

Solutions

1. Calculate the equivalent resistance of the resistance network shown in Fig. Q.1, and hence calculate the current drawn from the power supply and the power dissipated in the network. [4]

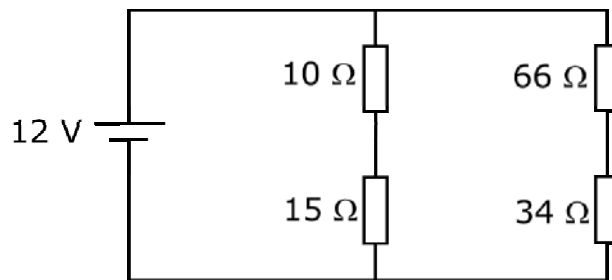


Fig. Q.1

$$1/R_{\text{eq}} = 1/(10+15) + 1/(66+34) = 1/25 + 1/100 = 0.05$$

$$\text{So } R_{\text{eq}} = 1/0.05 = 20 \Omega$$

$$I = V/R = 12/20 = 0.6 \text{ A}$$

$$\text{Power} = VI = 12 \times 0.6 = 7.2 \text{ W}$$

2. A load consisting of a 30Ω resistor in series with a $100 \mu\text{F}$ capacitor is connected across a 110 V 60 Hz single phase supply. Calculate the complex impedance of the load in Cartesian and polar form and hence calculate the magnitude and phase angle of the current. [6]

$$Z = R - jX_c = R - j/(2\pi fC) = 30 - j/(2\pi \times 60 \times 100 \times 10^{-6}) = 30 - j26.526 \Omega$$

$$= 40.04 \angle -41.48^\circ \Omega$$

$$I = V/Z = 110/(40.05 \angle -41.48^\circ) = 3.75 \angle 41.48^\circ \text{ A}$$

3. A squirrel-cage induction motor is stated by the manufacturer to have a rated power output of 2.5 kW at 950 rev min^{-1} when running from a 50 Hz supply

a) Calculate its torque at rated speed [2]

b) State the approximate speed at which the motor will run when it is providing no torque i.e. when it is running without load [1]

c) Calculate its speed (using the linear model) when driving a load requiring a torque of 20 Nm . [3]

$$\text{a) } T = P/\omega = 2500/(950 \times 2\pi/60) = 25.13 \text{ Nm}$$

$$\text{b) } 1000 \text{ rev min}^{-1}$$

$$\text{c) } N = 1000 - 20 \times (1000 - 950)/25.13 = 960.2 \text{ rev min}^{-1}$$

4. A strain gauge with a nominal value of 350Ω and a gauge factor of 2.2 is subjected to a tensile strain of 800 microstrain . By how much will its resistance change? [3]

$$\Delta R/R = G.F. \times \varepsilon \text{ so } \Delta R/ = R \times G.F. \times \varepsilon = 350 \times 2.2 \times 800 \times 10^{-6} = 0.616 \Omega$$

5. An inexperienced designer has created the circuit shown in Fig. Q.5. consisting of a NAND gate followed by an AND gate. It receives a series of pulses on input A and is intended to output pulses on output D only when B is low (digital 0).

(a) Draw a timing diagram showing the states of A, C and D the when input A is receiving a train of pulses and input B is low (digital 0).

[3]

(b) Draw a timing diagram showing the states of A, C and D the when input A is receiving a train of pulses and input B is high (digital 1), and hence explain briefly why the output is not as expected, including giving the name of this unexpected behaviour.

[5]

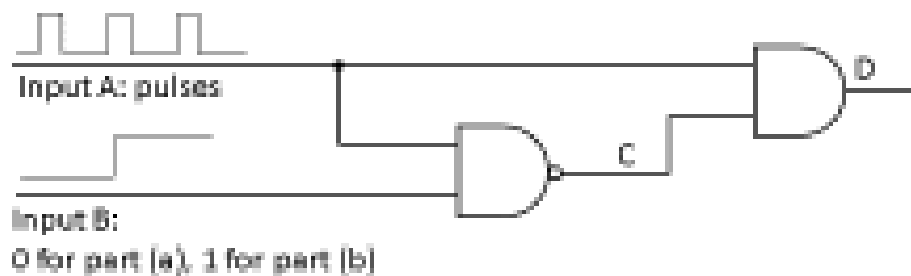
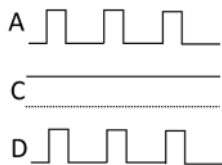
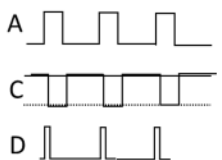


Fig. Q.5

a)



b)



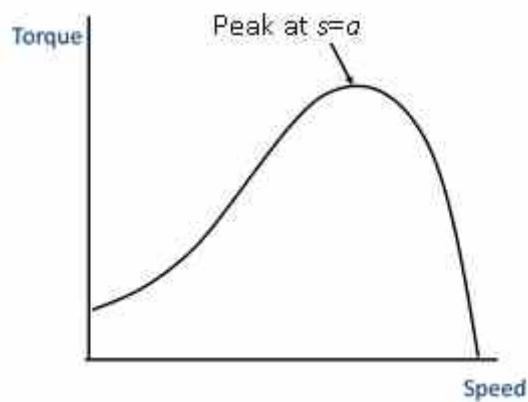
The input signal A will reach the AND gate shortly before the inverted signal C. For that fraction of a second, both signals will be high and the output from D will also be high. This situation is known as a race condition.

6. An analog-to-digital converter (ADC) is required to digitise a signal in the range 0-10 V with a resolution of 0.005 V. Determine how many bits of precision the ADC must have in order to achieve this resolution.

[3]

$$\ln(10/0.005)/\ln(2) = 10.96 \text{ so need 11 bits.}$$

7. a) An industrial load runs from a 415 V 50 Hz three-phase supply. It is star-connected and each phase of the load consists of a 50Ω resistor in series with a 200 mH inductor.
- Calculate the phase voltage (i.e. the line-to-neutral voltage). [2]
 - Calculate impedance of the each phase of the load in either Cartesian or polar form. [4]
 - Calculate the magnitude and phase of the line current. [4]
 - Calculate the power factor. [2]
 - Calculate the total power dissipated by the load. [3]
- (a) i) $415/\sqrt{3} = 239.6 \text{ V}$ [2]
- ii) $Z=R+jX_L=50 + j2\pi fL=50 + j2\pi \times 50 \times 200 \times 10^{-3} = 50 + j62.83 \Omega$
 $= 80.2 \angle 51.5^\circ$ [4]
- iii) $I = V/Z = 239.6/(80.2 \angle 51.5^\circ) = 2.98 \angle -51.5^\circ \text{ A}$ [4]
- iv) $\cos(-51.5^\circ) = 0.62$ [2]
- v) $P=3 |V_P| |I_P| \cos \phi = 3 \times 239.6 \times 2.98 \times 0.62 = 1331.4 \text{ W}$ [3]
- (b) A star-connected three-phase squirrel-cage induction motor with two pairs of poles per phase, running from the same supply as in (a), has a rotor resistance of 10Ω and a standstill rotor reactance of 50Ω referred to the stator windings.
- $a = R_r/X_r = 10/50 = 0.2$. Corresponds to the value of slip at peak torque; if loaded to this torque the motor will stall. [5]



- Calculate the torque when the motor is running at its rated speed of $1440 \text{ rev min}^{-1}$ using the theoretically-based model based on the motor's electrical parameters, and hence calculate its rated power output.

$$T = \frac{3p}{2\pi f} \times \frac{V^2 as}{X_R(a^2 + s^2)}, s = \frac{1500 - 1440}{1500} = 0.04$$

but note star conn. so $V = V_p = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6 \text{ V}$ (calculated earlier)

$$T = \frac{3 \times 2}{2\pi \times 50} \times \frac{239.6^2 \times 0.2 \times 0.04}{50(0.2^2 + 0.04^2)} = 4.216 \text{ Nm}$$

$$\text{Power} = T\omega = 4.216 \times 1440 \times \frac{2\pi}{60} = 635 \text{ W}$$

- (c) A loudspeaker has an impedance of magnitude 3Ω and is to be driven from a signal generator which is designed to be connected to a load with an impedance of magnitude 50Ω .

Calculate the turns ratio of the ideal transformer which will give the best match of the signal generator to the transformer, i.e. which will make the impedance of the load appear to be 50Ω when referred to the transformer primary.

[6]

$$Z_1 = (N_1/N_2)^2 Z_2$$

$$\text{So } N_1/N_2 = \sqrt{(Z_1/Z_2)} = \sqrt{(50/3)} = \sqrt{16.7} = 4.08 \text{ (~4 i.e. 4:1)}$$

8. (a) Write down the truth table of an AND gate, OR gate, XOR gate and a JK flip-flop.

marks]

[4

AND:

A	B	Q
0	0	0
0	1	0
1	0	0
1	1	1

OR:

A	B	Q
0	0	0

0	1	1
1	0	1
1	1	1

XOR:

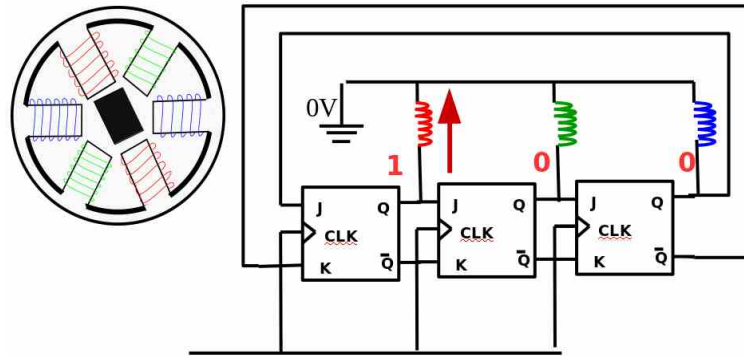
A	B	Q
0	0	0
0	1	1
1	0	1
1	1	0

JK:

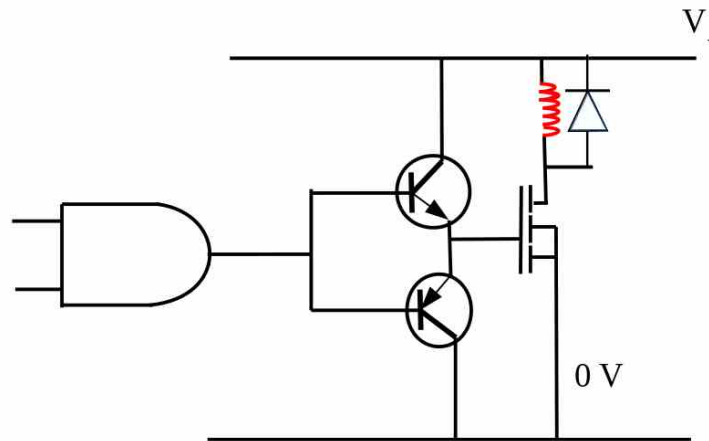
J	K	Q	Qbar
0	0	No change	No change
0	1	0	1
1	0	1	0
1	1	flip	Flip

- (b) Draw a diagram showing how a set of J-K flip-flops can be used to energise the windings of a switched reluctance motor in sequence, so that the motor moves around by one increment for each pulse received on the clock input. You may assume that the outputs of the J-K flip-flops are able to operate the windings directly (though in practice some further circuitry would be needed to make the signals sufficiently powerful).

[5 marks]



- (c) Draw a circuit diagram of a push pull pair showing how digital logic signal can be used to operate (i.e. switch on and off) a large solenoid. When the solenoid is turned off quickly, it could damage the push-pull pair circuit. Write down the equation describing this phenomenon. Add an extra component to your circuit diagram which would protect the circuit from this effect.



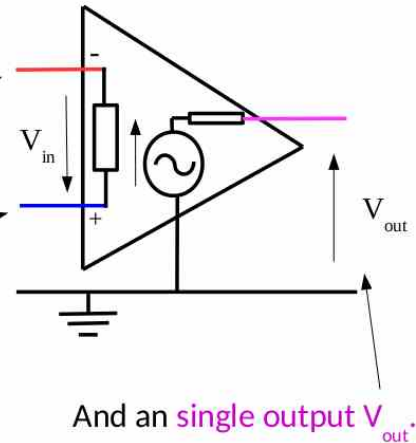
$$V = -L \frac{di}{dt}$$

[8]

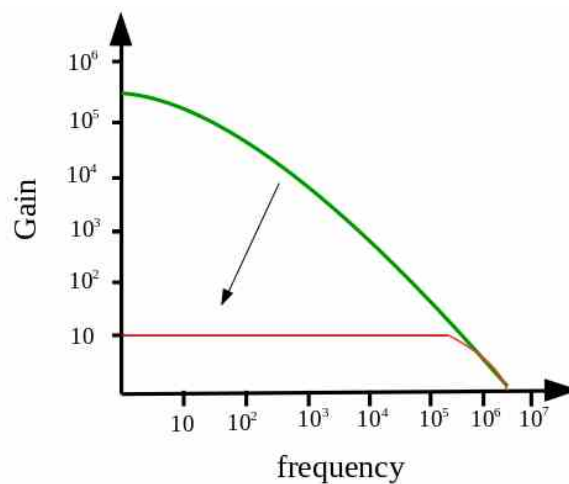
- (d) Draw the diagram for an operational amplifier, and label the inverting terminal, the non-inverting terminal, the output, the input resistance, the output resistance. Then write down the equation relating the voltage at the inverting terminal, non inverting terminal and the output. Using a diagram explain the difference between the open loop gain of an op-amp and the closed loop gain.

The Op-amp equivalent circuit has two inputs:

- Inverting input (-)
- Non-inverting input (+)



$$V_{out} = A(V_{+} - V_{-})$$



The open loop gain varies strongly as a function of frequency and is too high to be useful. Closed loop gain is flat as a function of frequency and is low enough to be useful.

[8]

- (e) The circuit shown in Figure Q.8(f) is to be used to average the results of three sensors. Demonstrate that the output voltage V_{OUT} is equal to the average of the input voltages V_1 , V_2 and V_3 but of opposite sign provided the open-loop gain of the op-amp is infinite.

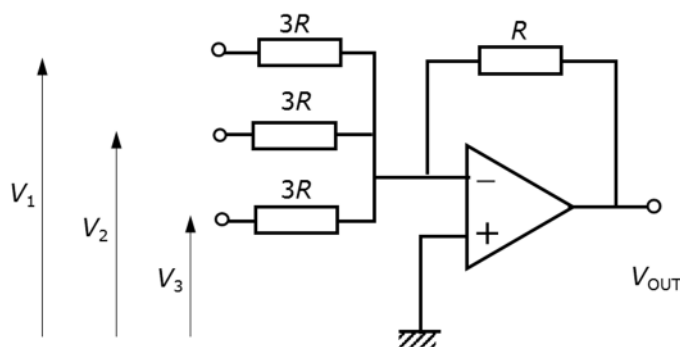


Fig. Q.8(f)

[10]

Using Kirchhoff's Voltage Law

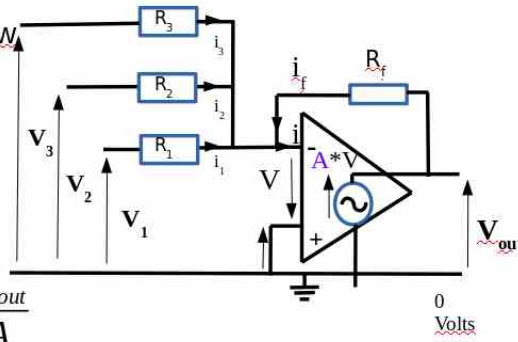
$$V_1 = i_1 R_1 - V$$

$$V_2 = i_2 R_2 - V$$

$$V_3 = i_3 R_3 - V$$

$$V_{out} = i_f R_f - V$$

If $R_o = 0$ and $V = \frac{V_{out}}{A}$



As A is very large, V tends to zero and the voltage equations may be simplified to:

$$V_1 = i_1 R_1$$

$$V_2 = i_2 R_2$$

$$V_3 = i_3 R_3$$

$$V_{out} = i_f R_f$$

1

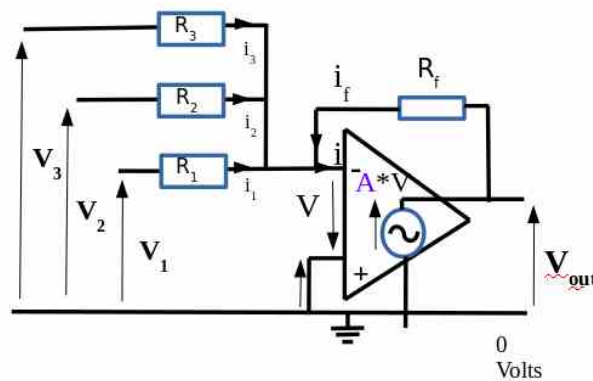
If the input resistance of the op-amp is high $i=0$.

Therefore we can write:

$$i_1 + i_2 + i_3 = -i_f$$

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} = \frac{-V_{out}}{R_f}$$

$$V_{out} = -R_f \left[\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \right]$$



END