

STRUCTURAL VIBRATION 1 SHEET 5

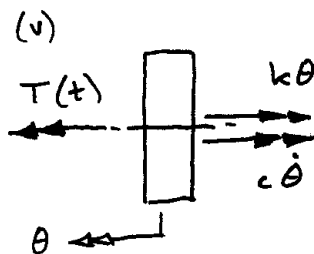
Q1 (iii) Equation of motion was derived in the lectures.

$$I \ddot{\theta} + k \theta = T$$

Put $T = T e^{i\omega t}$ and $\theta = \theta^* e^{i\omega t}$

$$\therefore \theta^* = \frac{T}{k - I\omega^2} = 0.0916 \text{ rad}$$

Note that θ^* is real since there is no damping.

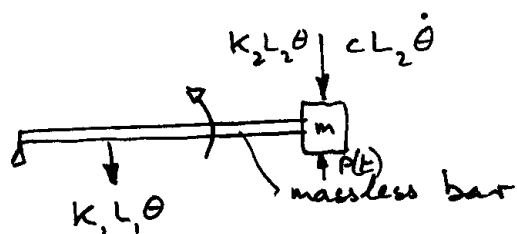


$$\leftarrow \ddot{\theta}: T - k\theta - c\dot{\theta} = I\ddot{\theta}$$

$$\text{or } I\ddot{\theta} + c\dot{\theta} + k\theta = T(t)$$

With the same substitutions, $\theta^* = \frac{T}{(k - I\omega^2) + i\omega c}$

Q2



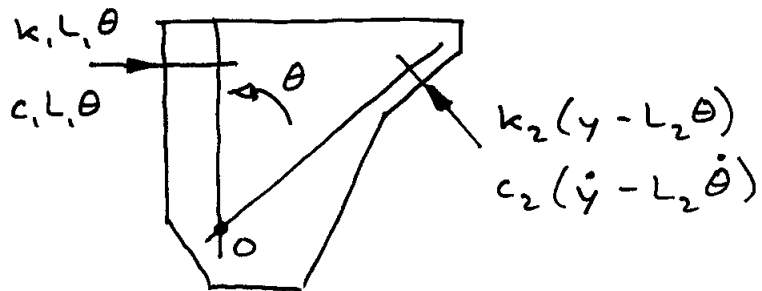
Equation of motion was derived in the lectures

$$mL_2^2 \ddot{\theta} + cL_2^2 \dot{\theta} + (k_1L_1^2 + k_2L_2^2) \theta = L_2 p(t)$$

Put $p(t) = P e^{i\omega t}$ and $\theta(t) = \theta^* e^{i\omega t}$

$$\text{Hence } H(\omega) = \frac{\theta^*}{P} = \frac{L_2}{(k_1L_1^2 + k_2L_2^2 - mL_2^2\omega^2) + i\omega cL_2^2}$$

Q3.



Equation of motion:

$$\ddot{\theta} \left[k_2 (y - L_2 \theta) + c_2 (\dot{y} - L_2 \dot{\theta}) \right] L_2 - [k_1 L_1 \theta + c_1 L_1 \dot{\theta}] L_1 = I_0 \ddot{\theta}$$

or

$$I_0 \ddot{\theta} + (c_1 L_1^2 + c_2 L_2^2) \dot{\theta} + (k_1 L_1^2 + k_2 L_2^2) \theta = k_2 L_2 y(t) + c_2 L_2 \dot{y}(t)$$

$$\text{Put } y(t) = Y e^{i\omega t} \text{ and } \theta(t) = \Theta^* e^{i\omega t}$$