Problems on Harmonic Excitations

Problem4

Objective: to train the students on quick and simple isolation design problems.

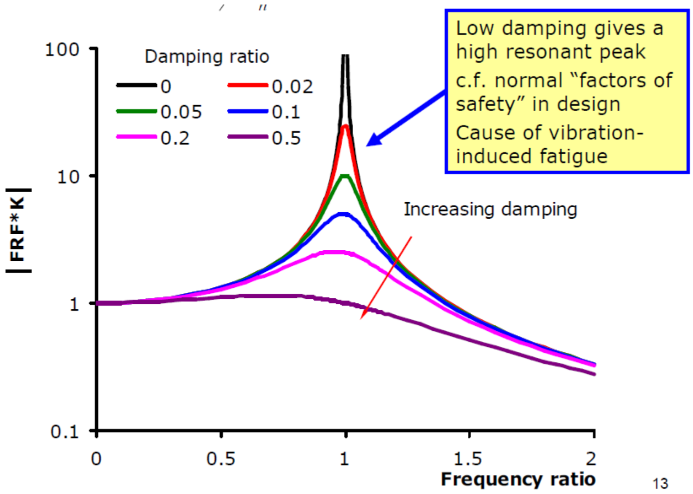
Level of difficulty: lower thanmedium

A machine of mass 250 kg has an excitation given by $850 \sin 100\pi t$ N, is to be mounted on the floor of a workshop using three mounting elements each of which has a stiffness k and a damping c. Obtain values for the stiffness and damping of the three suspension elements such that the maximum vertical displacement of the engine doesn't exceed 25 mm for the high frequency range.

Solution of Problem 1

Since

$$\frac{X}{F/k} = \frac{1}{\sqrt{(1-r)^2 + (2\gamma r)^2}}$$



Differentiating w.r.t the frequency ratio r we obtain the value of r that makes the amplitude ratio has its maximum value.

$$r = \sqrt{1 - 2\gamma^2}$$

This gives the maximum value of

$$\frac{X}{F/k} = \frac{1}{2\gamma\sqrt{1-2\gamma^2}}$$

From the family of curves shown in the notes and the solved example a lower value of the damping ratio has to be used to attenuate vibrations, now we take trial values for the damping ratio. Let's take $\gamma = 0.05$ then:

$$\frac{0.0025}{850/k} = \frac{1}{2 \times 0.05\sqrt{1 - 2 \times 0.05^2}}$$

Solving for k

$$k = 2176kN$$

This is the total stiffness and since the stiffness elements are in parallel, the stiffness for on element will be: $k_i = \frac{2176}{3} = 725kN$.

For damping calculations

$$c = \gamma\sqrt{2km}$$

$$c = 0.05\sqrt{2 \times 2176000 \times 250} = 1649.25\frac{Ns}{m}$$

$$c_i = \frac{1649.25}{3} = 549.75\frac{Ns}{m}$$

Problem5

Objective: To introduce the student to simple but serious design problems that is vibration based

Level of difficulty: above medium

A compressor rotor has a mass of 60 kg and mounted on a shaft that has a stiffness of 14000 kN, the rotor has an eccentricity of 1 mm and the clearance between the rotor and the housing 2 mm. The internal damping ratio of the rotor shaft system is 0.045 and the running speed of 6200 rev/min.

1- Determine whether the rotor will rub with the housing at the operating and critical speed or not.
2- If the rotor is to rub with the housing discuss the possible solutions to the rubbing problem.
3- Recommend a solution that will not require a modification of the rotor shaft system design.

Solution of problem 2

We first calculate the steady state amplitude and check whether it is less that the clearance or not at the operating speed 6200 rev/min

$$X = \frac{ar^2}{\sqrt{(1-r)^2 + (2\gamma r)^2}}$$

Since:

$r = \omega/\omega_n$ $\qquad \omega_n = \sqrt{\frac{k}{m}}$ $\qquad r = \frac{2\pi 6200/60}{\sqrt{\frac{1400000}{60}}} = 1.344$

$$X = \frac{0.001\ .\ 1.344^2}{\sqrt{(1-1.344)^2 + (2\ .0.045\ \ .1.344)^2}} = 2.023\ mm$$

$$X > clearance$$

What can make the situation worse when the rotor rotates at its **critical speed** which is equal to ω_n

In this case $r = 1$.

$$X = \frac{a}{\sqrt{2\gamma}} = \frac{0.001}{\sqrt{2.\ 0.045}} = 3.333\ mm$$

The possible solutions

The natural frequency has to be changed to yield a frequency ratio that will produce a smaller deflection than 2mm. this can be done by changing the rotor mass or the shaft stiffness.

$$X = \frac{ar^2}{\sqrt{(1-r)^2 + 2\gamma r^2}}$$

$$0.002 = \frac{0.001r^2}{\sqrt{(1-r)^2 + 2.0.045.r^2}}$$

Solving for *r* $r = 0.82$ $r = 1.4$

We chose $r = 1.3362$, why?

If the mass is to be changed

$$r = 1.4 = \frac{2\pi 6200/60}{\sqrt{\frac{1400000}{m}}} = \frac{649.3}{\sqrt{\frac{1400000}{m}}}$$

Solving for the mass m: $m = 65kg$

If the stiffness is to be changed:

$$1.4 = \frac{2\pi 6200/60}{\sqrt{\frac{k}{60}}} = \frac{649.3}{\sqrt{\frac{k}{60}}}$$

$$k = 1290\ kN$$

If it is not possible or practical to change *m* or *k* then **a damping mechanism** has to be added to the machine.

Problem 6

Objective: to train the students on simple and practical design problems with vibration in consideration.

Level of difficulty: medium

A motor of mass 160 kg is installed at the end of a metal beam that has $E = 210 \times 10^9 N/m^2$. The motor has unbalance of 0.55 kg and located at 20 cm from its shaft centre, this unbalance generates a force$F = A\omega^2 \sin \omega t + \phi$. If the operating speeds range is from 500 to 1200 rev/min. Obtain a suitable value of the area moment of inertia of the beam so that the maximum amplitude of vibration will not exceed 1 mm. Assume a damping ratio 0.1.

Hint notice that the force amplitude is a function of ω^2, this means that you have to modify the expression used for the steady state response amplitude.

Solution of problem 3

$$X = \frac{F}{\sqrt{(K - M\omega^2)^2 + C^2\omega^2}}$$

The amplitude of the unbalance force used in the previous equation is: $F = m_u e \omega^2$

Writing the amplitude equation in terms of the frequency ratio:

$$\frac{MX}{m_u e} = \frac{r^2}{\sqrt{(1-r)^2 + 2\gamma r^2}}$$

Substituting in the previous equation:

$$\frac{150 \times 0.001}{0.55 \times 0.20} = \frac{r^2}{\sqrt{(1-r)^2 + 2 \times 0.1 \times r^2}}$$

Solving for r $r_1 = 0.78$ $r_2 = 1.7$

The two values of r produce the same amplitude ratio $\frac{MX}{m_u e}$ both of them will produce different values for the natural frequency.

Remember $r = \frac{\omega}{\omega_n}$

$$r_1 = \frac{\frac{2\pi 500}{60}}{\omega_{n1}} < 0.78$$

$$r_2 = \frac{\frac{2\pi 500}{60}}{\omega_{n2}} < 1.70$$

For the cantilever beam :

$$\omega_n = \sqrt{\frac{3EI}{mL^3}}$$

ω_{n1} Produces $I = 6.07 \times 10^{-6}\ m^4$

ω_{n2} Produces $I = 2.23 \times 10^{-7}\ m^4$

Which one of them to choose, the answer depends on the situation, but for safety the larger *I* will make the beam withstand more bending stresses.

Problem 7

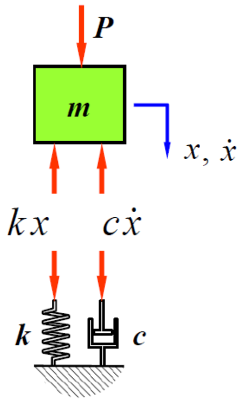
Objective: to train the students on solving simple isolation problems

Level of difficulty: above low

A machine of mass 125 kg is mounted on a support that has a spring of stiffness 900 kN/m and a damper of damping ratio of 0.2. The machine is acted upon by an unbalance that has amplitude of 400 N and rotating at a speed of 3000 rev/min. Determine the amount of force transmitted from the machine to the floor.

Solution of problem 4

From the free body diagram, the maximum force transmitted to the floor is:



$$|F_T| = \sqrt{(kX)^2 + C^2\omega^2X^2} = X\sqrt{k^2 + C^2\omega^2}$$

(1)

$$C = 2m\gamma\omega_n$$

To find the natural frequency we use the static deflection and the stiffness data given in the problem

$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{900000}{125}} = 26.83$ rad/s

$C = 2m\gamma\omega_n 2 \,.\, 125 \,.\, 0.2 \,.\, 26.83 = 1341.5$ N/s

$\omega = \frac{2\pi n}{60} = 2\pi \,.\, \frac{3000}{60} = 314.16$ rad/s

$$X = \frac{F}{\sqrt{(K - M\omega^2)^2 + C^2\omega^2}}$$

(2)

From equation 1 and 2

$$Ratio\ of\ the\ force\ transmitted = \frac{\sqrt{k^2 + C^2\omega^2}}{\sqrt{(K - M\omega^2)^2 + C^2\omega^2}}$$

$$Ratio\ of\ the\ force\ transmitted = \frac{\sqrt{(900000)^2 + (1341.5)^2\ 314.16^2}}{\sqrt{(90000 - 125\ .\ 314.16^2)^2 + 1341.5^2 314.16^2}}$$

$$Ratio\ of\ the\ force\ transmitted = 0.0811$$

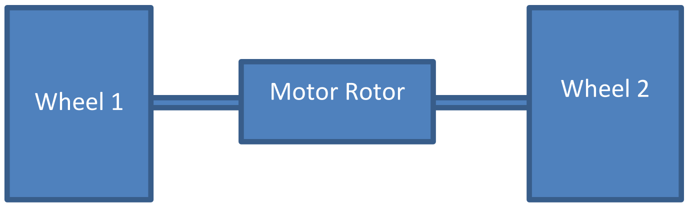
Amount of force transmitted=400x0.0811=32.44 N

Problem 8

Objective: to train the students on simple modelling of simple real industrial problems.

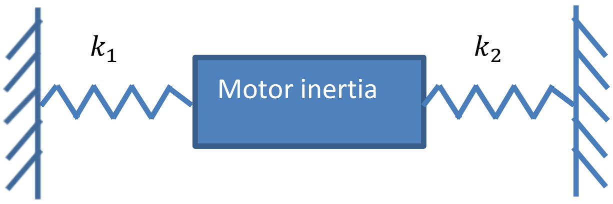
Level of difficulty: low but little tricky

An electric motor rotor has inertia of 0.025 kg.m is running at its normal speed. A suddenly applied electric harmonic introduced a harmonic torque output of frequency 80 Hz and amplitude of 190 N.m on the motor shaft. If the motor is carrying two heavy wheals one at each of its shaft ends determine the steady state response of the motor to the newly applied harmonic torque assuming that the motor shaft stiffness is 3600 N.m for each shaft side.



Solution of Problem 5

To solve this problem we have to build the physical model first. Since the motor shaft is flexible on both of its sides, the ends can be replaced by torsional spring elements. Since the two wheels are heavy the shaft ends can be replaced by a rigid support.



The equation of motion of the rotor can be easily written as:

$$J\ddot{\theta} + (k_1 + k_2)\theta = 190\cos 2\pi 80t$$

The steady state response for the undamped SDF system is given by:

$$\theta(t) = \frac{T}{((k_1 + k_2) - I\omega^2)}\cos 2\pi 80t$$

$$\theta(t) = \frac{190}{(3600 + 3600 - 0.025.(2\pi 80)^2)}\cos 2\pi 80t = 0.215^{\circ}\cos 502.65t$$