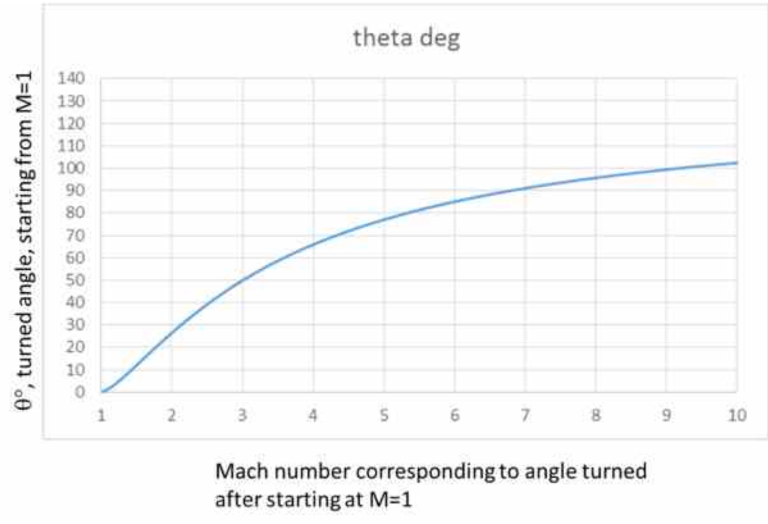
Small changes in velocity, pressure, density and Mach number are all proportional to the change in angle of the flow. Assuming that any turn of a supersonic flow began with a sound speed flow at zero degrees, the formula can be integrated:

$$\int_{M_1}^{M_2} \frac{\sqrt{M^2-1}}{1+\frac{\gamma-1}{2}M^2}\frac{dM}{M} = \int_{\theta_1}^{\theta_2} -d\theta$$

Integrate between M=1 and M to give:

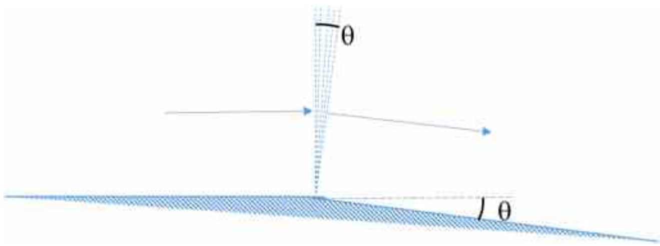
$$\theta = \sqrt{\frac{\gamma+1}{\gamma-1}}tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)} - tan^{-1}\sqrt{M^2-1}$$

The formula can be plotted and used in graphical form as follows.



To work out the Mach number change as a flow goes around a Prandtl-Meyer expansion fan, use the graph to find the Mach number at exit from a turn, which is the result of the integrated formula expressing the large number of infinitesimally small Mach waves at the corner.

The starting Mach number, say for example, 2, is the result of a turn starting from Mach 1, the angle being seen here to be approximately 25°. If this then turns by a further 10°, the exit Mach number will be at the point where the integral reaches the angle θ of 35°, i.e. approximately Mach 2.4. The expansion fan will appear under optical tests as shown approximately.



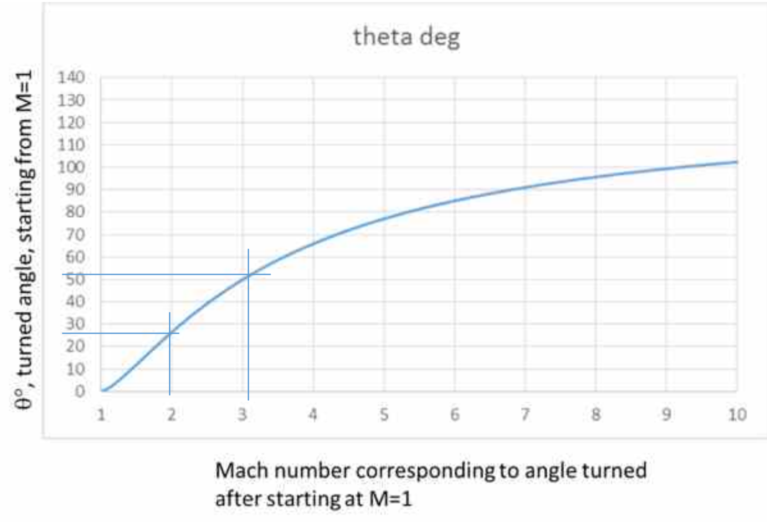
Example: a flow of air with γ = 1.4 at 280 K is expanded at a sharp corner by 25 degrees. The initial Mach number is 2; what is the final Mach number?

Ans: use the formula

$$\theta = \sqrt{\frac{\gamma+1}{\gamma-1}}tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)} - tan^{-1}\sqrt{M^2-1}$$

$$\theta = \sqrt{\frac{2.4}{0.4}} tan^{-1}\sqrt{\frac{0.4}{2.4}(2^2-1)} - tan^{-1}\sqrt{2^2-1} = 0.461\ radians\ or\ 26.4°$$

this is the starting angle of the flow, i.e. the angle that the flow approaches at, regardless of what it's actual value is, for the consideration of the calculation with Mach 2 entry the flow has already accelerated around a corner of 26.4° to get to the angle where it is. So this is the zero angle, the starting angle of the velocity vector turn. We now want the end angle to be 25° more than this, so 51.4°, and then we can solve for Mach number. Unfortunately solving the formula is quite hard and would be a trial and error method. So it is better to have a lookup table in a spreadsheet, or a graph that you can read from – both provided. Using the graph above:



the start point is at Mach 2, the angle is 26.4° as calculated, then find the end angle, 25° more, i.e. 51.4°, and the corresponding Mach number is approximately 3.1 to the resolution of the chart, and the spreadsheet confirms this value.

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e) Friction and heat transfer effects on flow in a tube

It can be shown that the relationship between flow in a pipe and the wall friction is:

$$4C_f\left(\frac{L^*}{D}\right) = \frac{1}{\gamma}\left[\frac{1-M^2}{M^2}\right] + \frac{\gamma+1}{2\gamma} ln \frac{(\gamma+1)M^2}{2+(\gamma-1)M^2}$$

For a given inlet Mach number, M, there is a length by which the flow will become critical (i.e. M = 1), for a given friction coefficient, C_f, which is the ratio of wall shear to dynamic pressure. It means that there is a length by which the flow will become sonic – a problem for long distance pipe flow, e.g. natural gas delivery.

Likewise to frictional flow in a tube, heat addition will lead to reaching critical condition:

$$\frac{q^*}{C_p T_0} = \frac{T_0^*}{T^*}\frac{T}{T_0}\frac{T^*}{T} - 1 = \frac{1}{1+\frac{\gamma-1}{2}M^2}\left[\frac{1+\gamma M^2}{M(1+\gamma)}\right]^2\left(1+\frac{\gamma-1}{2}\right)$$

Where T_0 is stagnation temperature at starting position, T_0* is the critical condition stagnation temperature, T is the static temperature initially and T* is the critical static temperature. This is called Rayleigh flow.

The following chart can be used to determine the pipe length from a starting condition until choked flow.

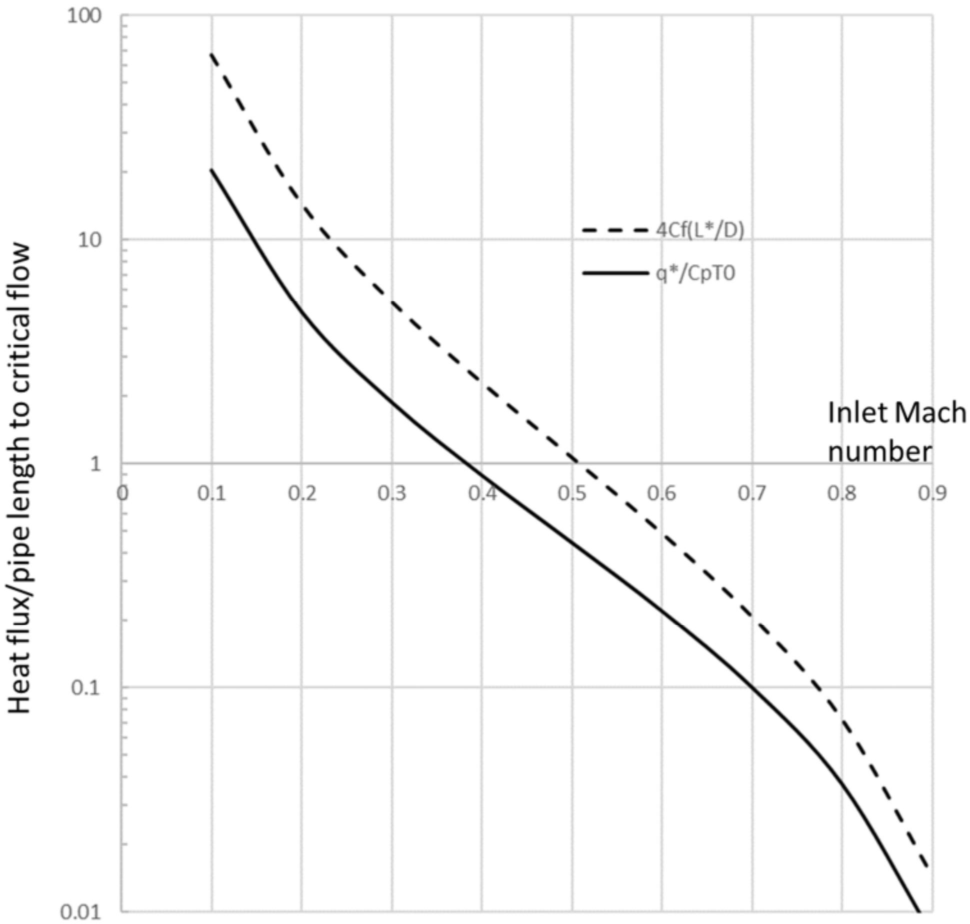

Figure 2 Effect of pipe wall friction factor (c_f) and heat transfer for heat flux to reach critical (q) for starting Mach number.*

Compressible flow questions

1. Sketch the velocity vectors that result from an oblique shock and state what happens to the vectors parallel and perpendicular to the shock wave.
2. Flow approaches a plane wall at an angle of 15° to the incoming flow, which has a Mach number of 3. Find the angle of the weak and strong shocks and hence the resulting Mach number. [Ans: 2.61, 0.42]
3. Describe the characteristics of Prandtl-Meyer expansion fans and why they occur in that way rather than an abrupt shock.
4. A flow of air at Mach 3 expands around a corner of angle 30°. Use the Prandtl-Meyer chart to determine the exit Mach number. [Ans: Mach ~5.3]
5. A pipe has a wall friction factor of 0.0015 and a diameter of 0.3 m. Use the chart for pipe flow with friction to find the length required to result in choked flow. [Ans: 525 m]
6. Flow of air in a pipe has stagnation temperature of 400 K and specific heat capacity at constant pressure of 1.005 kJ/kgK, and flows at Mach 0.4. Use the chart for critical specific heat addition to find the specific heat necessary to cause choking in the flow. [Ans: 362 kJ/kg]