Department of Mechanical, Materials & Manufacturing Engineering

DYNAMICS (VIBRATION)

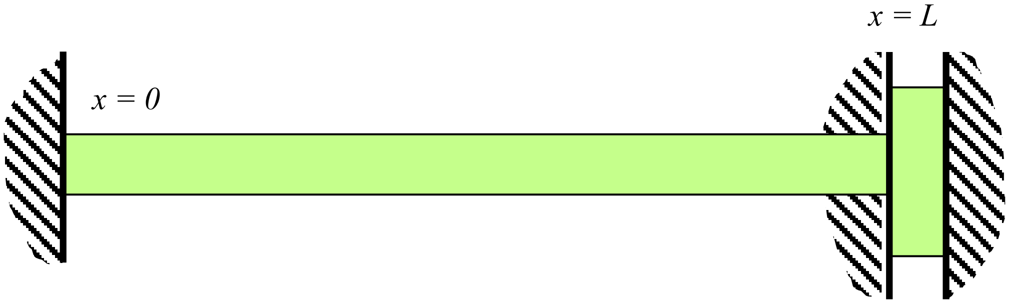
SHEET 3 : SHAFT WHIRL & BEAM VIBRATIONS

Take E = 207 GN/m^2, ρ = 7800 kg/m^3 for questions 2-5

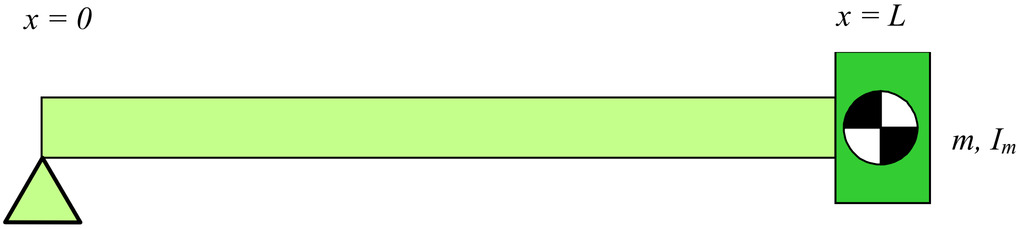
1. A uniform beam of length, L, is clamped at one end, x = 0, and pinned at the other, x=L, as shown in the figures below. For each beam find:
    a. Using boundary conditions determine the generalized matrix, $[Z]\{C\} = \{0\}$, that could be used to solve for undamped natural frequencies and mode shapes of the beam shown. You do not need to solve for the constants {C}, only show the generalized matrix and terms contained in [Z].
    b. Briefly sketch the resulting displacement of the beam for the first 3 mode shapes that you would expect.

i)

ii) The slider at x=L allows vertical motion, but not rotation.



iii) There is a point mass at the right hand side of the beam of mass m. Assume it can rotate and has rotational inertia, I_m.



iv) For the beam shown in iii) assume the mass on the right hand side of the beam does not rotate (i.e. I_m=0).

2. A 25 mm diameter shaft, 1.5 m long, is held by two roller bearings at one end (giving a “clamped” boundary condition) and by a self-aligning ball bearing at the other end (giving a “pinned” boundary condition). Find the critical speeds of the shaft.

2108, 6829, 14250, etc. (rev/min)

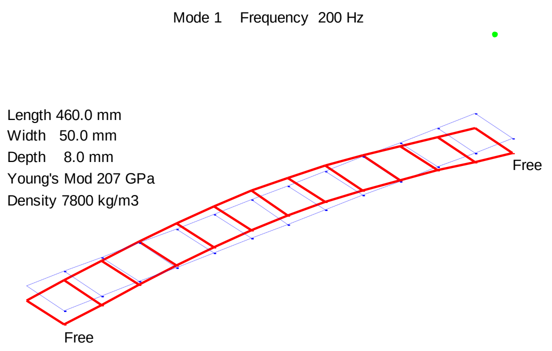
3. Derive the frequency equation for flexural vibration of a uniform beam that is free at both ends.

Using the values of the roots given in the handout, calculate the two lowest natural frequencies for a free-free beam of length 460 mm and a rectangular cross-section 50 mm by 8 mm.

Repeat the calculations using the Matlab script febeam.m, which you can download from the module’s WebCT site.

$$\lambda^2\left(\cos\lambda L\ \cosh\lambda L - 1\right) = 0\ ;$$

200 Hz, 551.8 Hz



4. A beam of length, L, is clamped at the end $x = L$. At $x = 0$, a spring of stiffness, k, is pin-jointed to the beam and provides lateral restraint. Find the frequency equation in determinant form.

$$\begin{vmatrix} 0 & -\lambda^2 & 0 & \lambda^2 \\ -EI\,\lambda^3 & k & EI\,\lambda^3 & k \\ \sin\lambda L & \cos\lambda L & \sinh\lambda L & \cosh\lambda L \\ \lambda\cos\lambda L & -\lambda\sin\lambda L & \lambda\cosh\lambda L & \lambda\sinh\lambda L \end{vmatrix} = 0$$

5. Derive the frequency equation for flexural vibration of a uniform beam that is pinned at one end and free at the other. Derive the expression for the mode shape function.

$$\lambda\left(\tan\lambda L - \tanh\lambda L\right) = 0\ ; \qquad Y_r(x) = \sin\lambda_r x + \frac{\sin\lambda_r L}{\sinh\lambda_r L}\sinh\lambda_r x$$