University of Nottingham
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LECTURE 3B

3-phase AC & Induction Motors

Electromechanical Devices
MMME2051

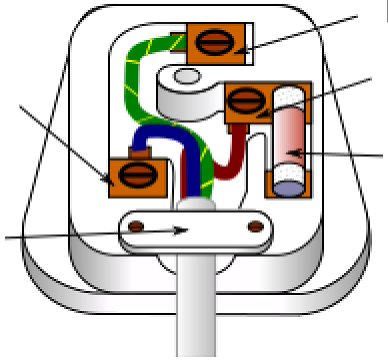
Module Convenor – Surojit Sen

Learning Outcomes

- 3-phase AC
  - **Star** v **Delta**
  - **Line** v **Phase**
- Induction Motor
  - Operation Principle
  - **Stator** & **Rotor**
  - Concept of **Electromagnetism** (**Fleming's** Left & Right Hand Rule)
  - **Synchronous** & **Asynchronous**

Domestic Electricity Supply

- We know that mains electricity supply in the UK is $240V\ 50Hz$ – what does this mean?
- If you measure the voltage (using a Voltmeter or Multimeter), you will read a voltage with $V_{rms} = 240V$ & $f = 50Hz$
- $\boldsymbol{v = 240\sqrt{2}\cos 2\pi 50t}$
- Electricity supply cable has 3 cores:
  - **Live** – Supply line
  - **Neutral** – Reference line
  - **Earth** – Direct connection to earth $(0V)$



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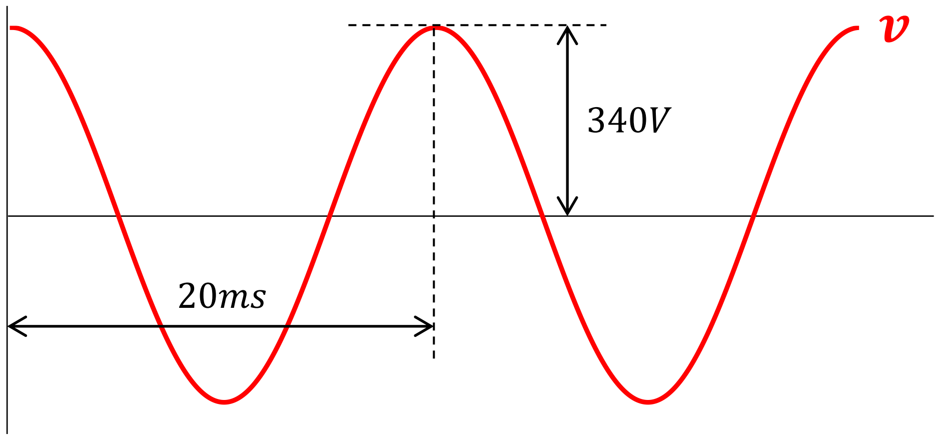
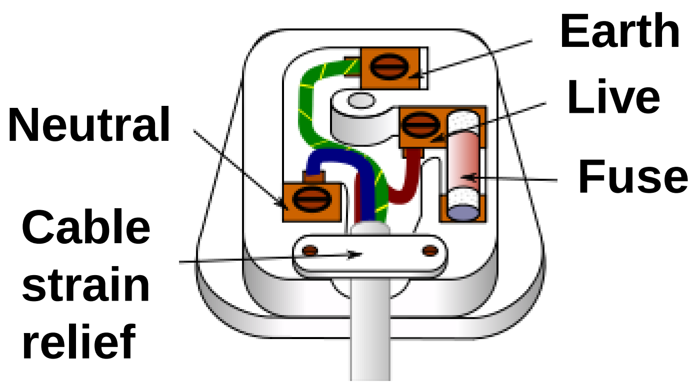
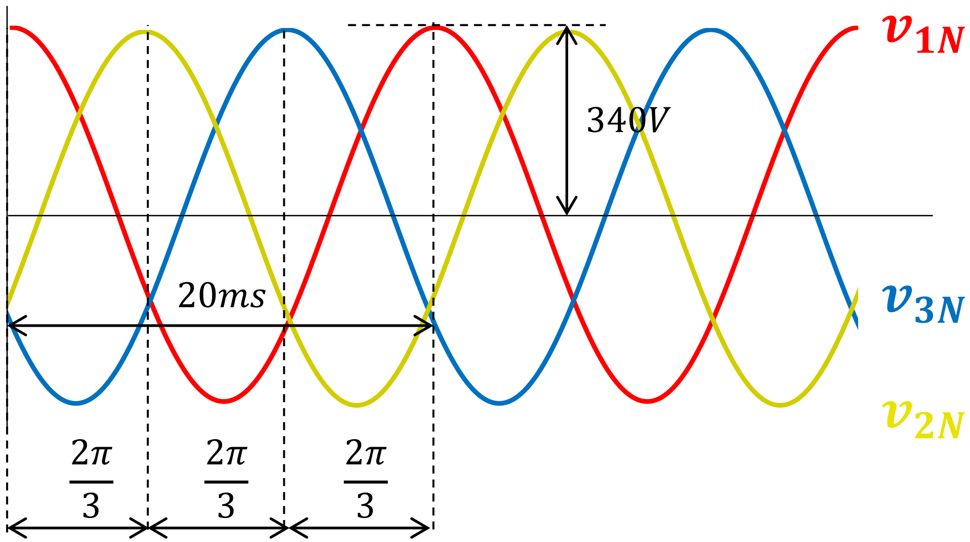
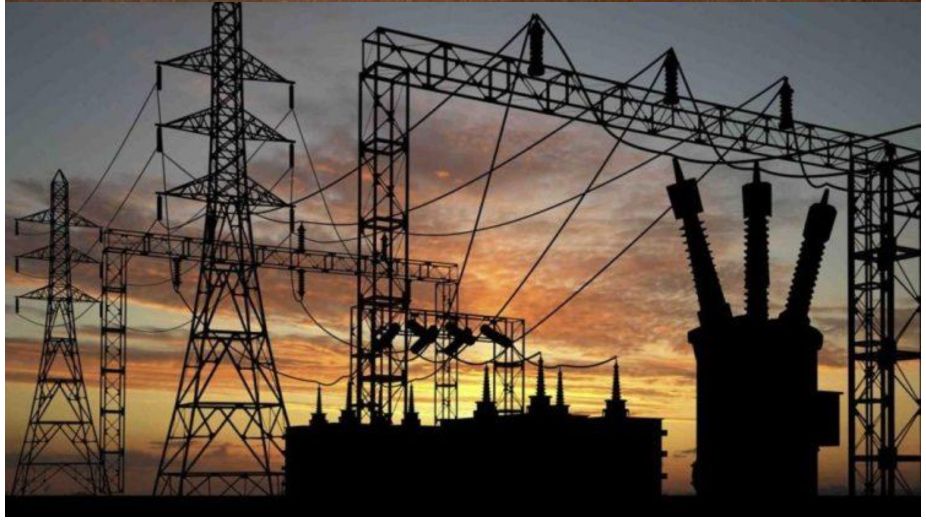




Electricity Grid

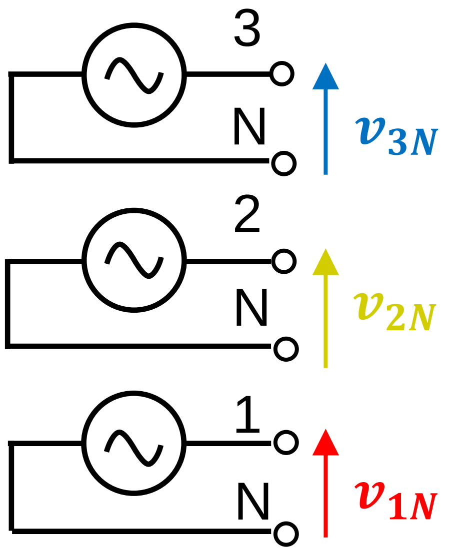
- However, power stations (like the Ratcliffe-on-Soar) generate $415V$ $50Hz$ 3-phase
- $\boldsymbol{v_{1N} = 240\sqrt{2}\cos 2\pi 50t}$
- $\boldsymbol{v_{2N} = 240\sqrt{2}\cos\left(2\pi 50t - \frac{\pi}{3}\right)}$
- $\boldsymbol{v_{3N} = 240\sqrt{2}\cos\left(2\pi 50t + \frac{\pi}{3}\right)}$





Electricity Grid

- Each phase may be regarded as a **separate** $\mathbf{240V\ 50Hz}$ **AC supply**



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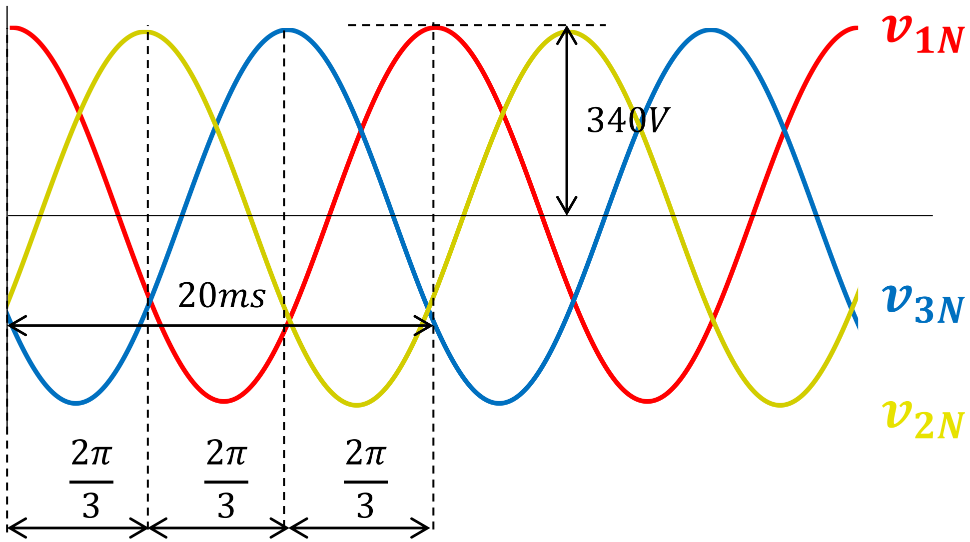
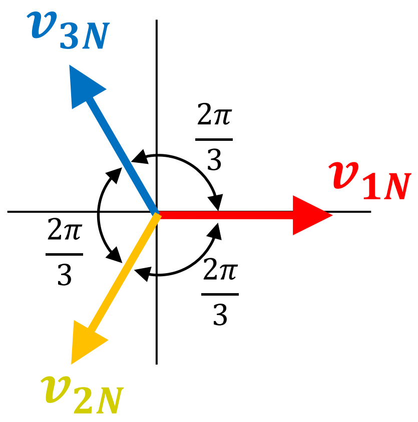
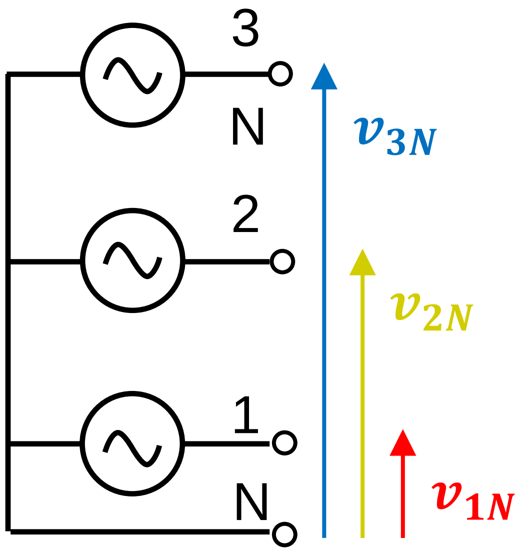
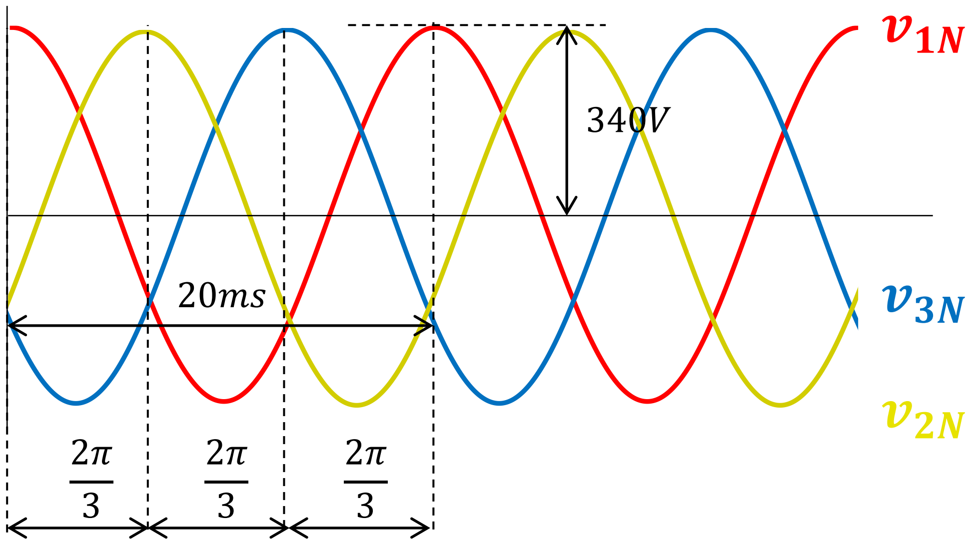


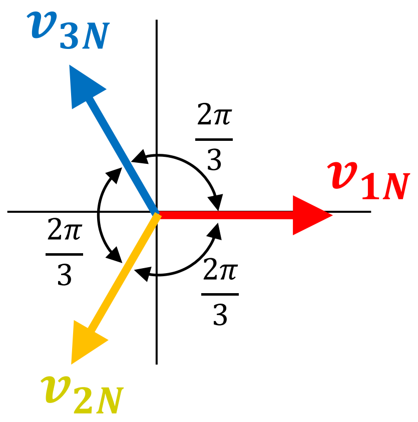


Electricity Grid

- In practice, the three supplies share a **common reference**, i.e., **neutral** point
- $v_{1N} = 240\sqrt{2}\cos 2\pi 50t$
- $v_{2N} = 240\sqrt{2}\cos(2\pi 50t - \frac{\pi}{3})$
- $v_{3N} = 240\sqrt{2}\cos(2\pi 50t + \frac{\pi}{3})$







Electricity Grid

- This arrangement is called **star connection** – common neutral for all phases
- $v_{1N} = 240\sqrt{2}\cos 2\pi 50t$
- $v_{2N} = 240\sqrt{2}\cos\left(2\pi 50t - \frac{\pi}{3}\right)$
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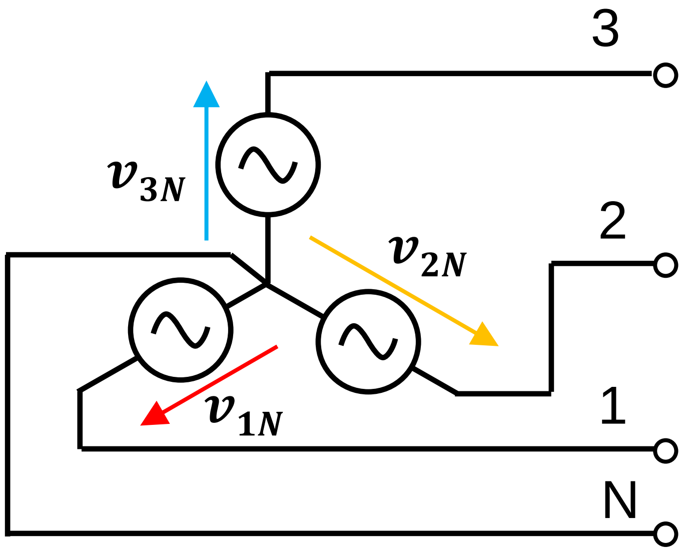
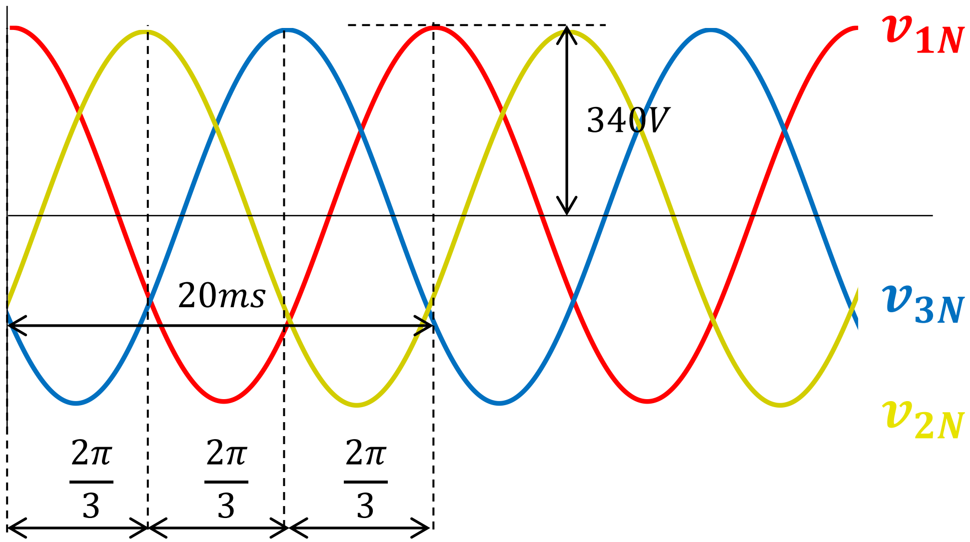
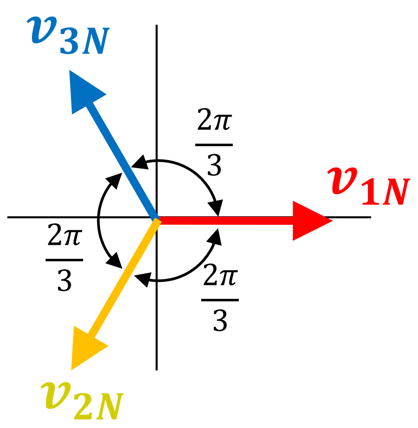






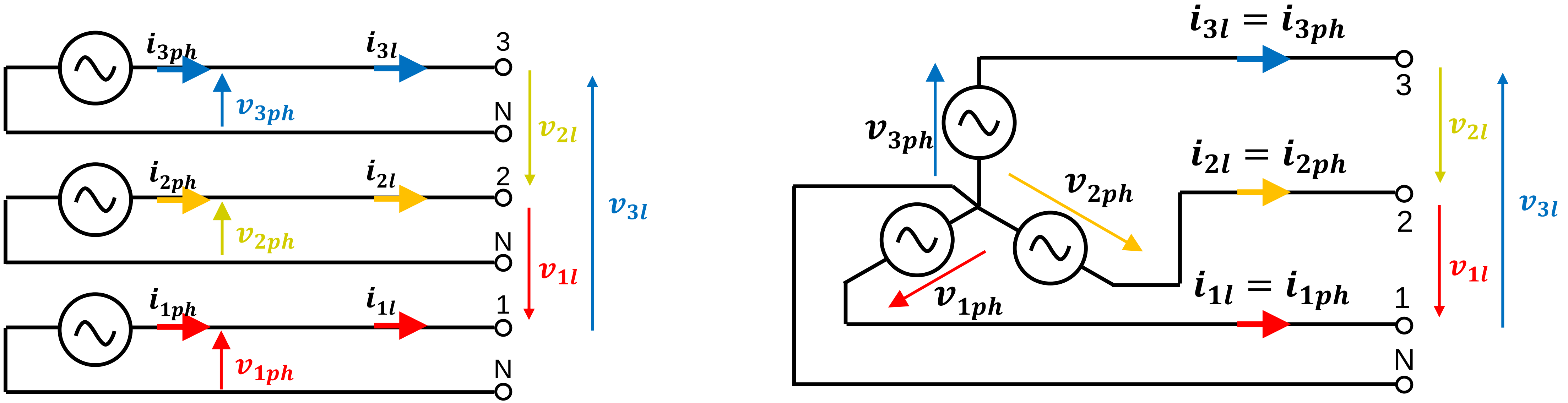
Electricity Grid

But why 415V?

- Because 3-phase supplies are often referred in **line voltage** form (not **phase voltage**)
- Line Voltage means voltage of a phase with respect to the next phase in sequence

Concept of Line v Phase (voltage & current)

- **In any 3-phase entity (source or load), you have Line & Phase variables**
- This applies to both voltage (**line/phase voltage**) and current (**line/phase current**)
- **Phase Voltage** – Voltage across any phase
- **Line Voltage** – Voltage between two live lines (in appropriate phase sequence)
- **Phase Current** – Current through any phase
- **Line Current** – Current through any live line

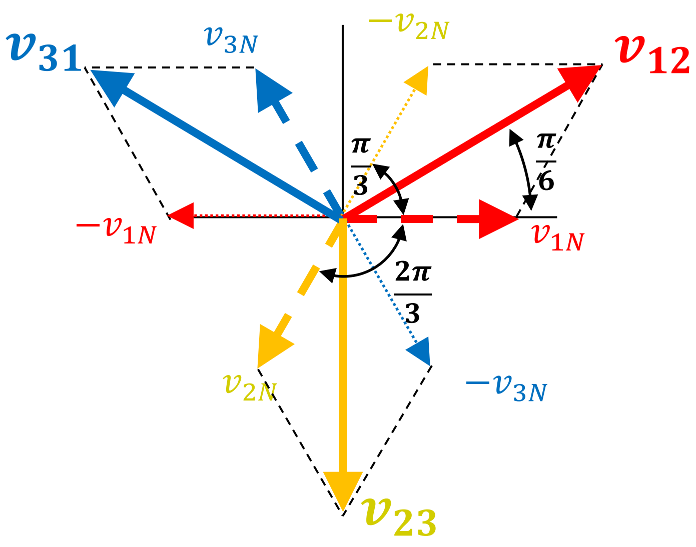


Relationship between phase and line voltages for star-connected device?

Line v Phase Voltage

Line Voltage

Voltage between any two live lines, i.e., $v_{12} = v_{1N} - v_{2N} = \sqrt{3}v_{1N}$



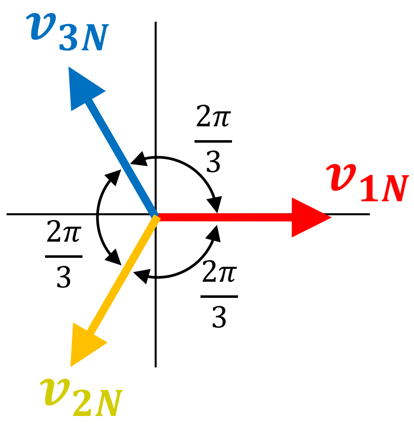
$|v_{12}| = |v_{23}| = |v_{31}| = \sqrt{3}|v_{1N}| = \sqrt{3}|v_{2N}| = \sqrt{3}|v_{3N}|$

$$|Line\ Voltage| = \sqrt{3} \times |Phase\ Voltage|$$

$$415V = \sqrt{3} \times 240V$$

Phase Voltage

Voltage of a phase with respect to the common neutral point



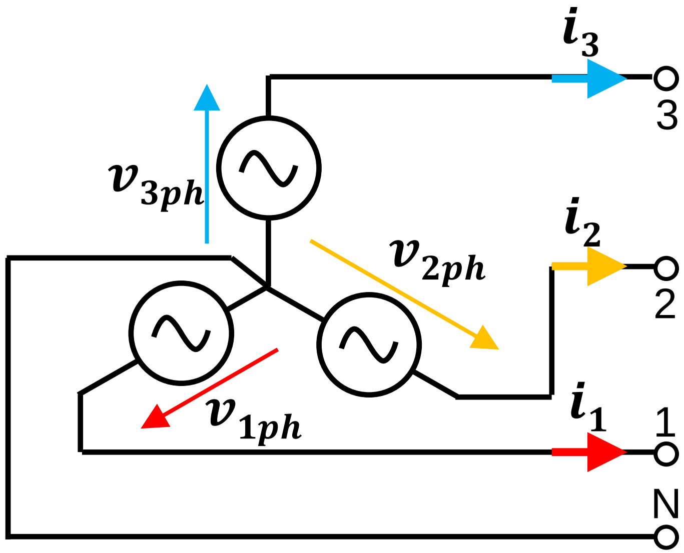
Exercise 1 – Prove $v_{12} = \sqrt{3}v_{1N}$

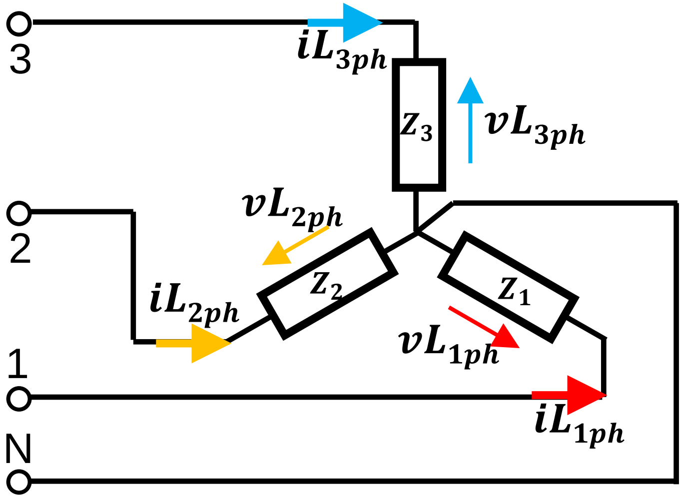
Exercise 2 – Prove the phase offset of v_{12} is 30° advanced from v_{1N}

(hint: you know the phase voltage in the polar form, magnitude & angle of 120 deg. Convert to cartesian and subtract. Then convert it back to polar form)

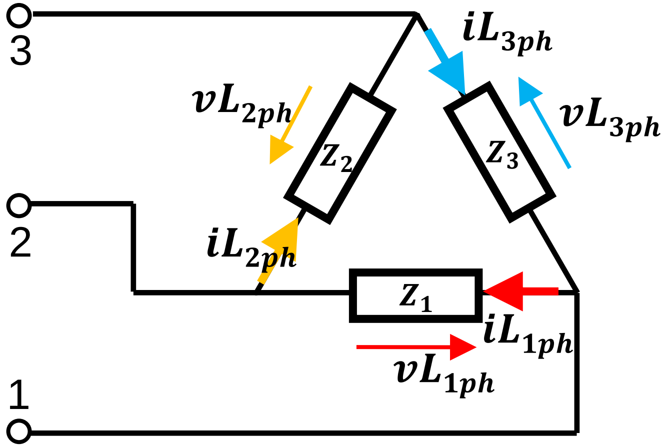
Star v Delta Load

- We discussed 3-phase power source – what about **3-phase load**?
- We can connect a 3-phase load in two configurations – **star** & **delta**
- **Let us find out voltage and current values**





STAR



DELTA

Star Load

- When you connect two 3-phase devices, you match the line variables

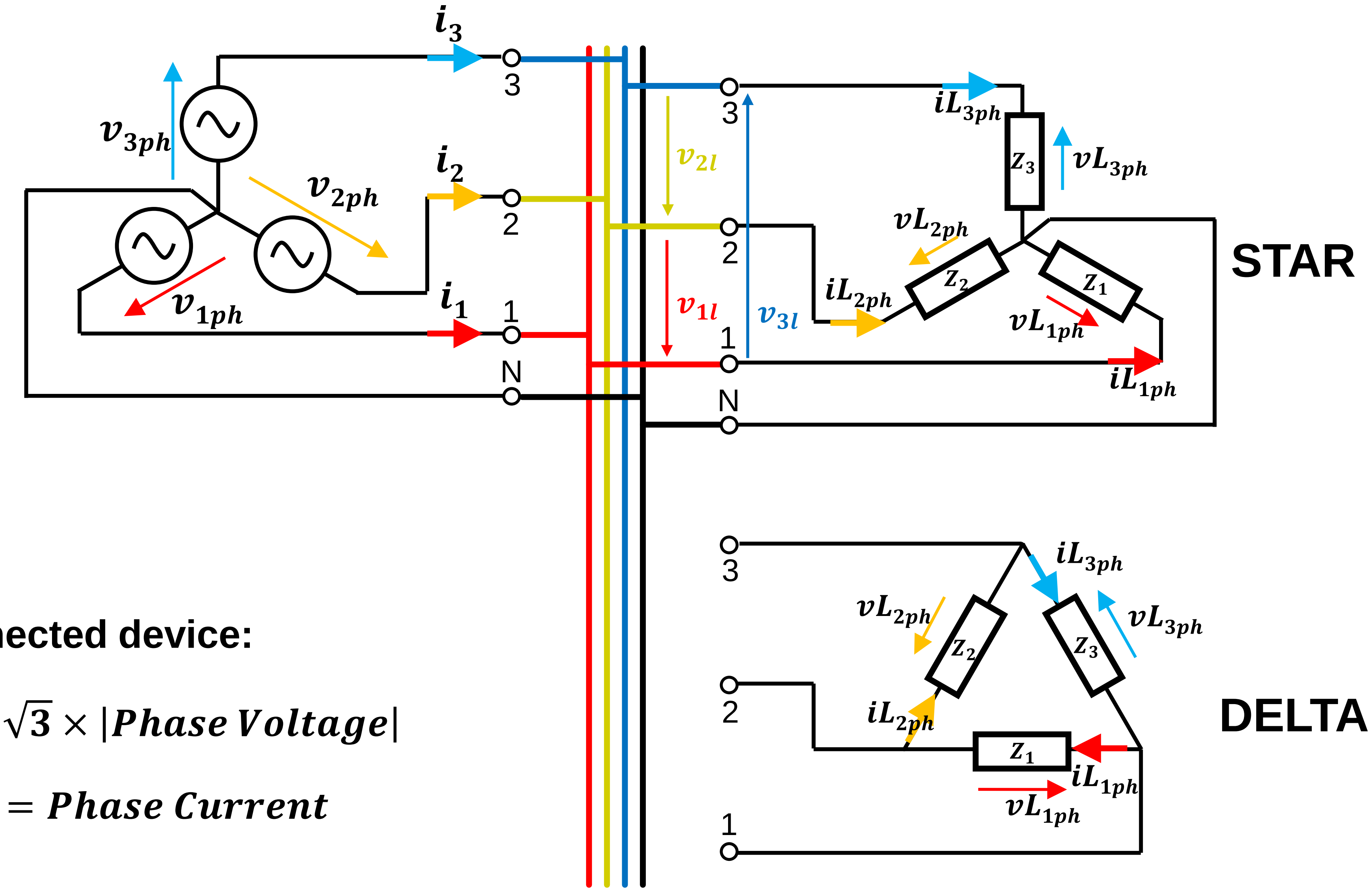
$$vL_l = v_l \ \& \ iL_l = i_l$$

- We also know

$$vL_l = \sqrt{3} vL_{ph}$$

- Lastly, line and phase currents are equal

$$iL_l = iL_{ph}$$



Star-connected device:

$$|Line\ Voltage| = \sqrt{3} \times |Phase\ Voltage|$$

$$Line\ Current = Phase\ Current$$

Delta Load

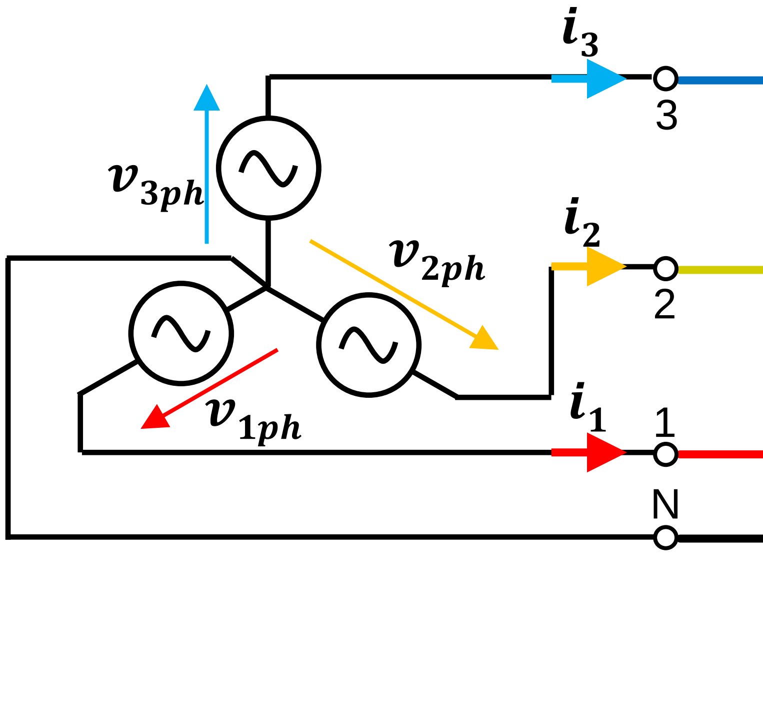
- When you connect two 3-phase devices, you match the line variables

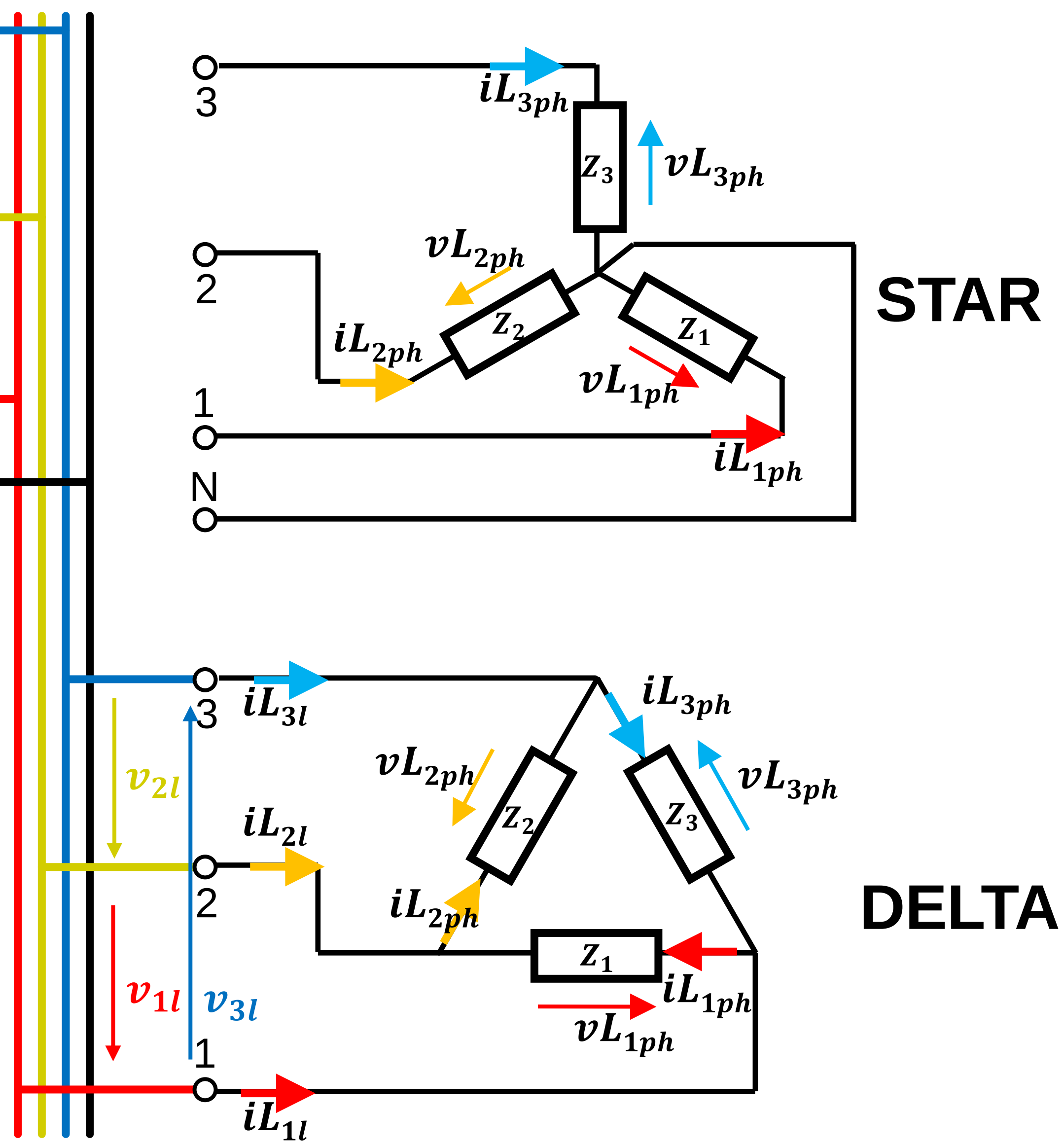
$$vL_l = v_l \;\&\; iL_l = i_l$$

- We can see (on load side) that the phase voltage is equal to line voltage

$$vL_l = vL_{ph}$$

- Lastly, line and phase currents are NOT equal (follows the same relationship of line/phase voltage in star)





$iL_{l12} = i_{ph1} - i_{ph2}$ Similarly, $iL_{l23} = i_{ph2} - i_{ph3}$ and $iL_{l31} = i_{ph3} - i_{ph1}$

Delta-connected device:

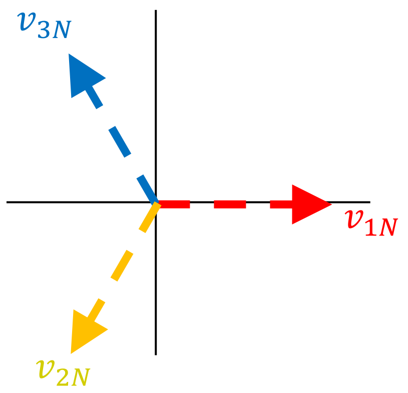
$$Line\ Voltage = Phase\ Voltage$$

$$|Line\ Current| = \sqrt{3} \times |Phase\ Current|$$

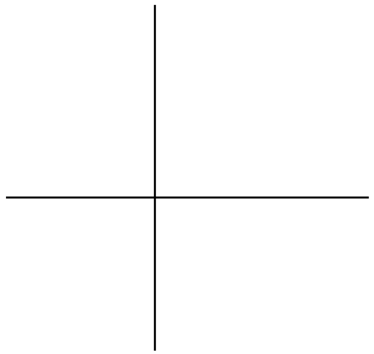
The Phasor Perspective

Source (star)

Let us start with defining the Phase Voltage



Load (star)

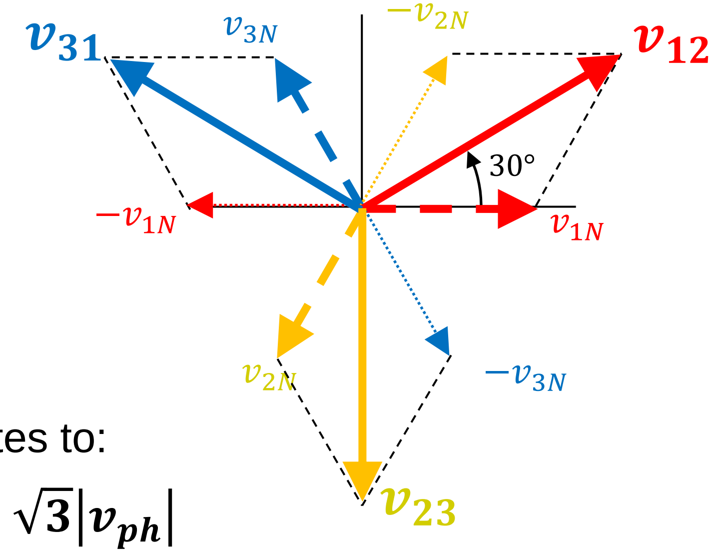


The Phasor Perspective

Source (star)

Calculate the Line Voltage

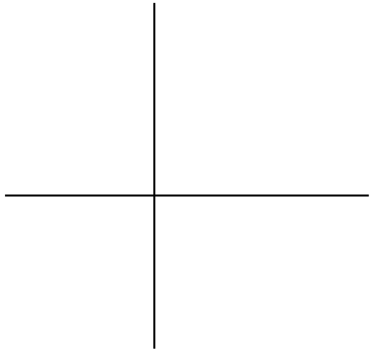
$$v_{12} = v_{1N} - v_{2N}$$



This translates to:

$$|v_l| = \sqrt{3}|v_{ph}|$$
$$\angle v_l = \angle v_{ph} + 30°$$

Load (star)



The Phasor Perspective

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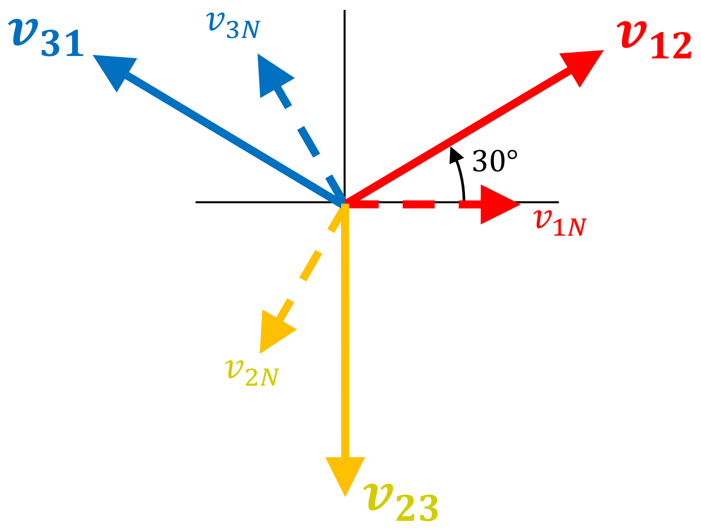
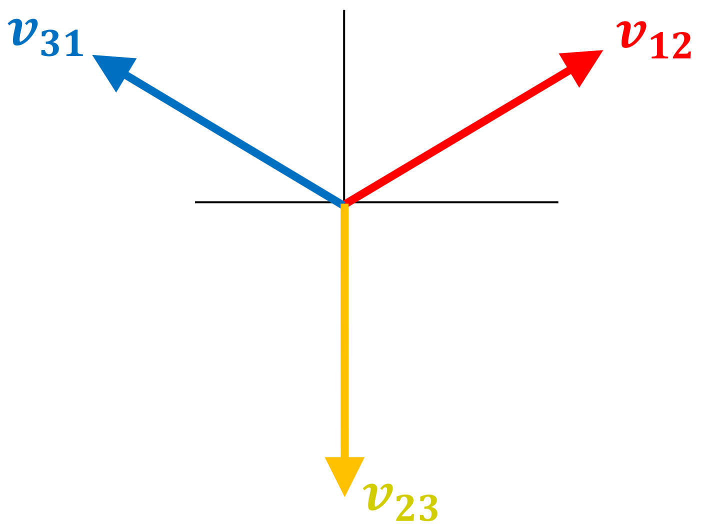


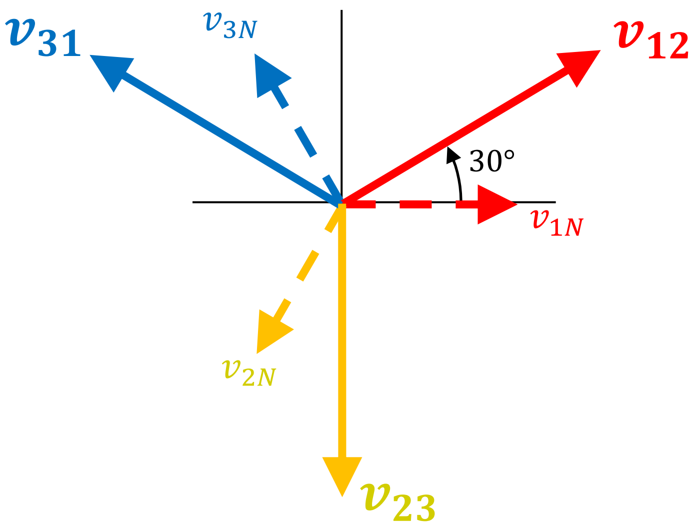
Load (star)

Match the Line Voltage with source



The Phasor Perspective

Source (star)



Load (star)

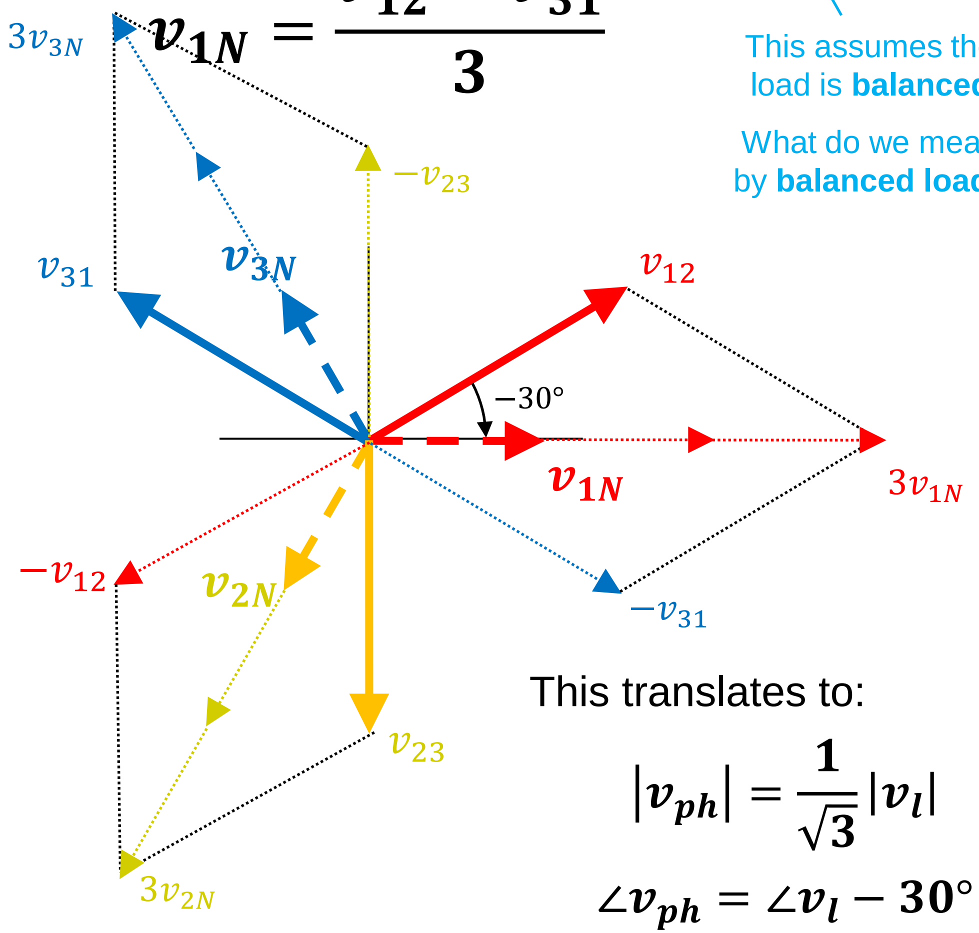
Calculate the Phase Voltage

$v_{12} = v_{1N} - v_{2N}$ & $v_{31} = v_{3N} - v_{1N}$ & $v_{12} + v_{23} + v_{31} = 0$

$$v_{1N} = \frac{v_{12} - v_{31}}{3}$$

This assumes the load is **balanced**

What do we mean by **balanced load**?



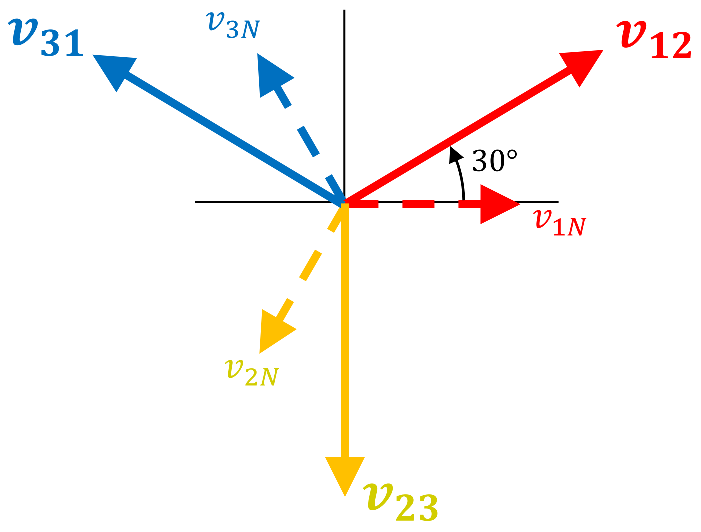
This translates to:

$$|v_{ph}| = \frac{1}{\sqrt{3}}|v_l|$$

$$\angle v_{ph} = \angle v_l - 30°$$

The Phasor Perspective

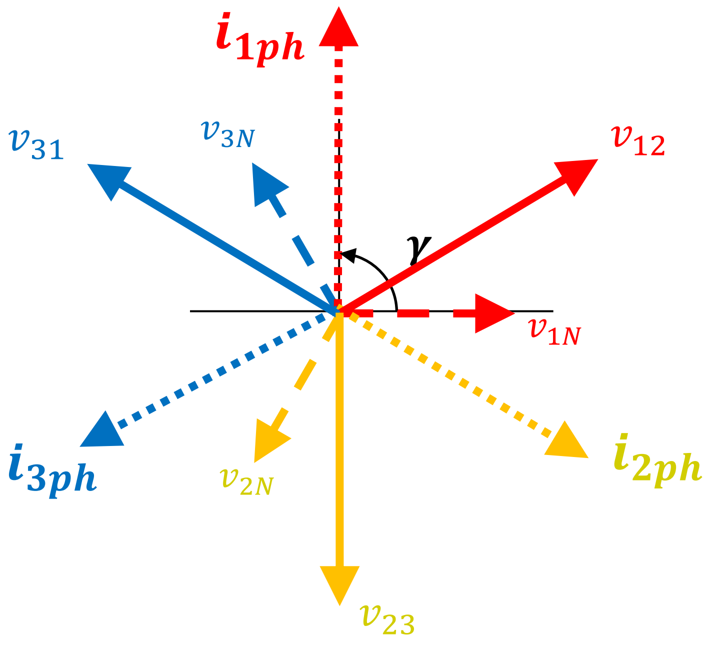
Source (star)



Load (star)

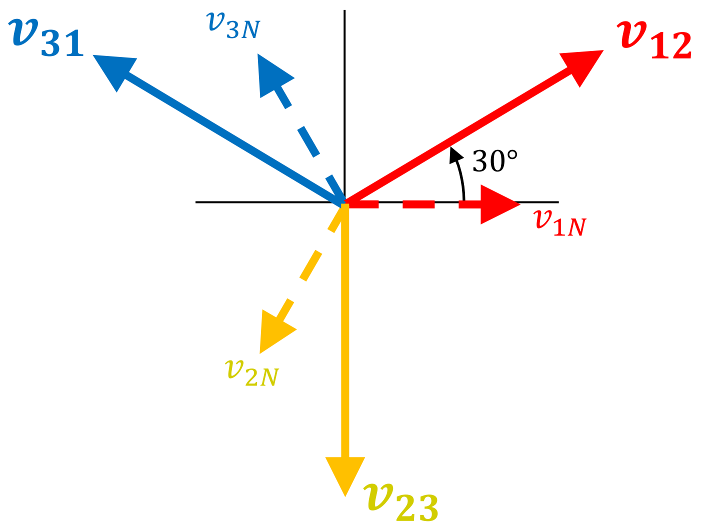
Calculate the Phase Current

$$i_{1N} = \frac{v_{1N}}{Z_1}$$



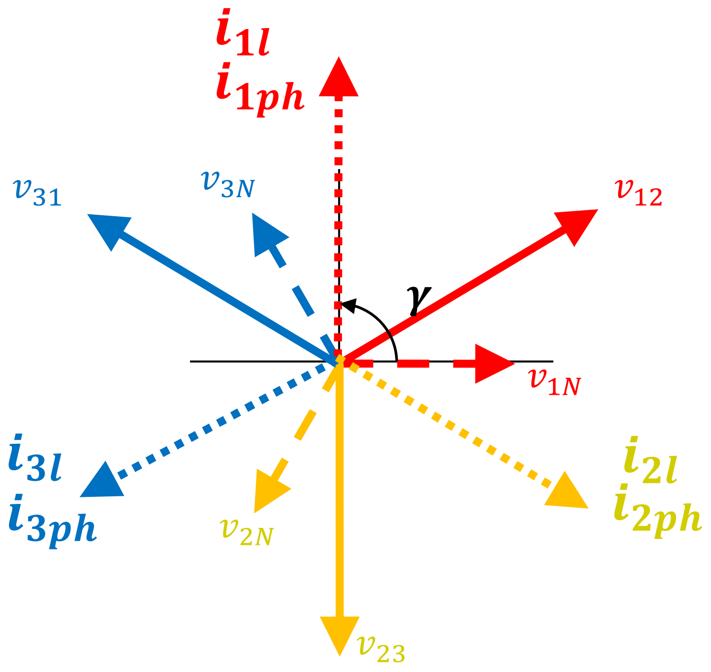
The Phasor Perspective

Source (star)



Load (star)

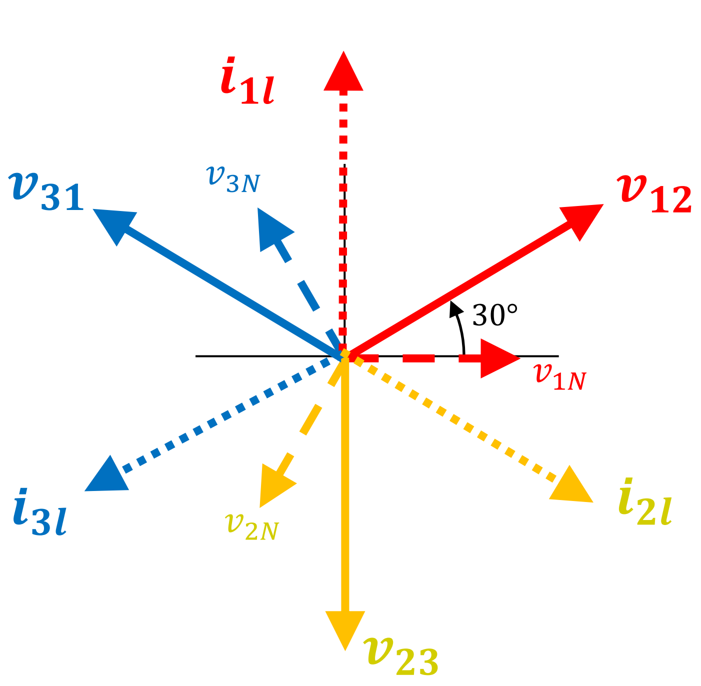
Line and Phase current is same for star arrangement



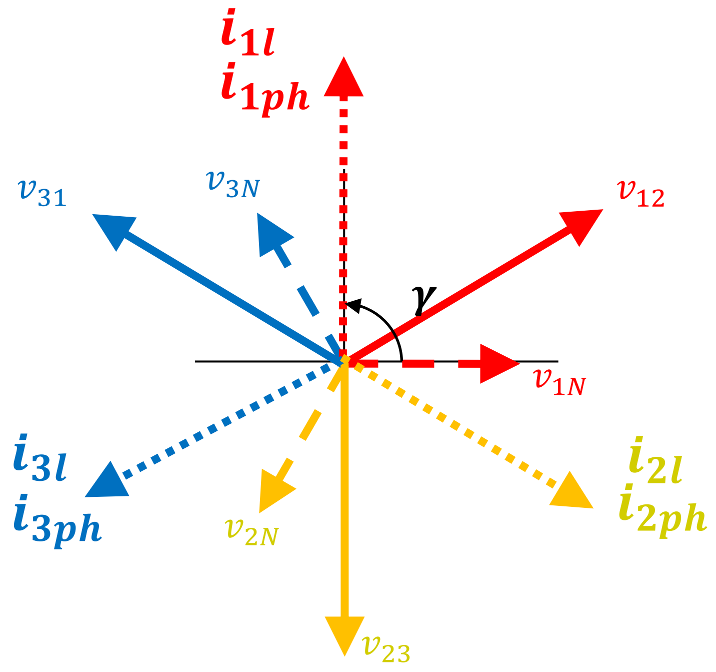
The Phasor Perspective

Source (star)

Match the Line Current



Load (star)



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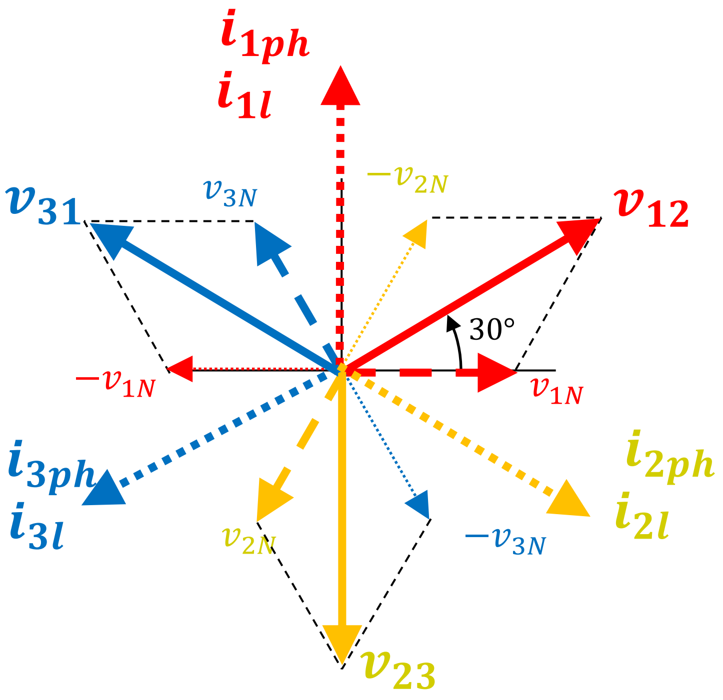

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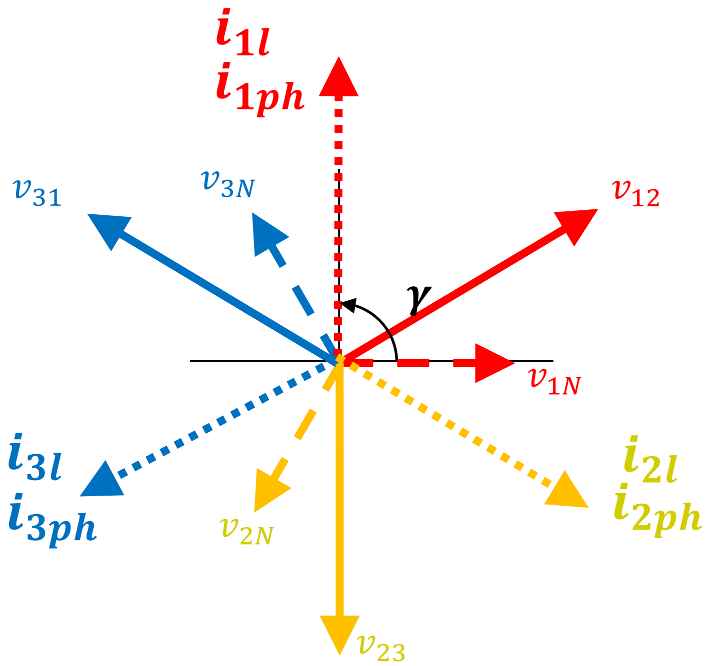


Balanced Load

- 3-phase devices (source and load) are usually balanced
- This means that the impedance (complex value) of all three legs in a 3-phase load is equal

$$Z_1 = Z_2 = Z_3$$

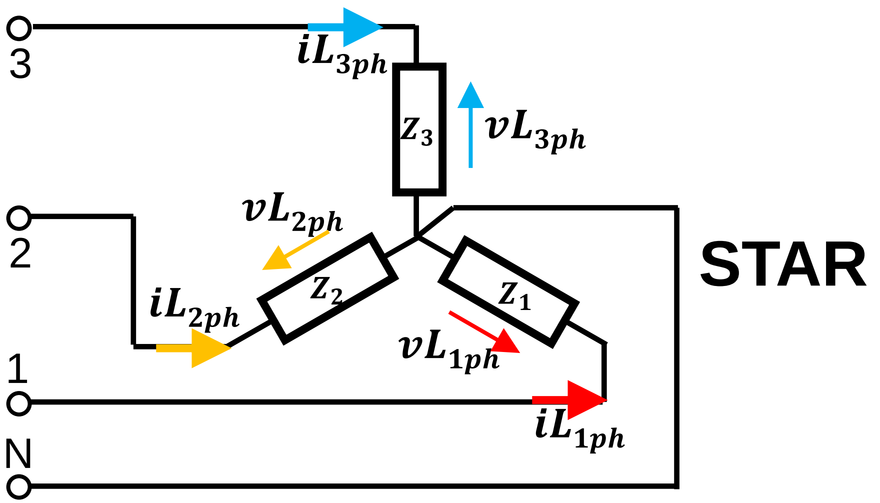
- In case of a load, this means that the voltage is same (except the phase angles, which are set apart by 120°)

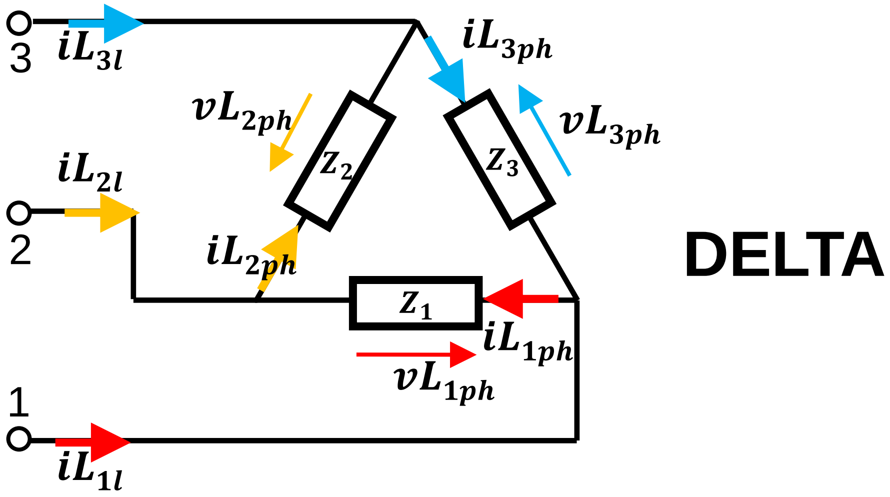
$$v_{1N} = V\cos 2\pi f t$$

$$v_{2N} = V\cos(2\pi f t - 120°)$$

$$v_{3N} = V\cos(2\pi f t + 120°)$$

- Balanced load (and source – which is almost always true) ensures that the **line/phase currents have equal magnitudes** (phase angles spaced apart by 120°) and **neutral current is zero**





3-phase Power

- Total power dissipated in a single-phase load

$$P = VI\cos\gamma$$

- Hence, power dissipated in one phase of a balanced 3-phase load

$$P = V_{ph}I_{ph}\cos\gamma$$

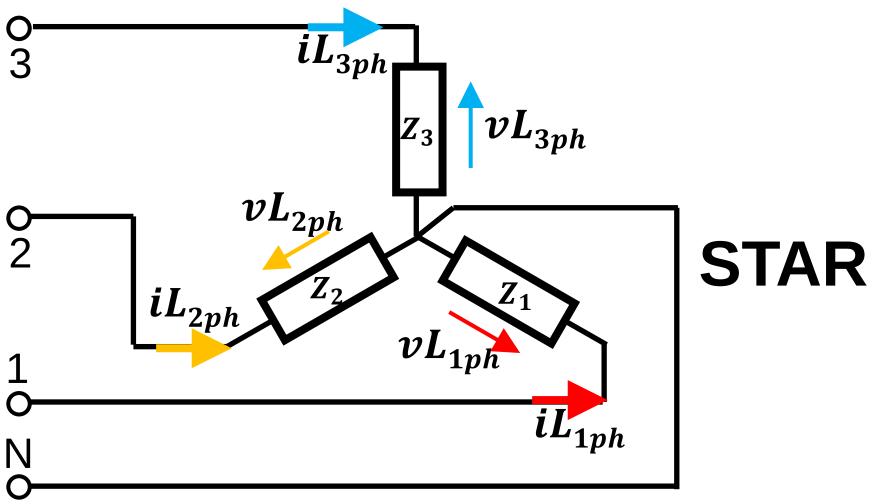
- Total power dissipated in three phase with be 3x this (balanced load)

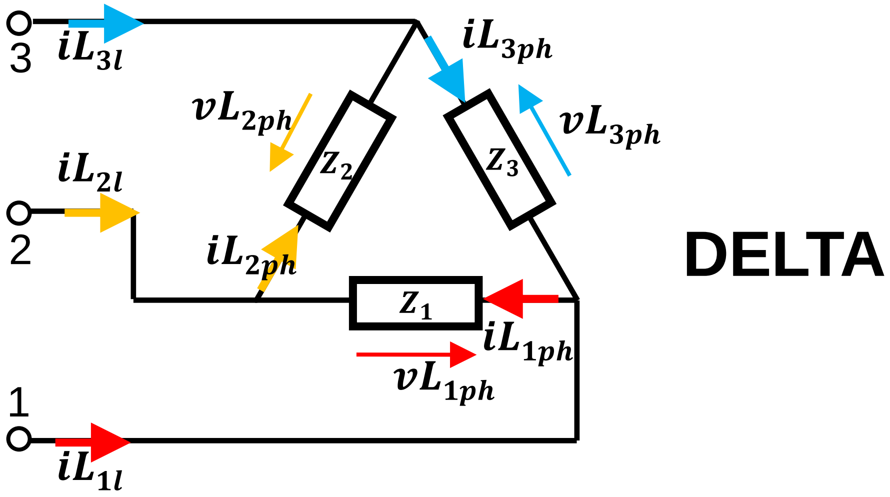
$$P = 3V_{ph}I_{ph}\cos\gamma$$

- In **star arrangement**, $V_l = \sqrt{3}V_{ph}$ and $I_l = I_{ph}$, hence

$$P = \sqrt{3}V_lI_l\cos\gamma$$

- This is the same case for **delta arrangement** as well!





3-phase Summary

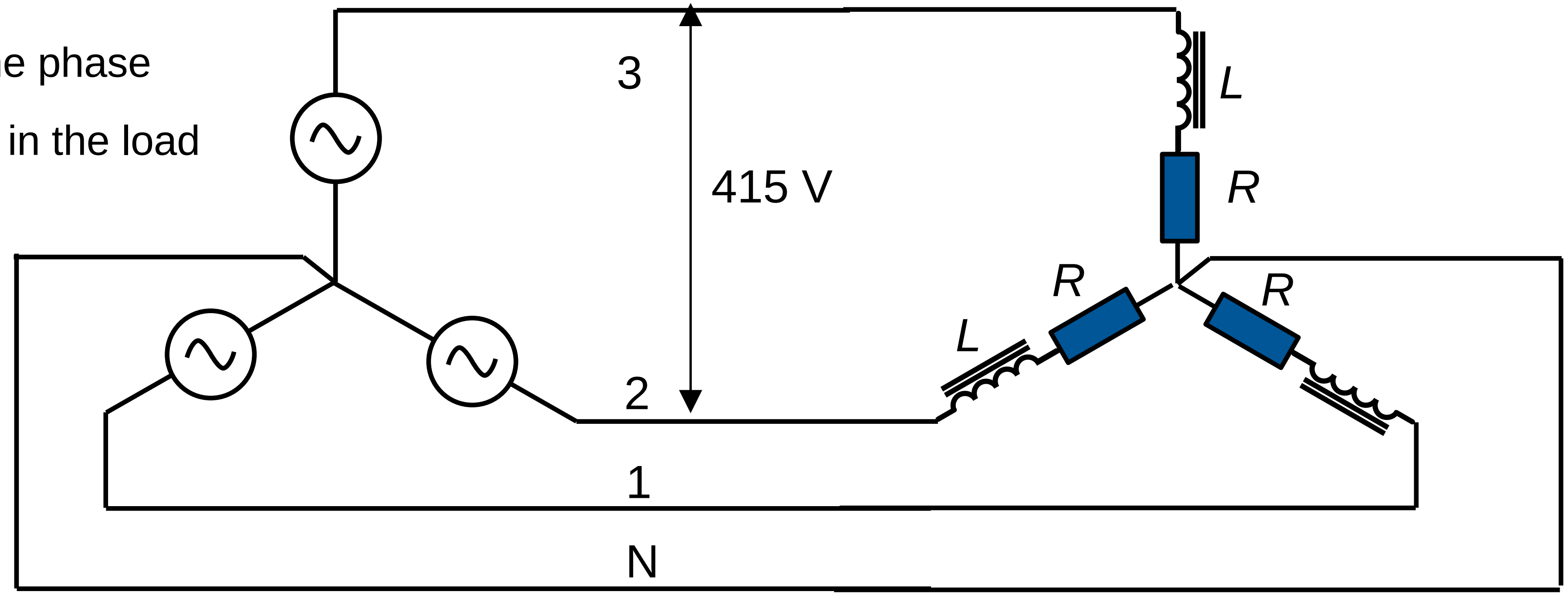
- Understand the concept of Line and Phase variables:
  - **Phase Voltage** – Voltage across any phase
  - **Line Voltage** – Voltage between two live lines (in appropriate phase sequence)
  - **Phase Current** – Current through any phase
  - **Line Current** – Current through any live line
- In **Star** Load:
  - Line Voltage is $\sqrt{3} \times$ Phase Voltage and $30°$ advanced
  - Line and Phase Currents are identical
- In **Delta** Load:
  - Line Current is $\sqrt{3} \times$ Phase Current and $30°$ advanced
  - Line and Phase Voltages are identical
- When you connect two 3-phase devices (e.g., load to source), you **match the line variables**
- Be very **careful with the directions**! Note that the line current for source was pointing outward, and for load, inward.

Example

A balanced 3-phase 415V (line voltage) 50Hz supply feeds a star-connected load of which each phase comprises a 470Ω resistor connected in series with a 1H inductor.

Calculate:

- Line Current
- Power dissipated in one phase
- Total power dissipated in the load



Example

$$
\begin{aligned}
Z &= R + j(X_{\mathrm{L}} - X_{\mathrm{C}}) \\
&= R + j(2\pi f L + 0) \\
&= 470 + j \times 2\pi \times 50 \times 1 = \mathbf{470 + j314\ \Omega}
\end{aligned}
$$

Find phase voltage

$$V_L = \sqrt{3} \times V_{ph} \text{ so } V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = \mathbf{239.6\ V}$$

Ohm's law for impedances

$$
\begin{aligned}
I_P &= I_L = \frac{V_P}{Z} = \frac{239.6}{470 + j314} = \frac{239.6 \ \times (470 - j314)}{(470 + j314)(470 - j314)} = \\
&= \frac{239.6 \ \times (470 - j314)}{470^2 + 314^2} \\
&= 0.352 - j0.235\text{A} = \mathbf{0.423\angle - 33.7°}
\end{aligned}
$$

$$
\begin{aligned}
\text{Power in one phase} &= V_{\mathrm{ph}} I_{\mathrm{ph}} \cos\gamma \\
&= 239.6 \times 0.423 \times \cos(-33.7°) \\
&= \mathbf{84.32\ W} \\
\text{Total power} &= 3V_{\mathrm{ph}} I_{\mathrm{ph}} \cos\varphi \\
&= 3 \times 84.32 \\
&= \ \mathbf{252.95\ W}
\end{aligned}
$$

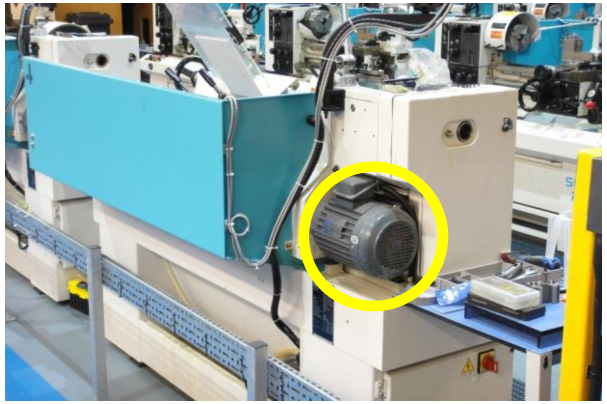
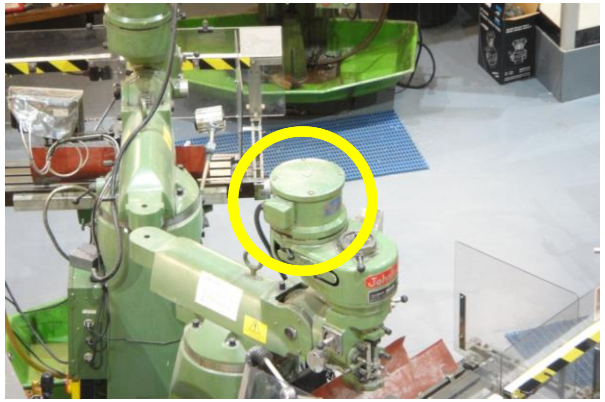
Learning Outcomes

- 3-phase AC
  - **Star** v **Delta**
  - **Line** v **Phase**
- Induction Motor
  - Operation Principle
  - **Stator** & **Rotor**
  - Concept of **Electromagnetism** (**Fleming's** Left & Right Hand Rule)
  - **Synchronous** & **Asynchronous**

Induction Motor

What is an Induction Motor?

- Widely used motor in the industry for powers about a few tens of watts
- Very cost-effective, reliable and rugged design
- Also used at home in slightly modified form to run off single phase supply



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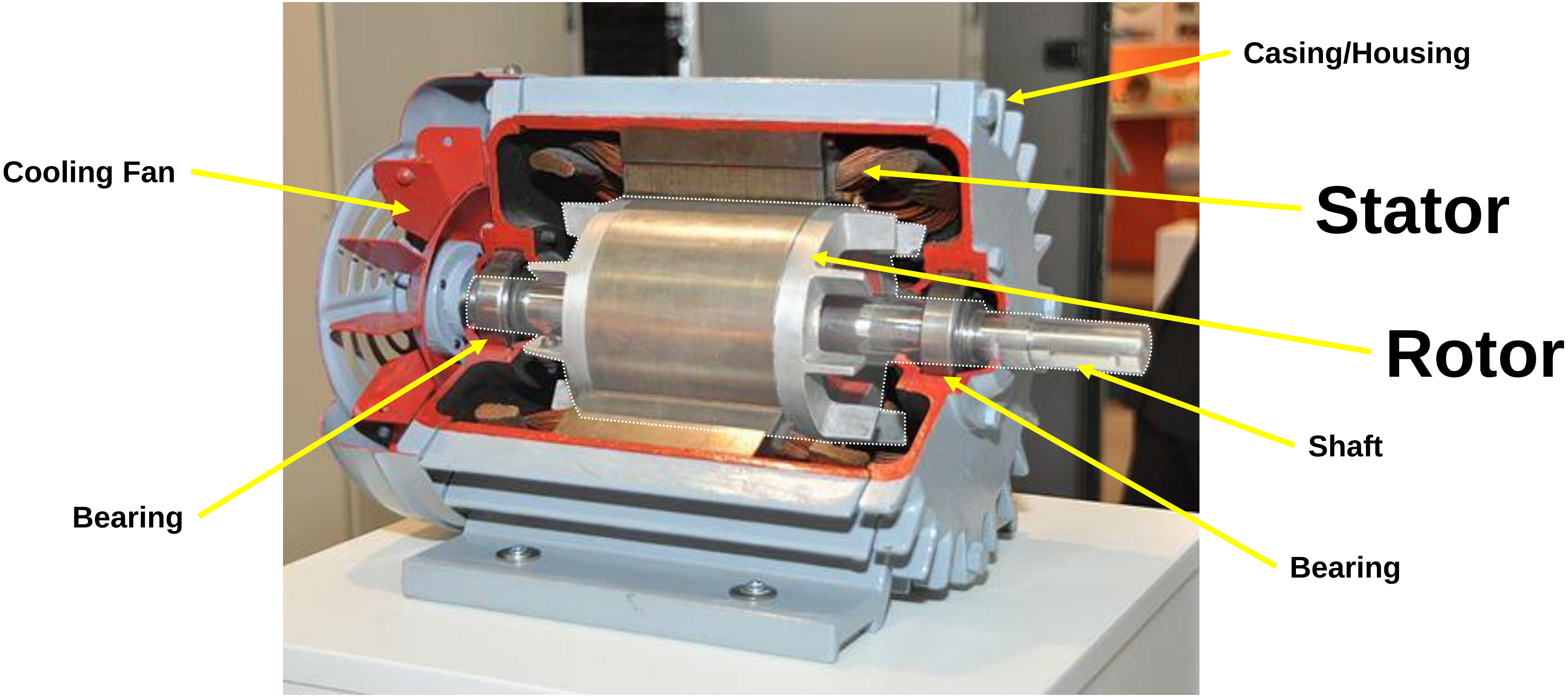
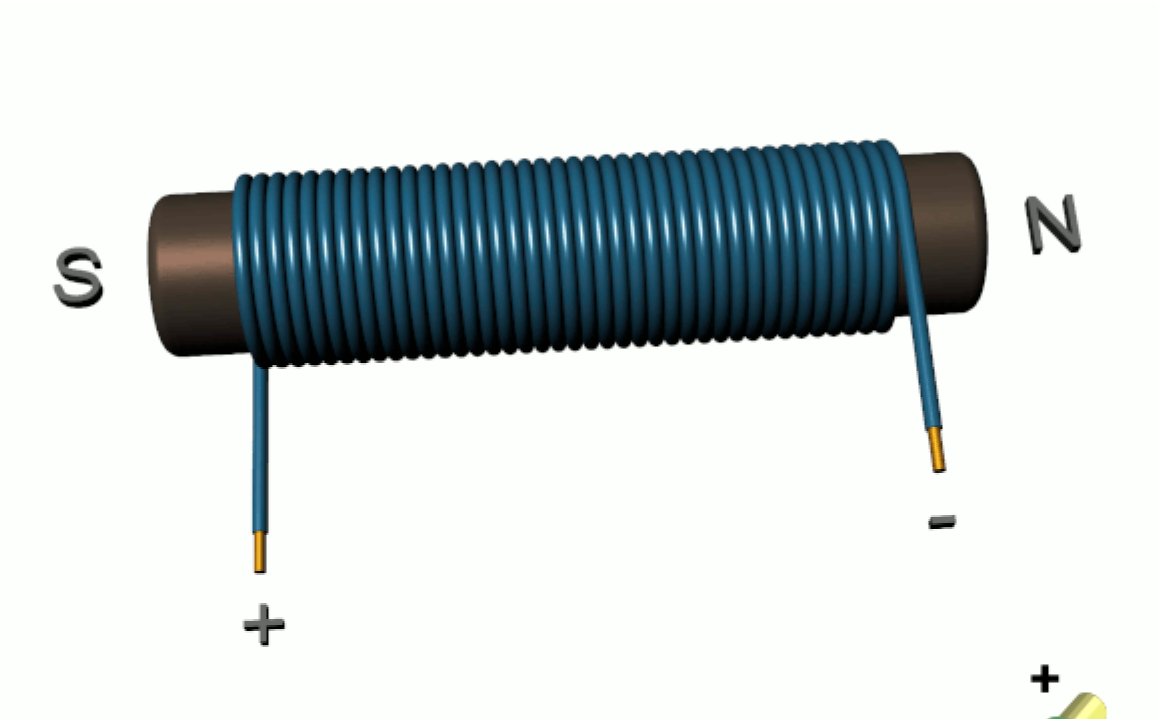
Induction Motor



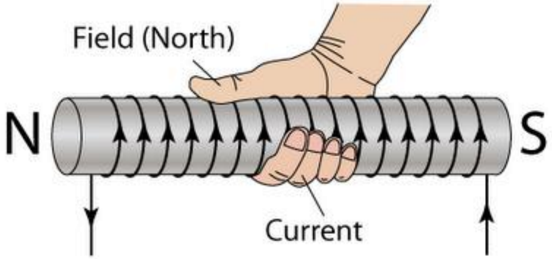
Stator

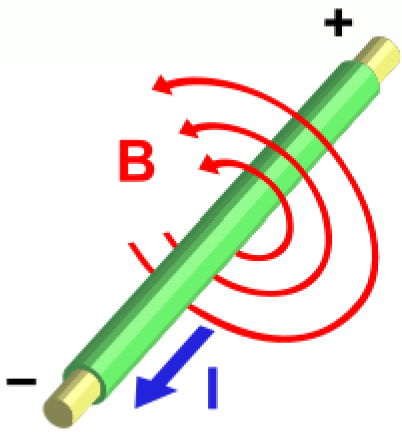
Electromagnetism

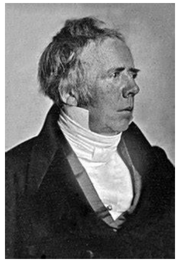
The physics of using electricity to produce magnet, and vice versa



If you take a rod (made of magnetic material), wind a coil of wire around it, and pass some current through the coil, the core becomes a magnet



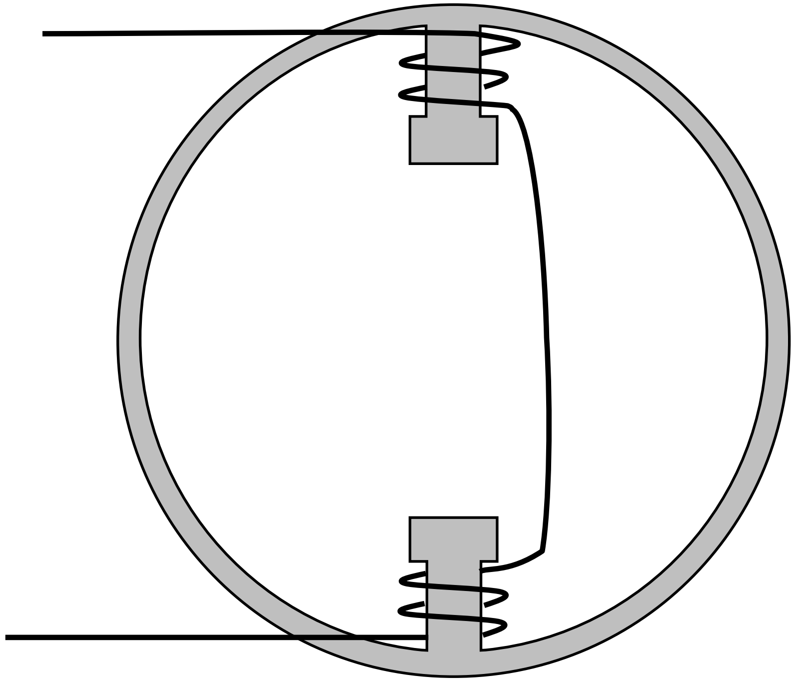




Hans Christian Ørsted was a Danish physicist and chemist who discovered that electric currents create magnetic fields, which was the first connection found between electricry and magnetism

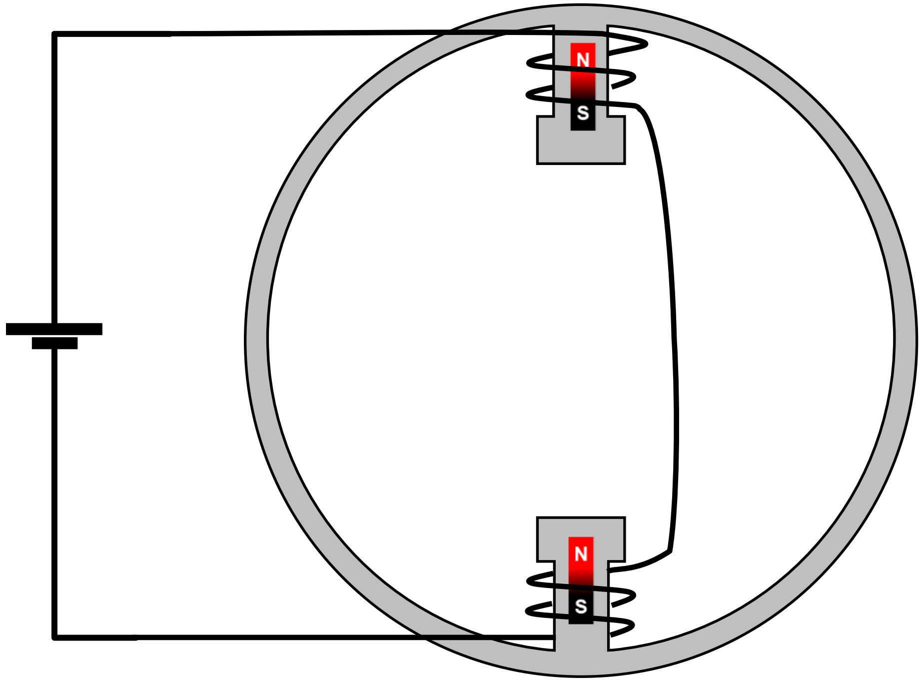
Stator

Electric motors have a stator (stationary part) with either permanent magnets or wound poles (electromagnets)



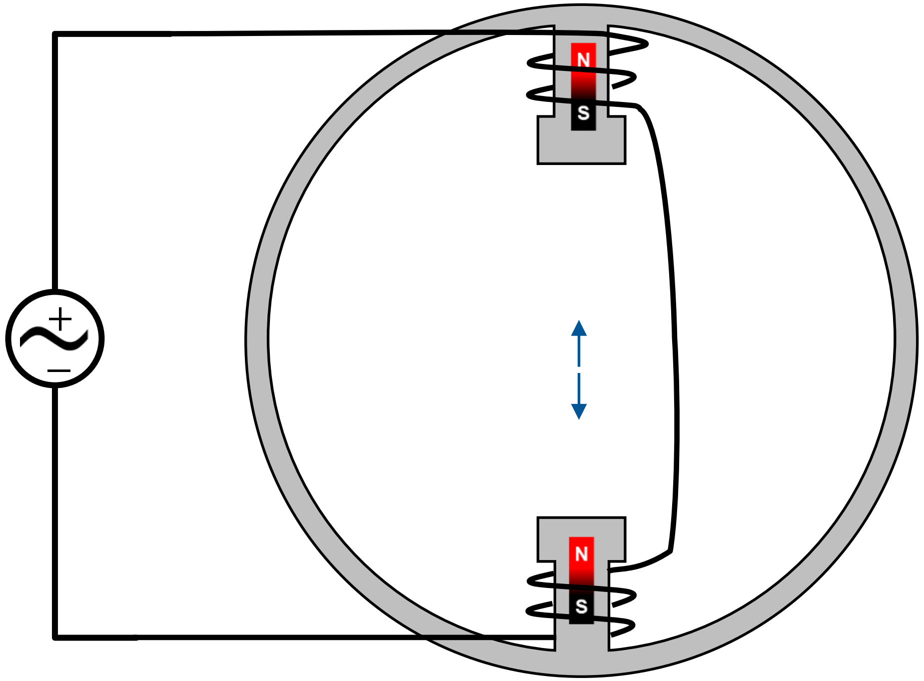
Stator

If such a pair of poles is energised with DC, they become magnetised



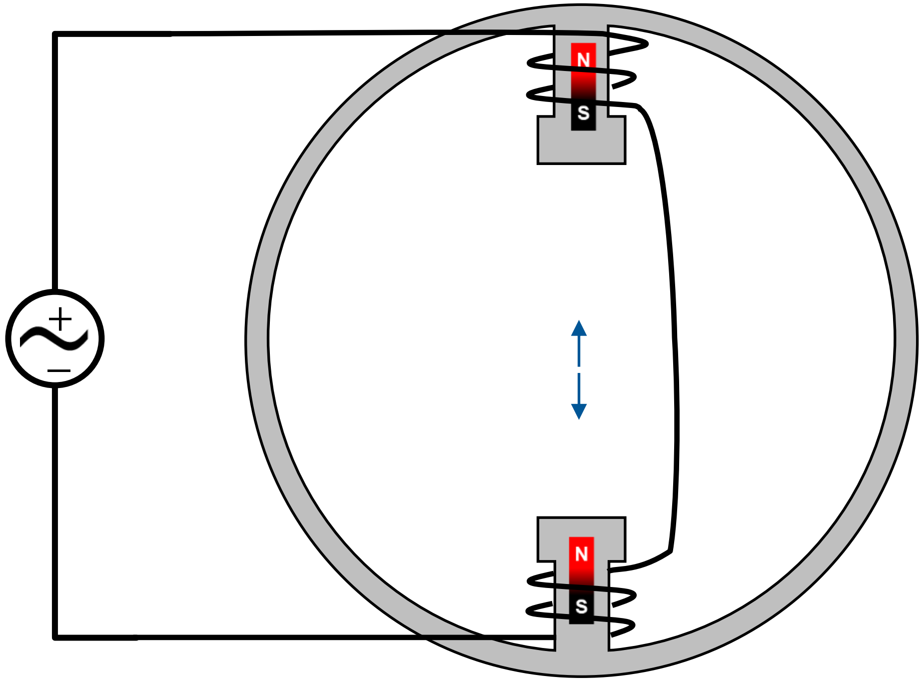
Stator

If such a pair of poles is energised with AC, we get a sinusoidally varying magnetic field



Stator

If such a pair of poles is energised with AC, we get a sinusoidally varying magnetic field

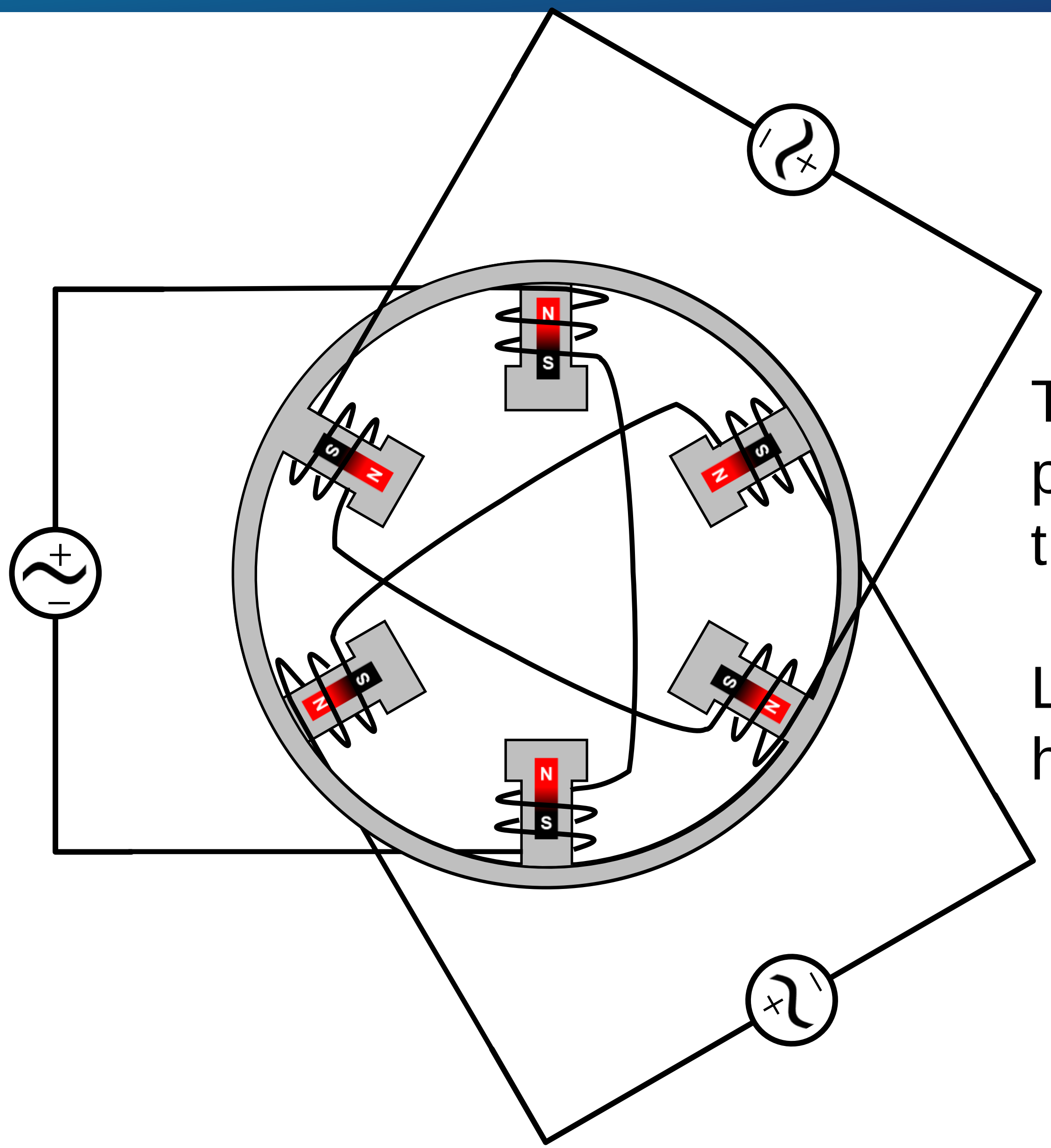


But, with a single pair of poles and single phase AC, we only get a single dimension of motion

What we want is a 2-dimensional rotation motion!

Stator

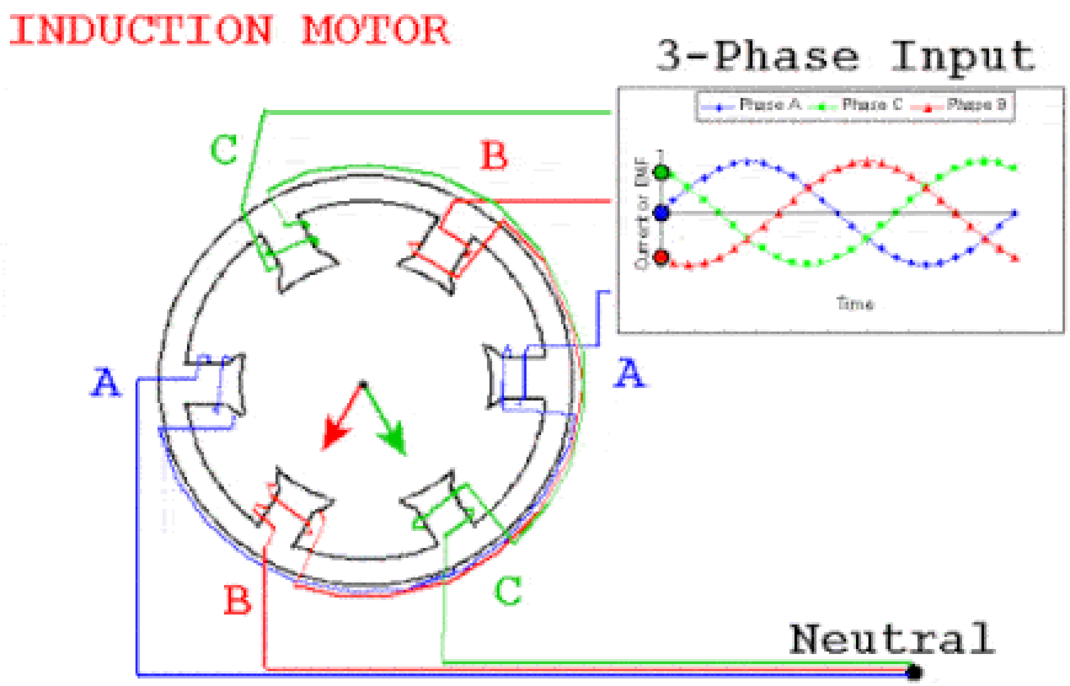
So we basically need to have the "pole pairs" multiplied thrice and spaced equally apart in a total revolution, i.e., 120° apart



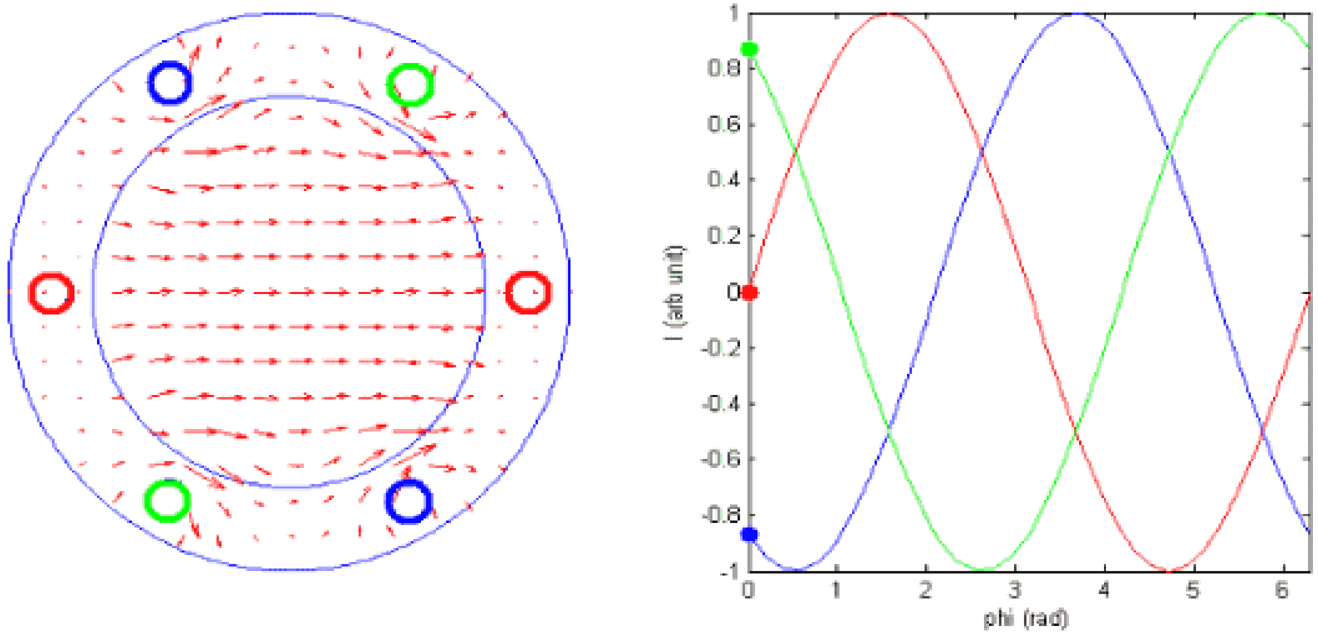
Then, we apply a 3-phase AC voltage to these pole pairs

Lets watch what happens then!

Stator



https://axljoann.blogspot.com/2021/05/3-phase-induction-motor-hitachi-three.html

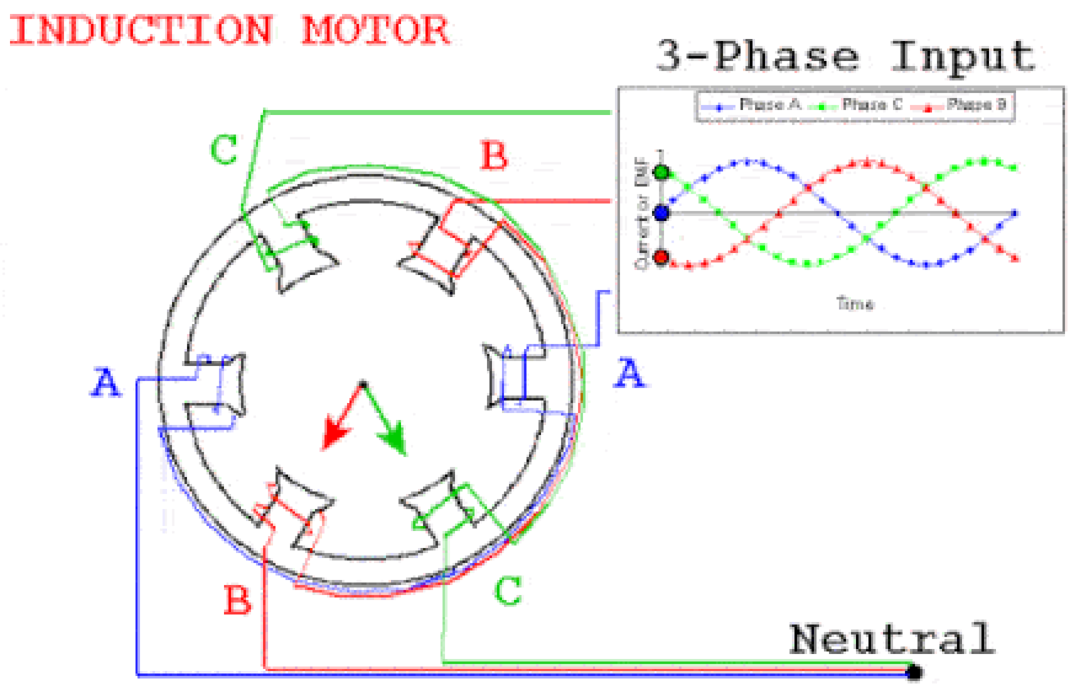


https://medium.com/@abhisheksingh73017/how-an-induction-motor-starts-real-answer-from-an-engineer-65f2fd7fa5b1

Rotating Magnetic Field

Requires AC (smooth motion) and 3-phase (direction stability)

Stator



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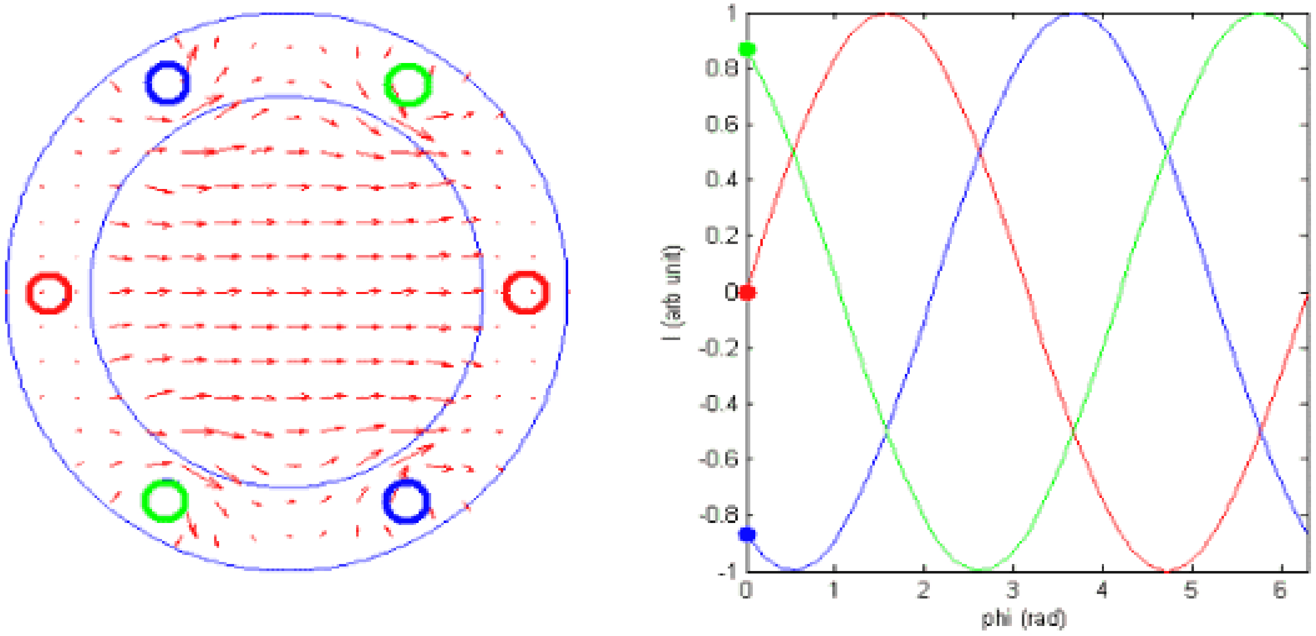

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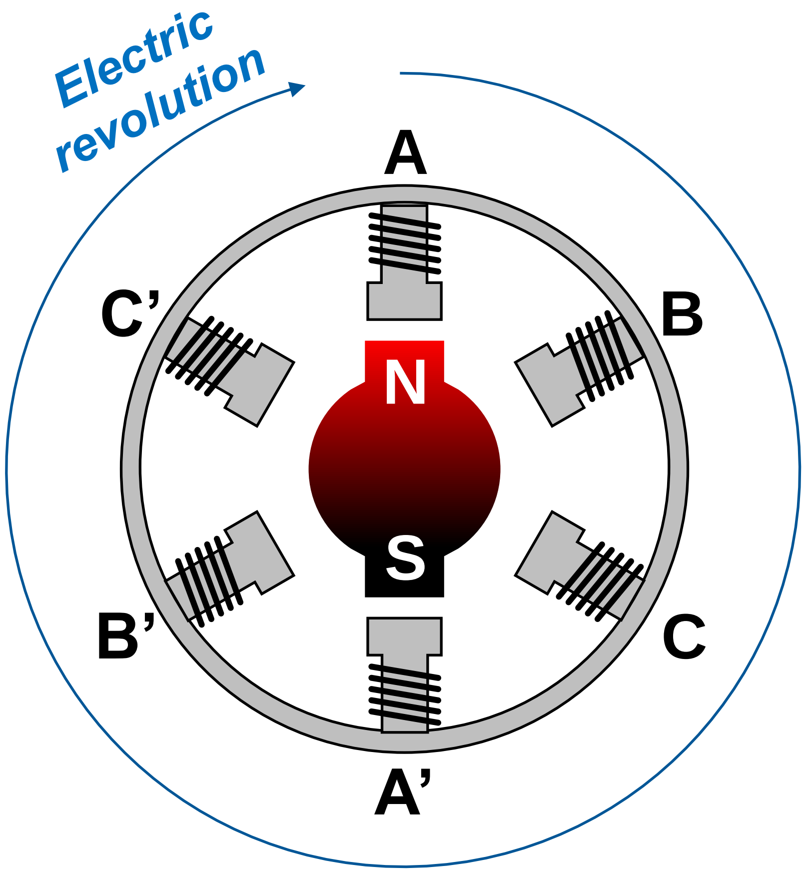
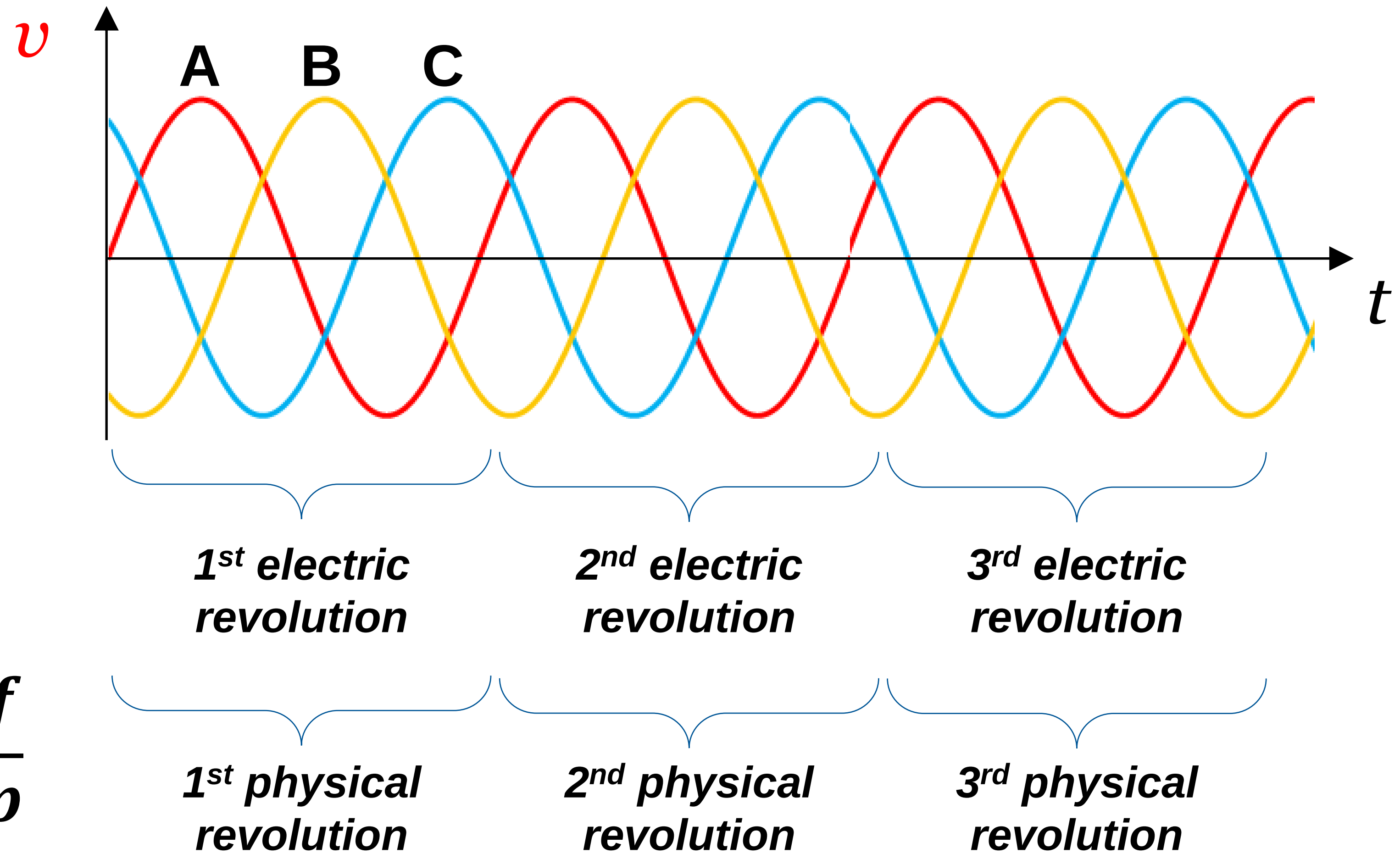
The speed of rotation is called "synchronous speed" which is nothing but the 3-phase AC frequency!

$$n_s(Hz) = f \text{ or } n_s(RPM) = 60 \times f$$

Multiple Pole Pairs

Dividing the physical "angular space" into multiple sets of 3-phase

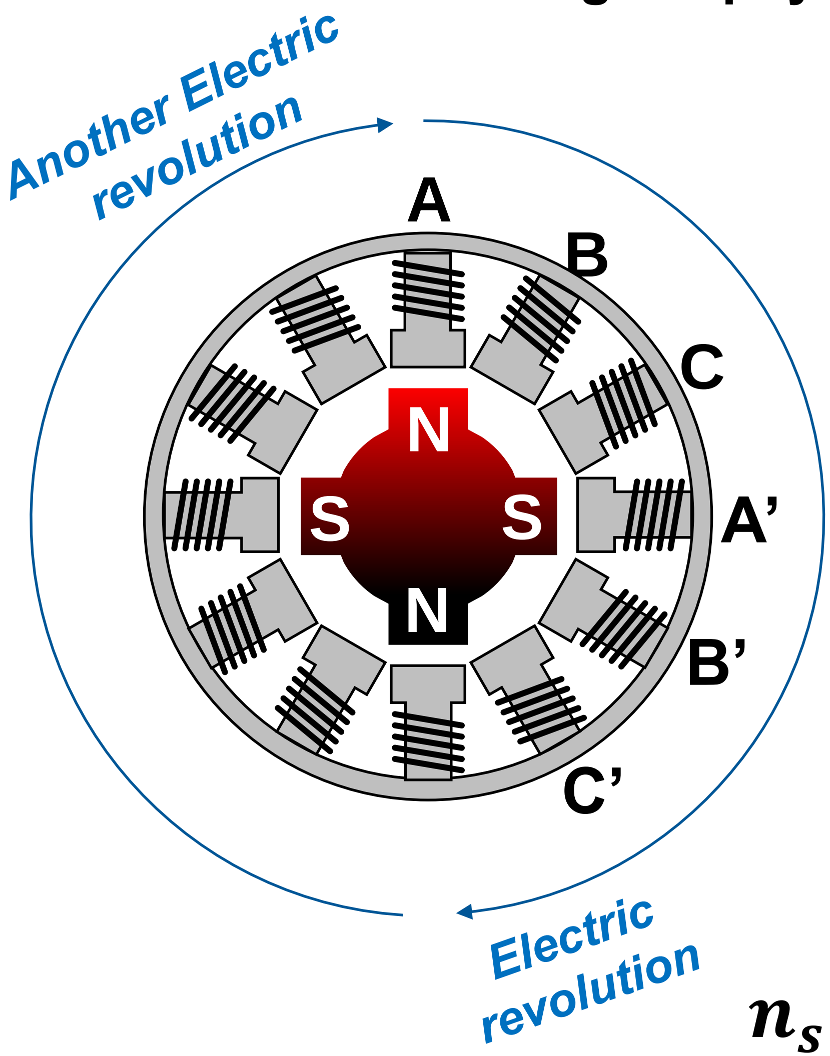




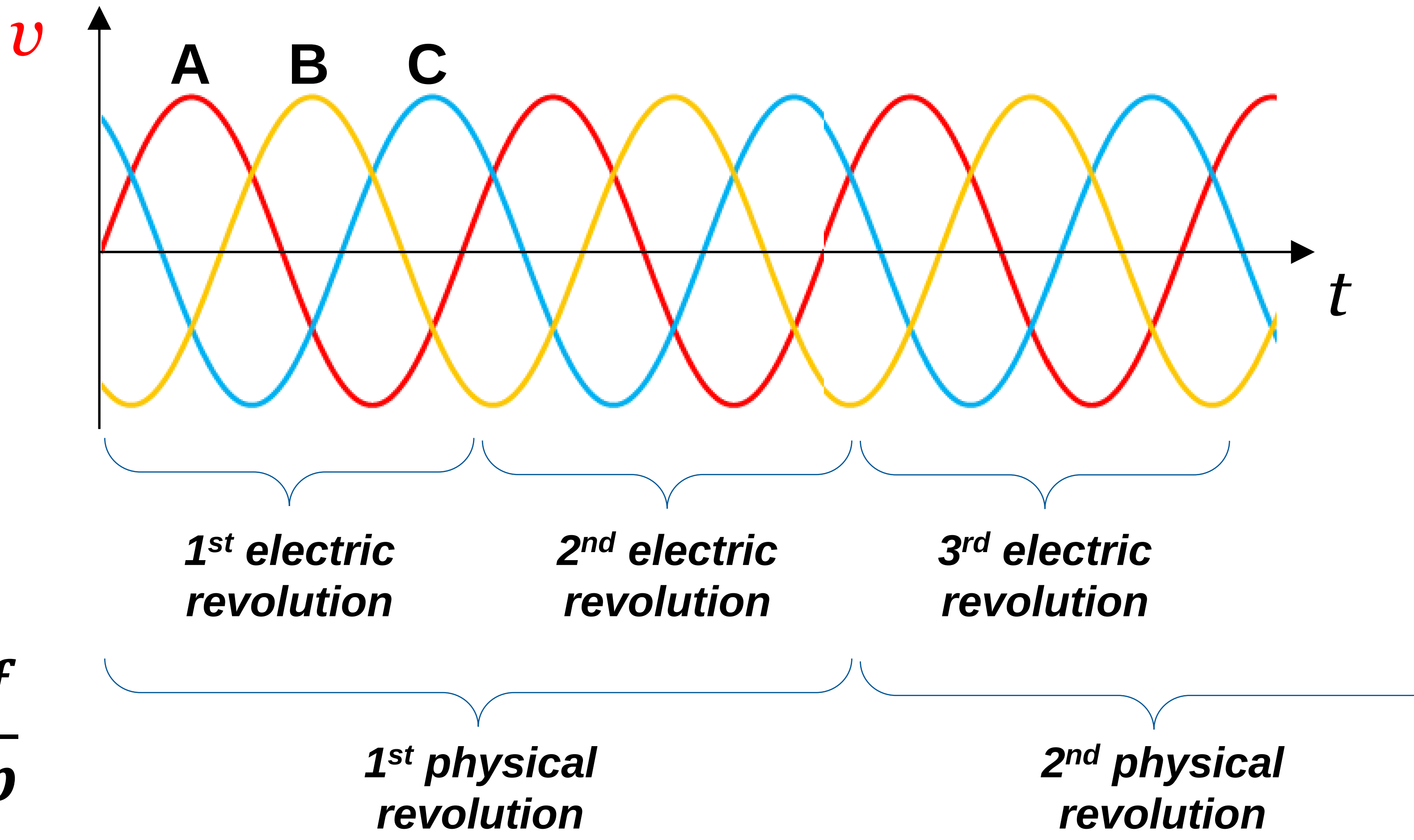
$$n_s = \frac{f}{p}$$

Multiple Pole Pairs

Dividing the physical "angular space" into multiple sets of 3-phase

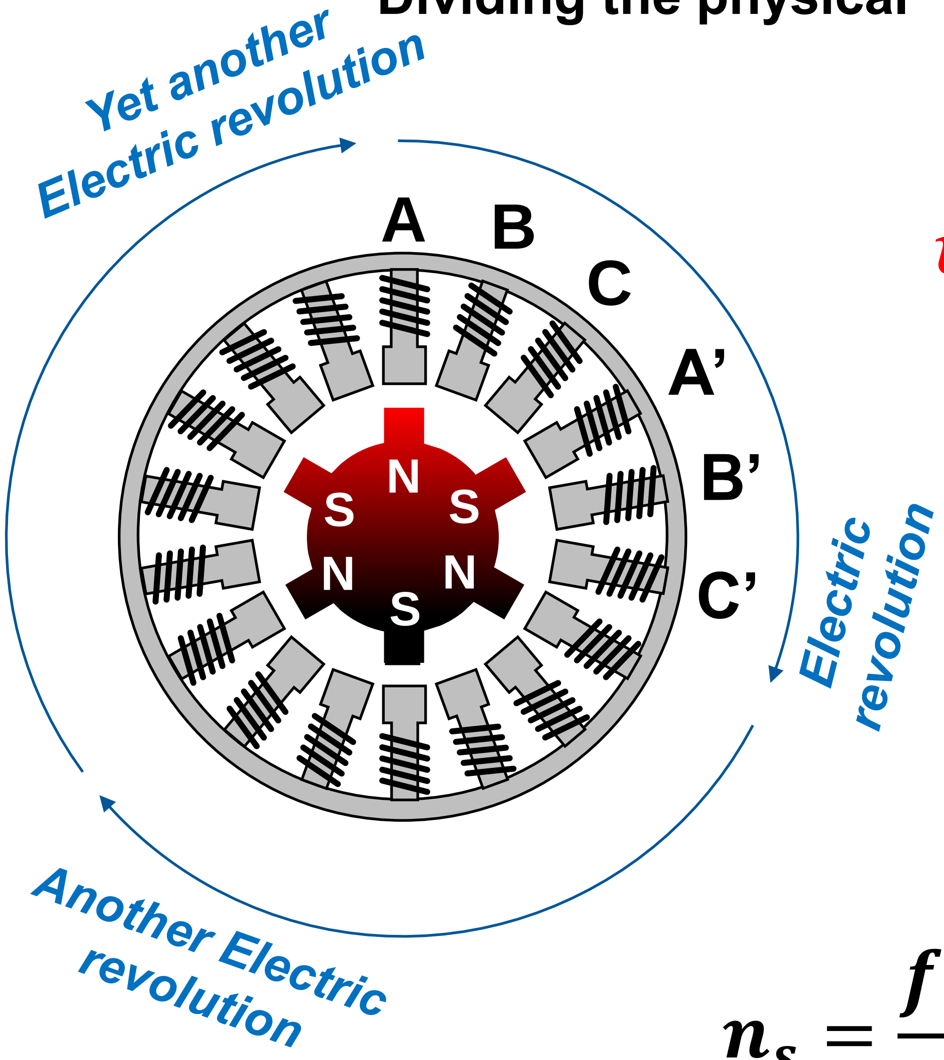


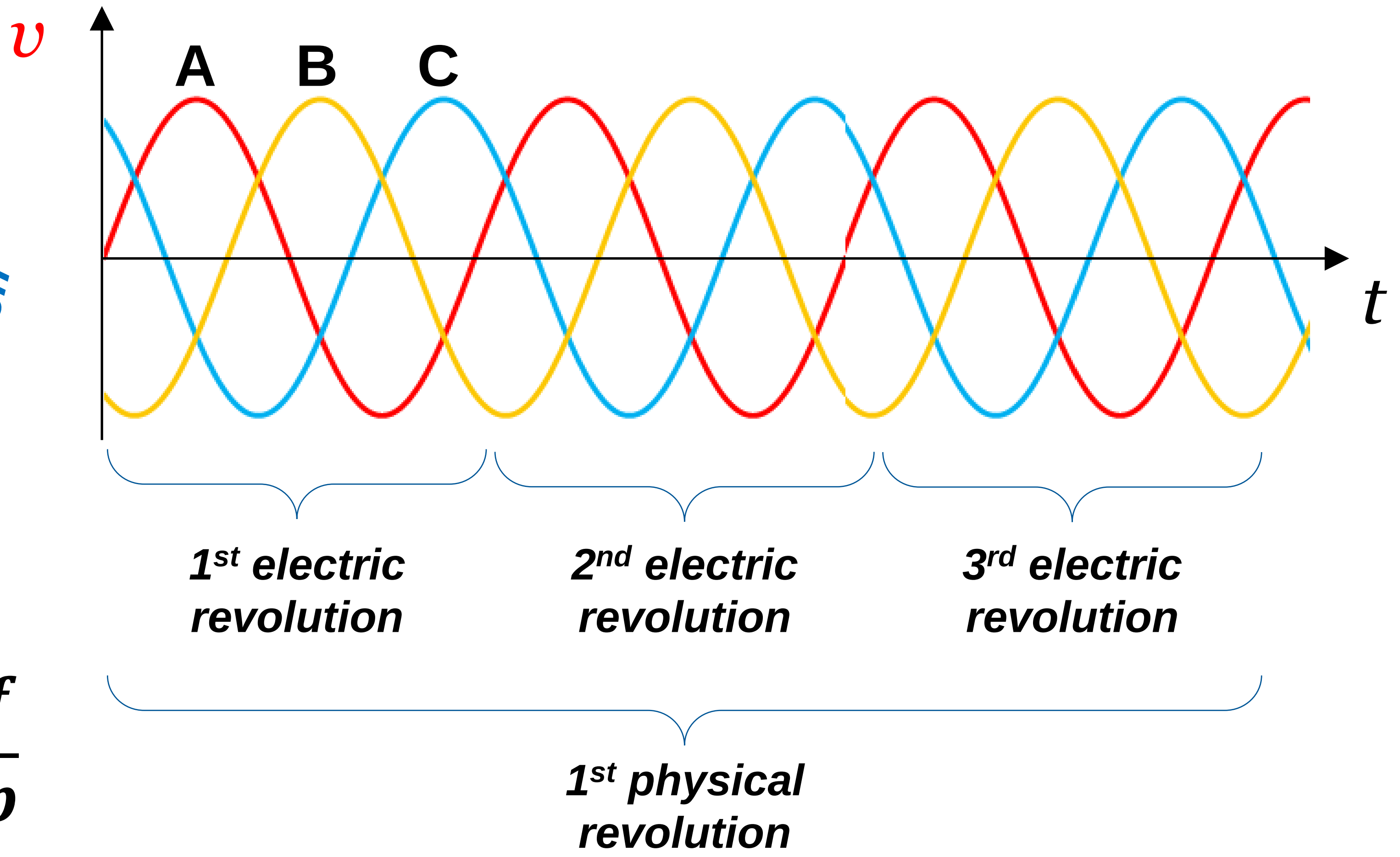
$$n_s = \frac{f}{p}$$



Multiple Pole Pairs

Dividing the physical "angular space" into multiple sets of 3-phase





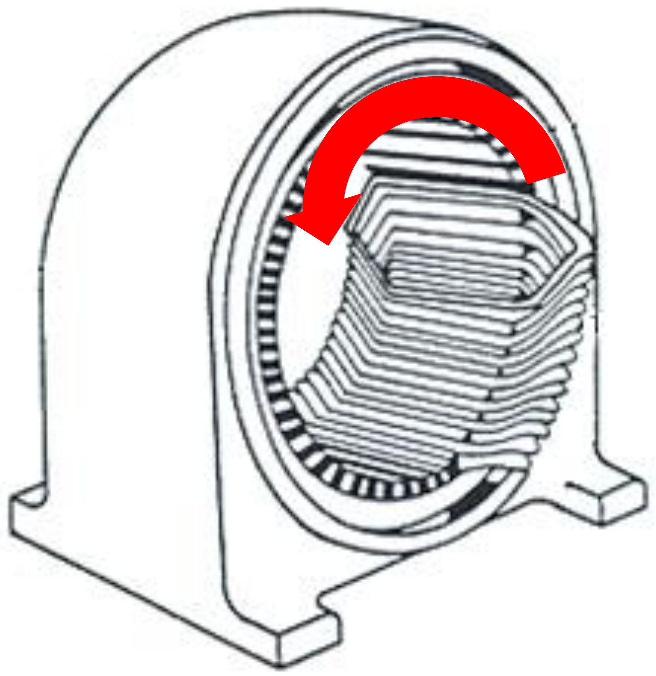
$$n_s = \frac{f}{p}$$

Multiple Pole Pairs

Number of Poles per phase	P (number of pole pairs per phase)	Sync Speed	Sync Speed for $50\ Hz$ standard AC supply (RPM)
2	1	$n_s = \frac{f}{p}$ (Hz) $n_s = \frac{60 \times f}{p}$ (RPM)	3000
4	2		1500
6	3		1000

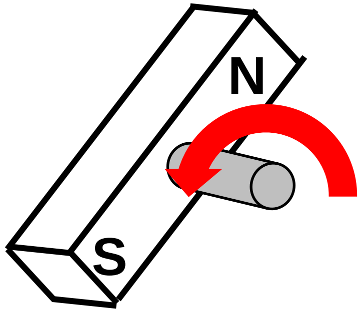
Rotor

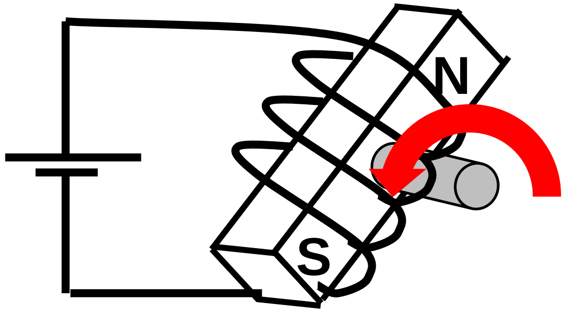
OK, so now we have a stator which is neatly producing a **rotating magnetic field** inside the central hollow space



That's great, but **how to make motion**?

- We can put a **permanent magnet** in this "field" and attach it to a shaft? This "real magnet" would want to follow this "imaginary magnet"
- Or we can think of a smart logic to **induce another "imaginary magnet"** in the rotor – saves us money/hassle in getting permanent magnets!



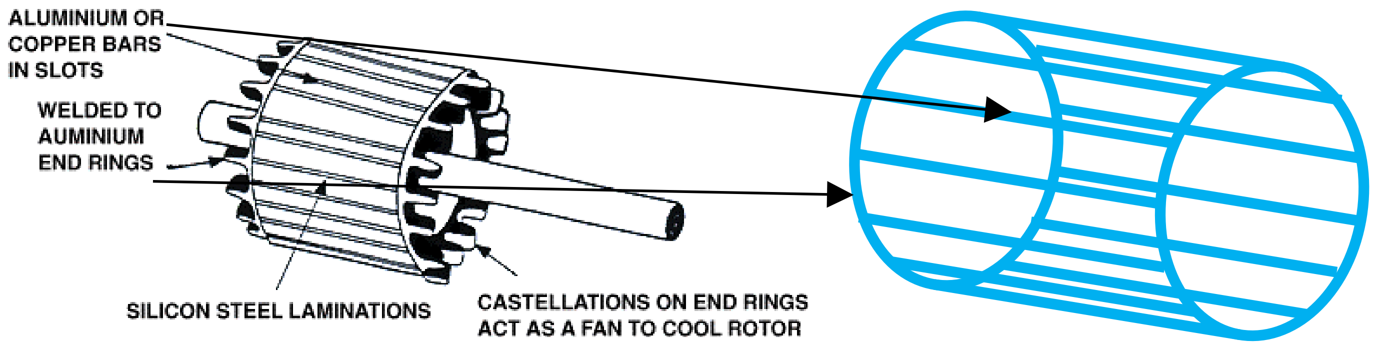


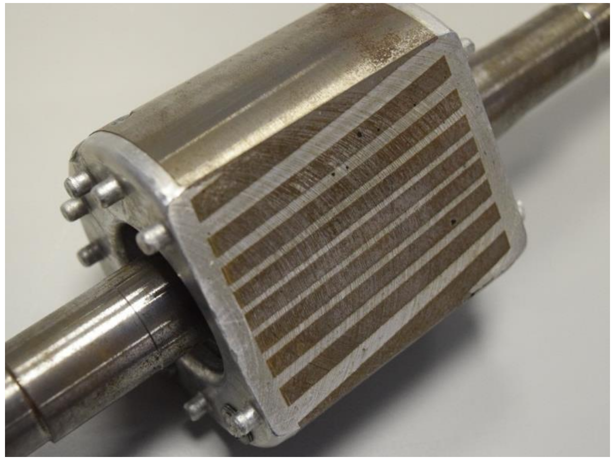
Rotor

How to induce another "imaginary magnet" in the rotor using the field produced by the stator?

We insert a set of shorted-out conductors forming the rotor – **squirrel-cage rotor**

Before we attempt to understand how this works, we need to learn a bit more about Electromagnetism





Electromagnetism and Fleming's Left/Right Hand Rules

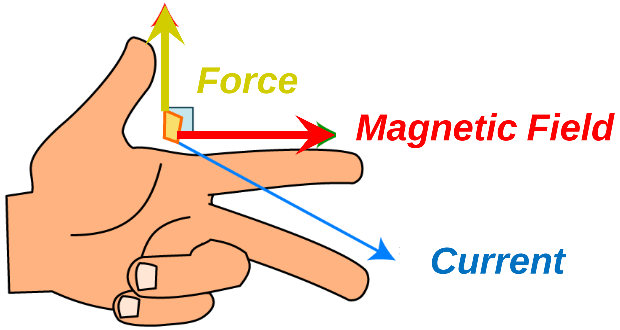
What is Electromagnetism?

Mathematical relationship between:

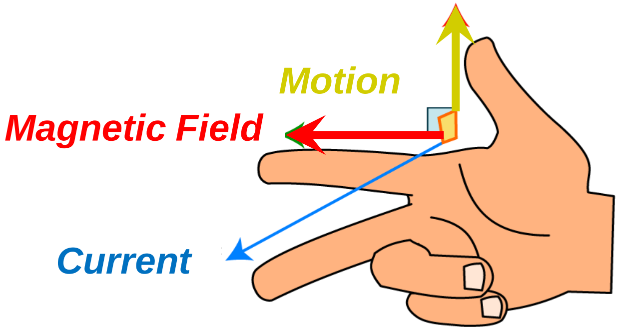
- **Current**
- **Magnetic Field**
- **Motion/Force**

Sir John Ambrose Fleming was an English electrical engineer and physicist who invented the first radio transmitter (among other things). He lectured at the University of Nottingham in 1881 (it was not called that then though!)

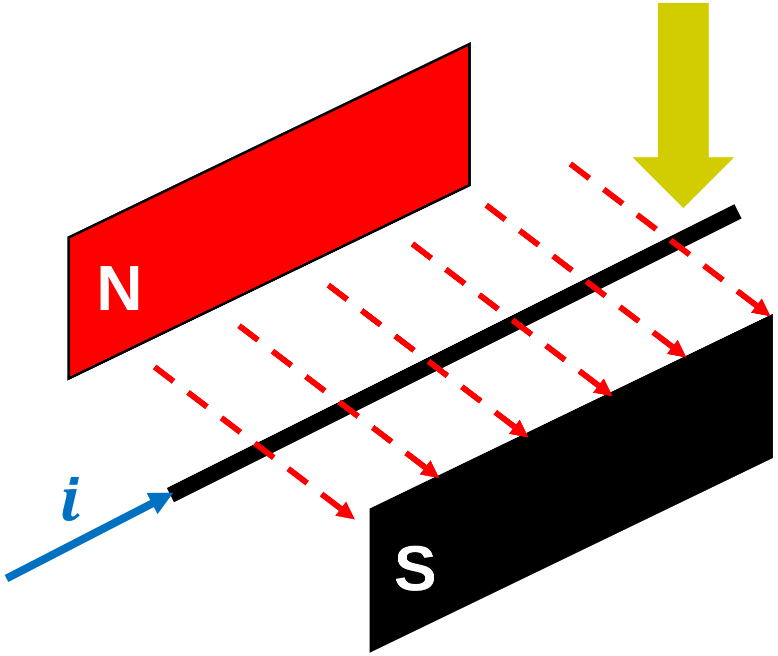
Left-Hand Rule (Motors)

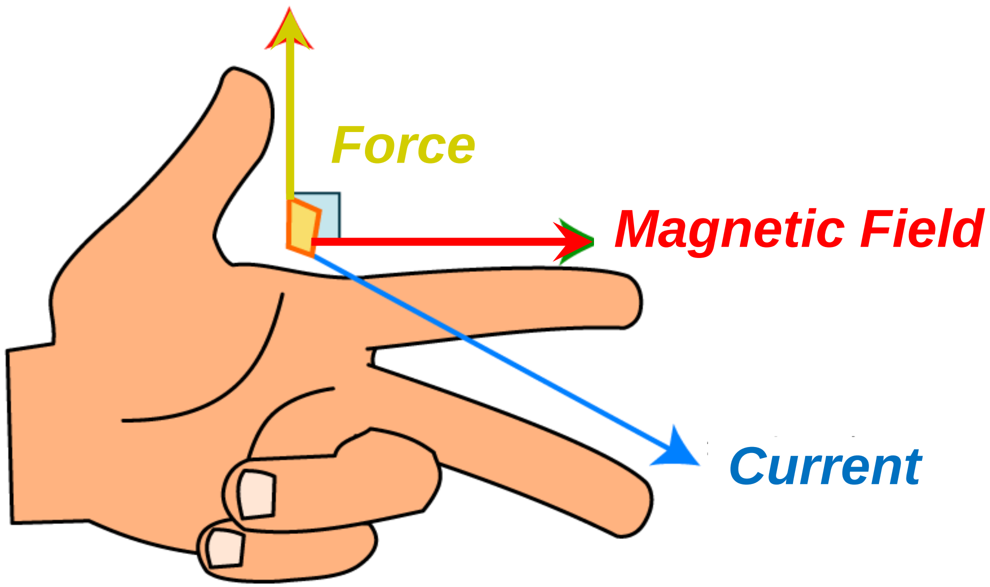


Right-Hand Rule (Generators)



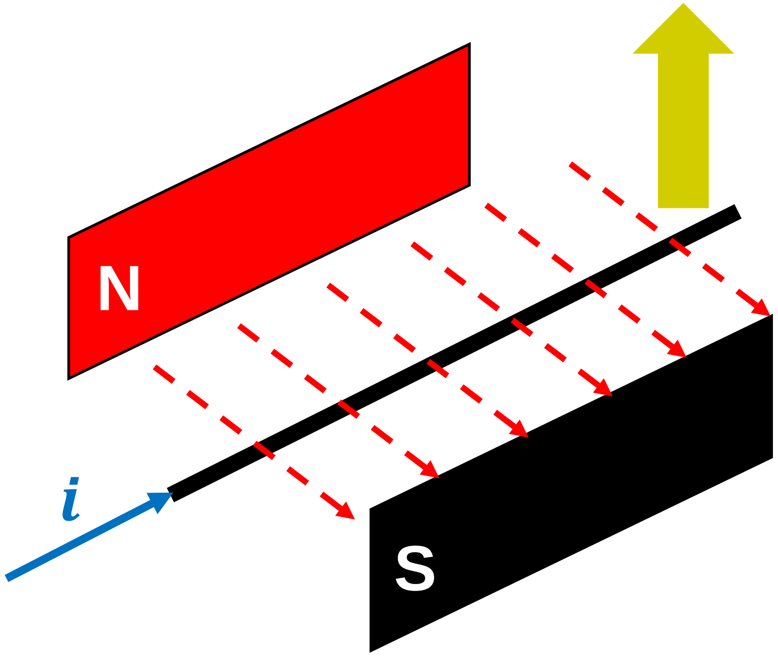
Left-Hand Rule (Motors)

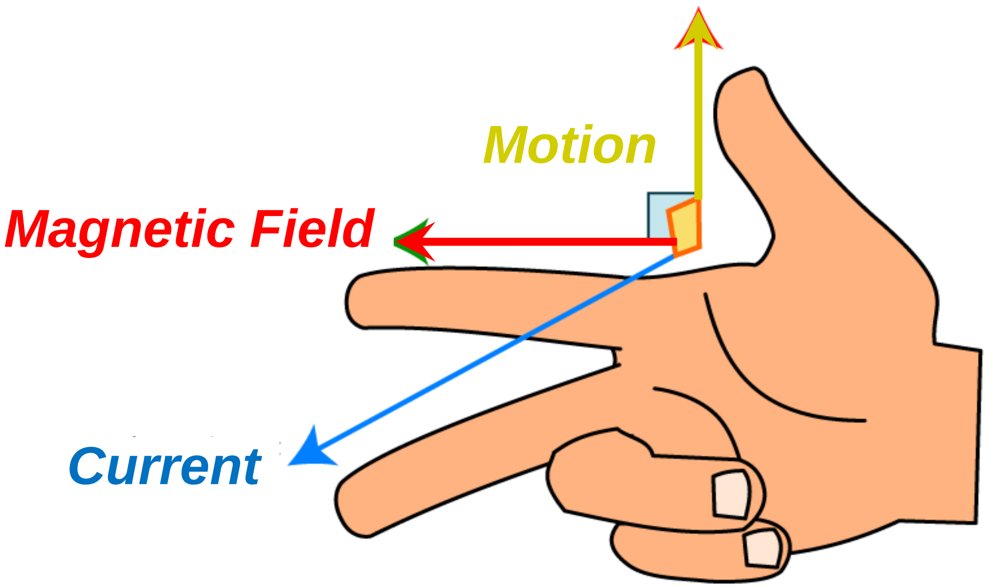




A current-carrying conductor in a magnetic field experiences a force/thrust

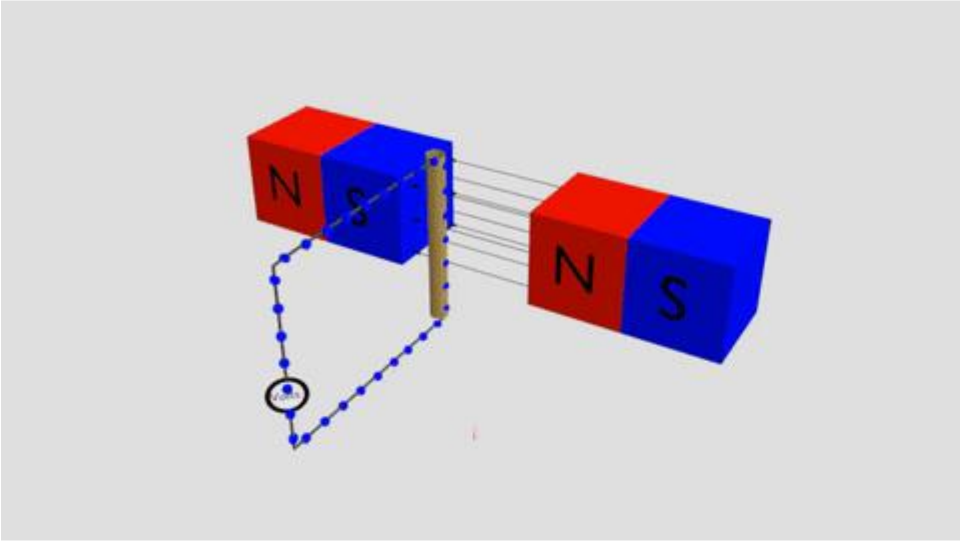
Right-Hand Rule (Generators)

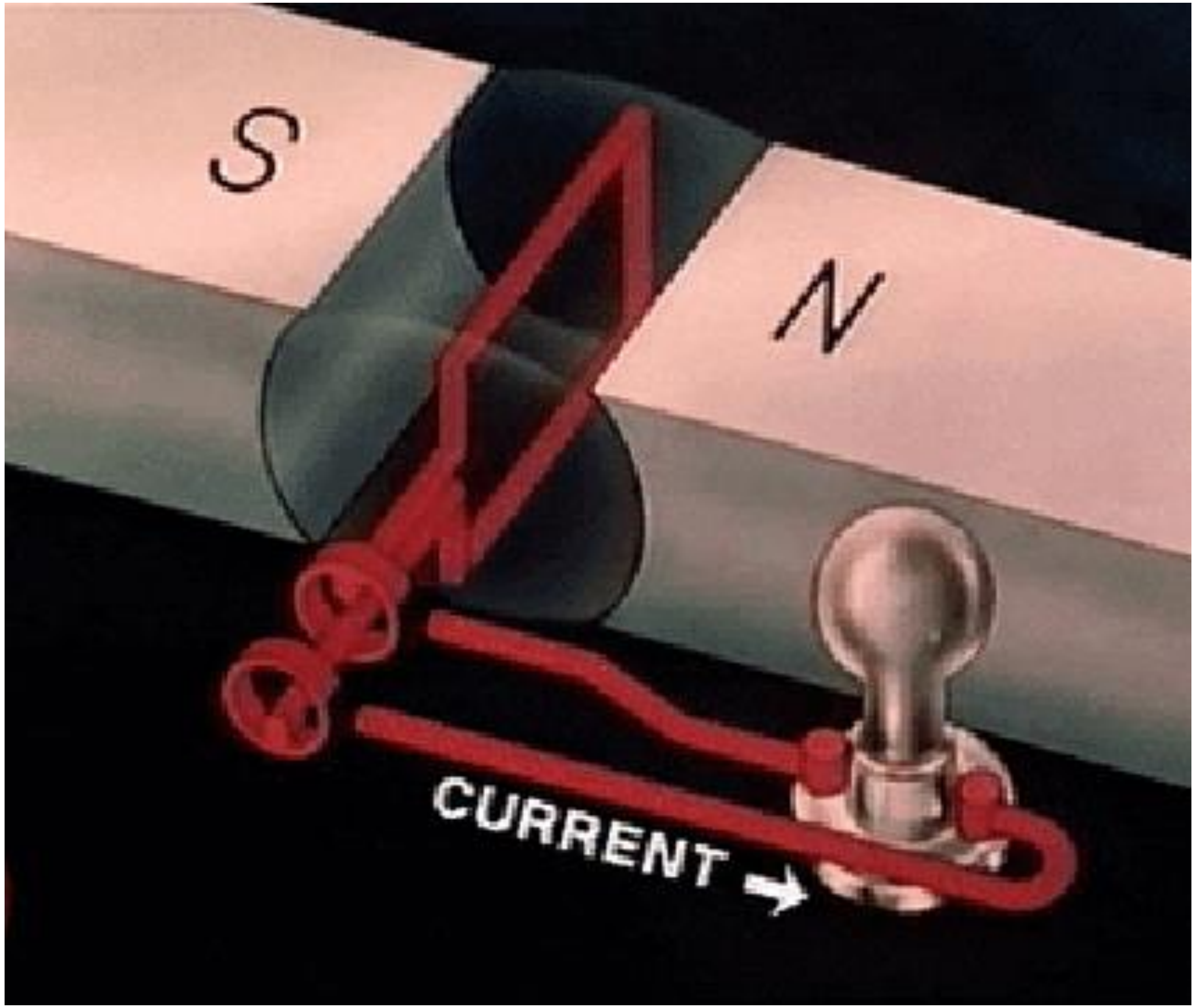




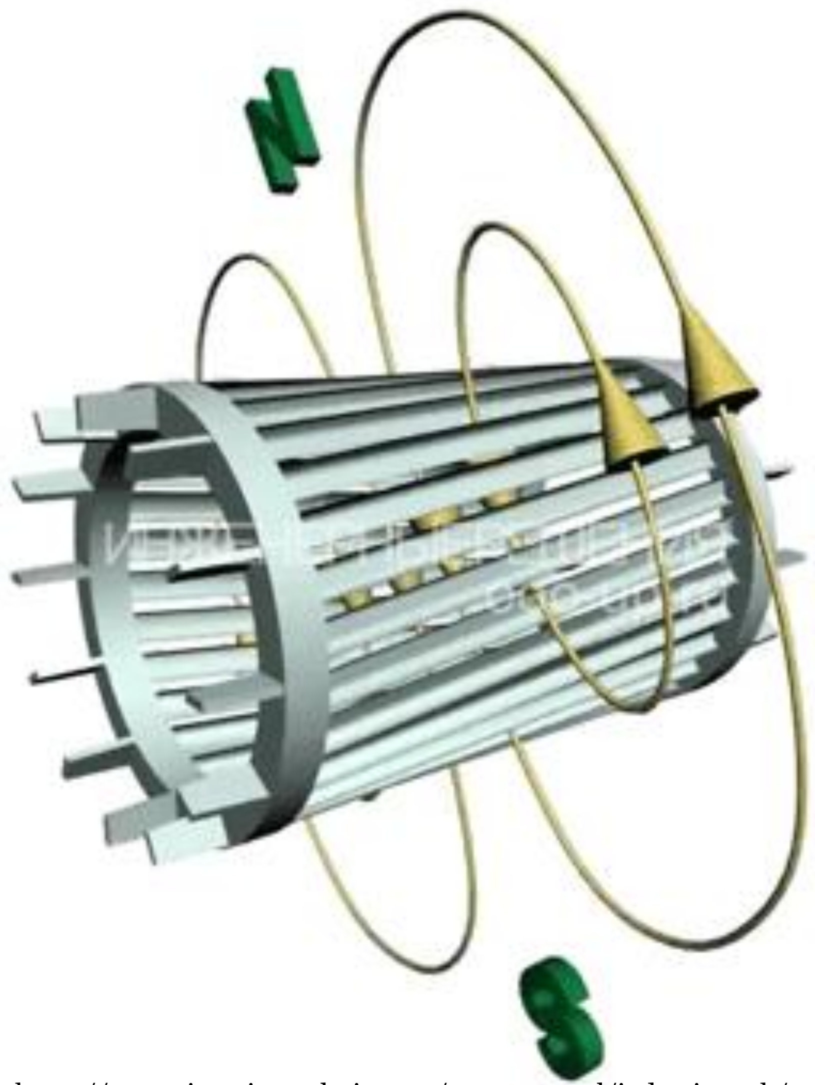
A conductor moving in a magnetic field generates a voltage across itself (current produced if circuit was to be completed)

Electromagnetism and Fleming’s Left/Right Hand Rules





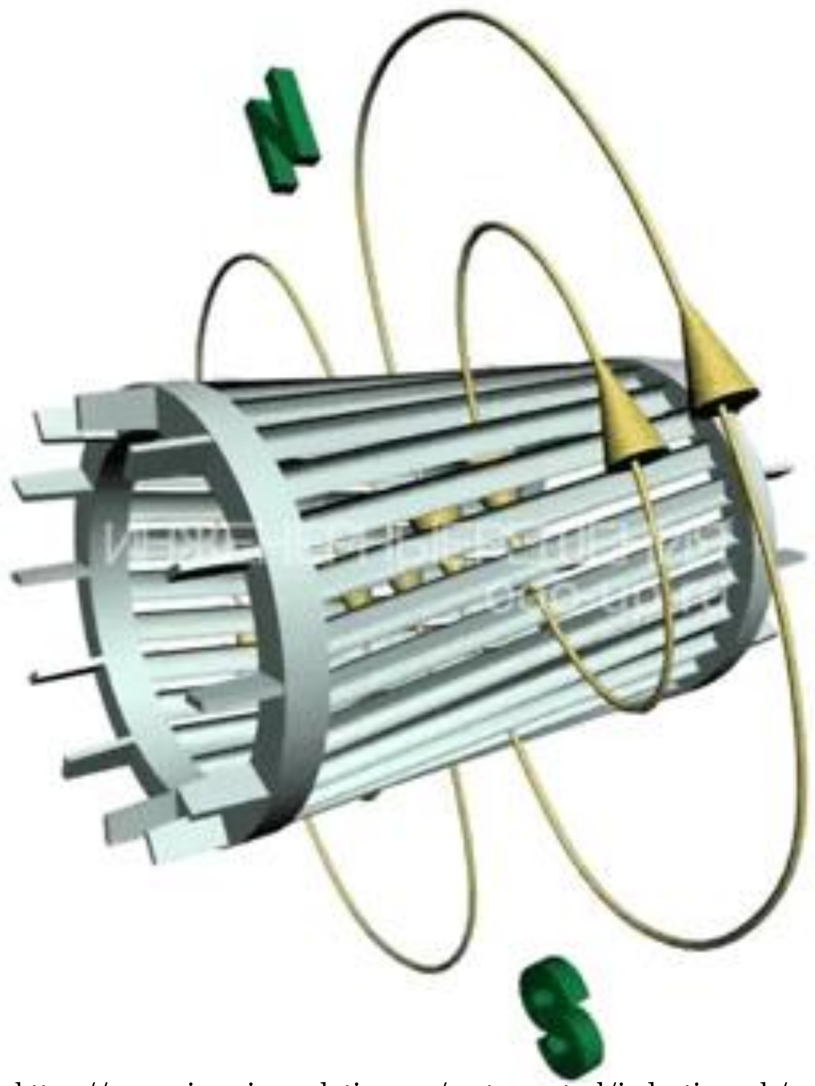
Rotor



https://en.engineering-solutions.ru/motorcontrol/induction3ph/

- Rotating **Magnetic Field** produced by the stator is continually **cutting a conductor**
- An **EMF gets generated** (RH Rule)
- In a squirrel cage rotor, everything is shorted! Hence, **current flows**
- Now the conductor is a **current-carrying** conductor
- Current-carrying conductor experiences a **force** in the **magnetic field** (LH Rule)
- **Rotor rotates!**

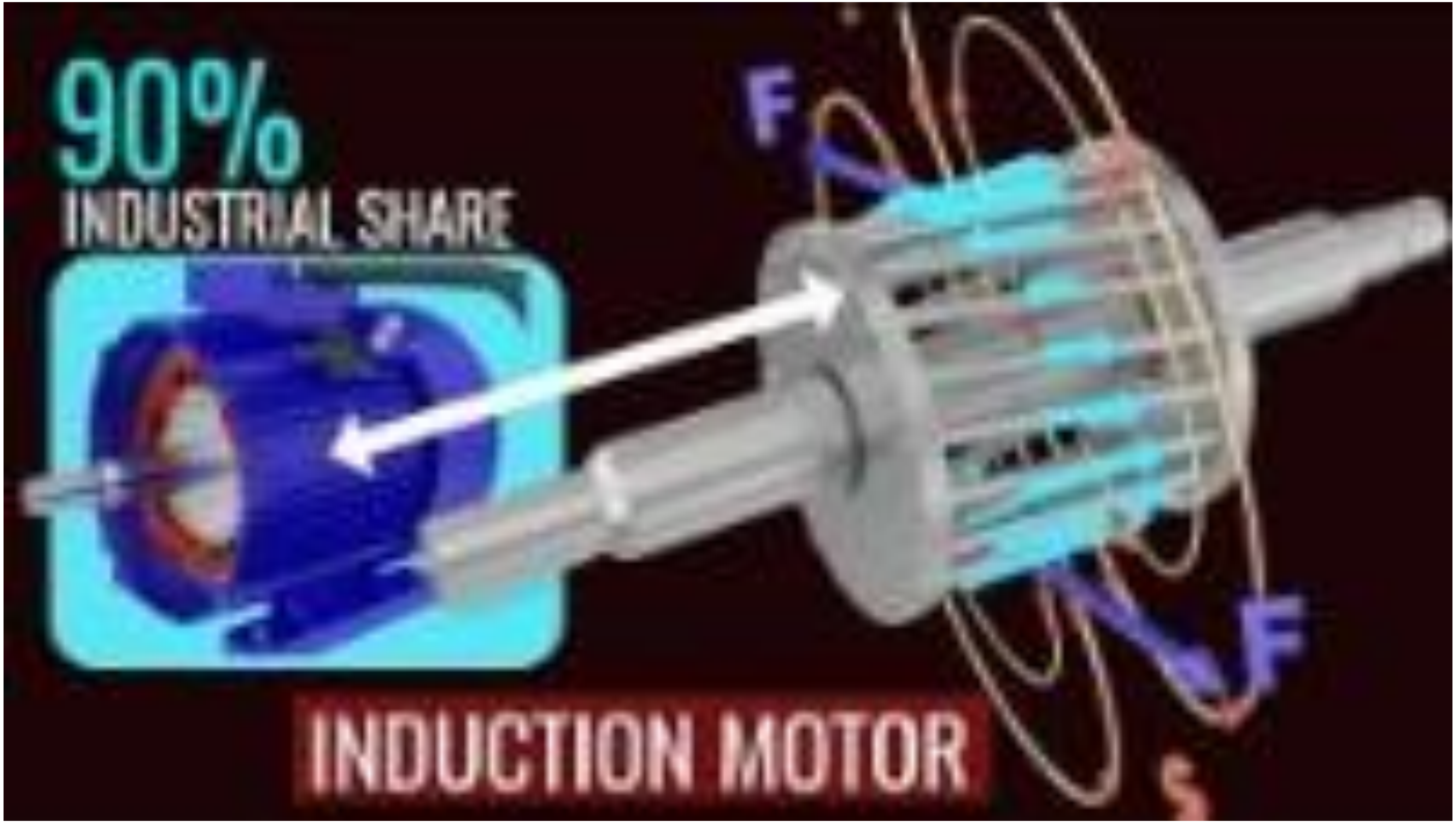
Rotor



https://en.engineering-solutions.ru/motorcontrol/induction3ph/

- **Synchronous** Speed = Speed of the **rotating magnetic field**, i.e., **stator** field, i.e., **input supply**
- Recall RH Rule:
  - field must **cut the conductor,** to
  - **generate EMF**, to
  - **generate current**, to
  - **generate force**
- **Rotor needs to slip** (allowing the cutting) to produce any torque
- **Higher slip = higher torque**

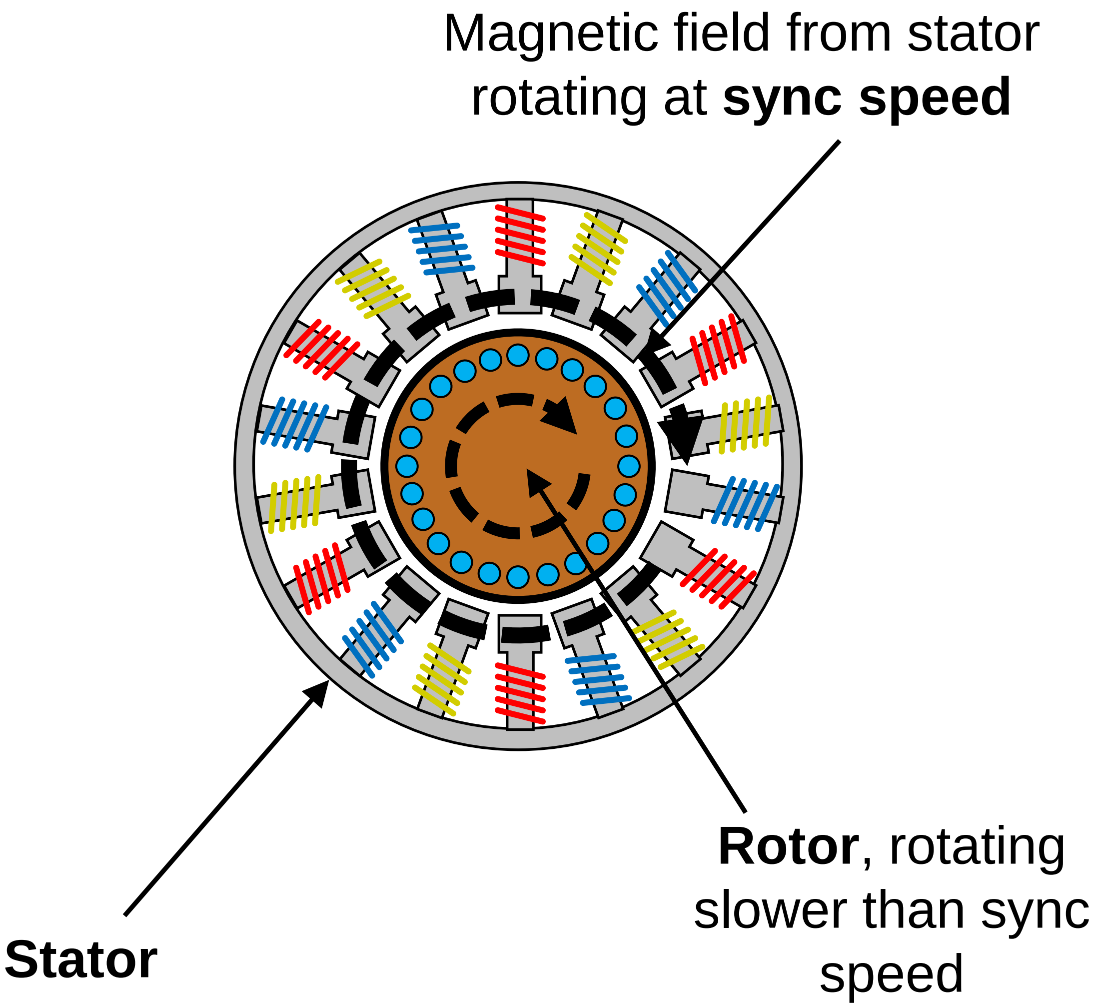
Operating Principle of the Induction Motor



Operating Principle of the Induction Motor

The induction motor consists of the:

- Stator with magnetic field rotating around its inside
- Rotor which is inside the stator, experiencing the rotating magnetic field
- Output shaft is in one piece with rotor
- When rotor is pulled around by field, it drives the shaft, and we get useful power out



Characteristic Equation of an Induction Motor

$$T = \frac{3p}{2\pi f} \times \frac{V^2 as}{X_R(a^2 + s^2)}$$

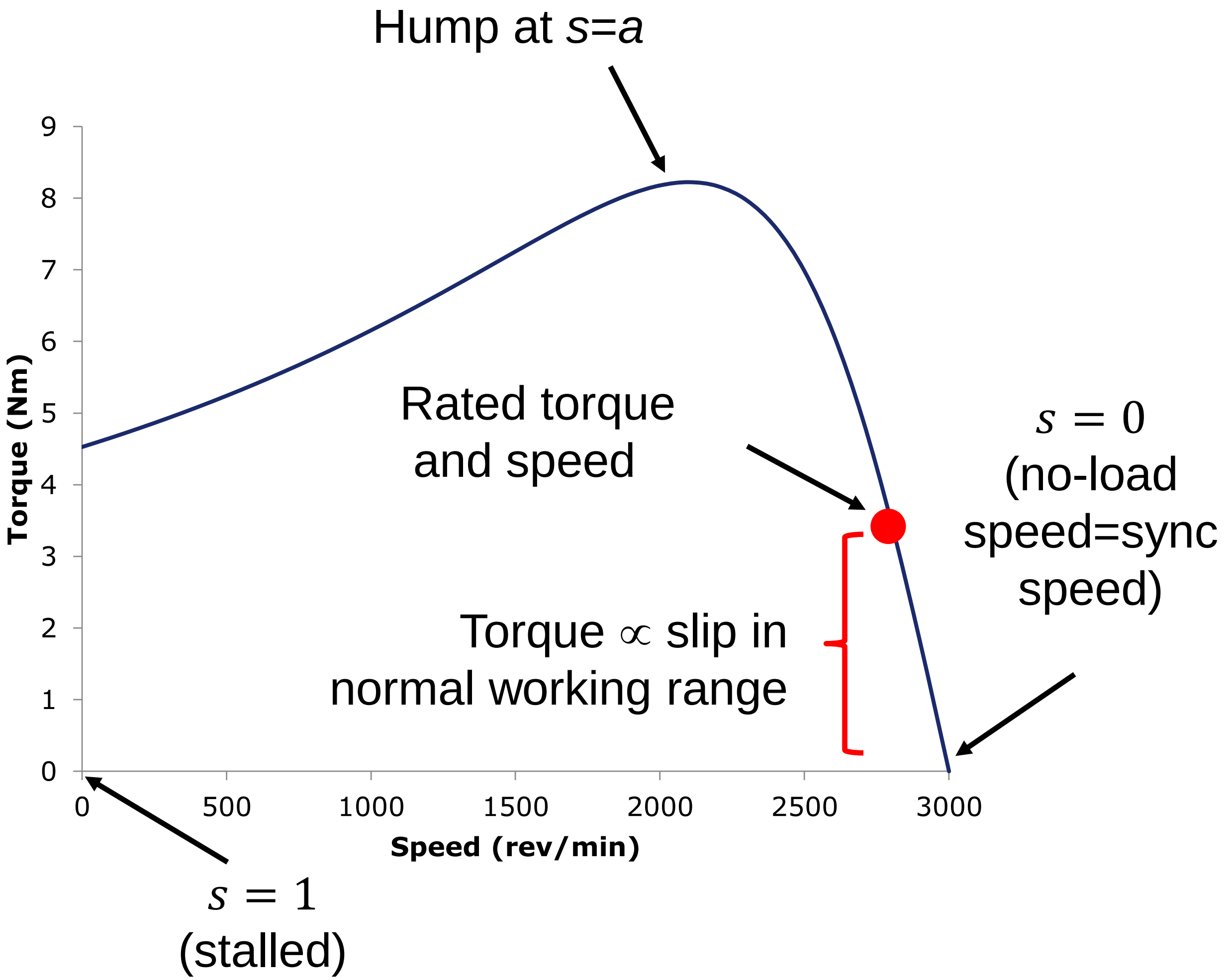
- T –Torque in star-connected motor
- p – Pole pairs per phase
- f – Supply frequency
- V – Supply phase voltage
- $a = \frac{R_R}{X_R}$ – Resistance-to-reactance ratio of rotor
- $s = \frac{n_s - n}{n_s}$ – Per-Unit slip (n_s – Sync Speed)
- n – Actual speed of rotor (same unit as sync speed)
- X_R – Reactance of Rotor

Characteristic Equation of an Induction Motor

- No-load speed = synchronous speed
- Torque $\propto$ slip (approx.) for small torques
- Torque-speed characteristic has "hump" at $s = \frac{R_R}{X_R} = a$
- Under running conditions slip is small e.g. 5%
- By setting $\frac{dT}{ds} = 0$, can show that maximum ("pull-out") torque is

$$T_{max} = \frac{3p}{4\pi f}\frac{V^2}{X_R}$$

- Motor stalls if load torque T reaches $T_{\max}$



Example

A 3-phase star-connected 415V 2 pole 50Hz induction motor has a rotor resistance 1.2Ω/phase and rotor standstill reactance 6 Ω per phase.

Driving a mechanical load the motor runs at 2900 rev min^{-1}.

Calculate

a) per unit slip

b) torque

c) mechanical output power

a) 2 poles per phase, so

$$p = \text{no of pairs of poles} = 1$$

$$f = 50\text{Hz, so}$$

$$n_s = 60\frac{f}{p} = \frac{60 \times 50}{1} = 3000 \text{ rev min}^{-1}$$

$$s = \text{ per-unit slip } = \frac{n_s - n}{n_s}$$

$$= \frac{3000 - 2900}{3000} = \mathbf{0.0333}$$

Example

A 3-phase star-connected 415V 2 pole 50Hz induction motor has a rotor resistance 1.2Ω/phase and rotor standstill reactance 6 Ω per phase.

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Calculate

a) per unit slip

b) torque

c) mechanical output power

$b)\ \frac{R_R}{X_R} = \frac{1.2}{6} = 0.2$

$$T = \frac{3p}{2\pi f} \times \frac{V^2 as}{X_\mathrm{R}(a^2 + s^2)}$$

But note star connection, so

$$V = V_P = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6\mathrm{V}$$

$$T = \frac{3 \times 1}{2\pi \times 50} \times \frac{239.6^2 \times 0.2 \times 0.0333}{6(0.2^2 + 0.0333^2)}$$

$$= \mathbf{14.81\ Nm}$$

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Driving a mechanical load the motor runs at 2900 rev min^{-1}.

Calculate

a) per unit slip

b) torque

c) mechanical output power

c) Recall from Dynamics module in 1st year:

Mechanical Power (W) = Torque (Nm)×Angular Velocity (RPM)

$$\omega = 2900 \times \frac{2\pi}{60} = 303.7\mathrm{rad\ s^{-1}}$$

$$\mathrm{P} = T\omega$$

$$P = 14.81 \times 303.7 = 4500\ \mathrm{W} = \mathbf{4.5\ kW}$$

Summary

- 3-phase AC
  - **Star** v **Delta**
  - **Line** v **Phase**
- Induction Motor
  - Operation Principle
  - **Stator** & **Rotor**
  - Concept of **Electromagnetism** (**Fleming's** Left & Right Hand Rule)
  - **Synchronous** & **Asynchronous**

Attendance

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