MMME2046 Dynamics: Control Lecture 5

Improving Transient and Steady-State Performance. Stability of Feedback Systems

Next week

- 8/3/2021
  - The return of DB3!

Lecture Objectives:

- Demonstrate how velocity feedback and PID controls can improve performance
- Develop basic understanding of stability and behaviour in third and higher order systems
- Introduce and use the Routh-Hurwitz stability criteria

Summary Based on Case Studies

- Hydraulic Position Control System: 1st order

$$G(s) = \frac{\mu}{1 + Ts}$$

- Electro-Mechanical Position Control System: 2nd order

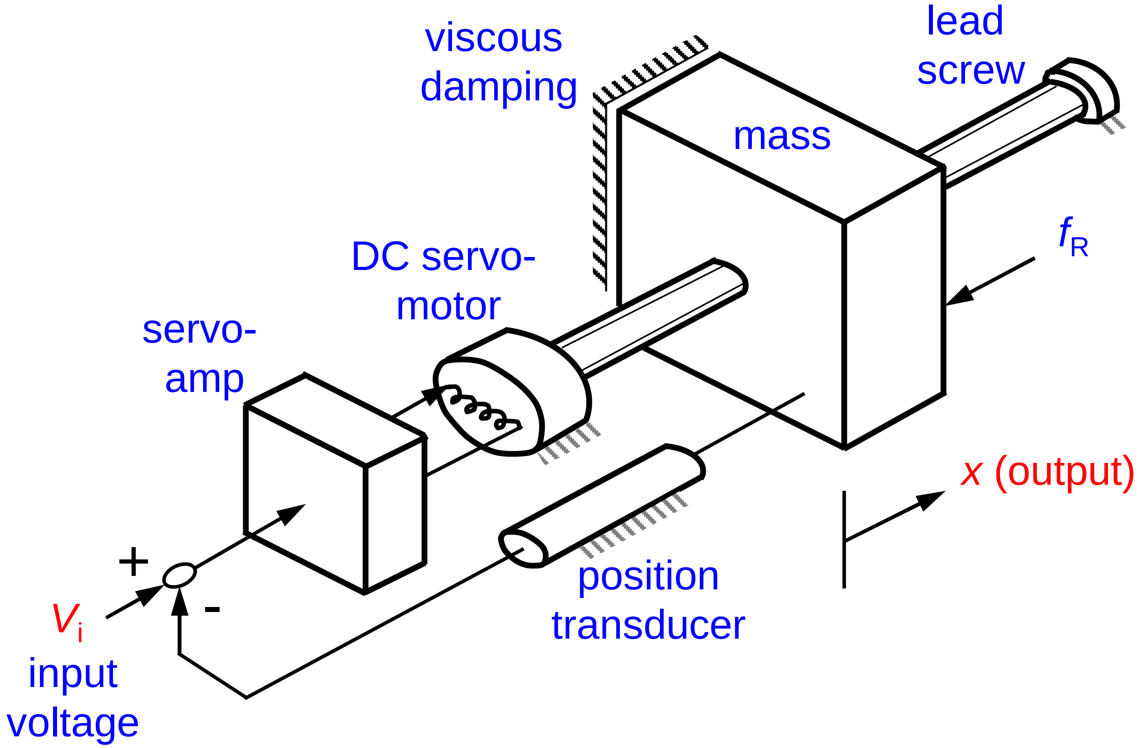
$$G(s) = \frac{\omega_{\mathrm{n}}^2}{s^2 + 2\gamma\omega_{\mathrm{n}} s + \omega_{\mathrm{n}}^2}$$

- Steady-State Response/Error: Final Value Theorem

Result dependent upon type of input (step, ramp, etc.)

- Transient Response: Poles (roots of the C.E.)

Last Week's Example: Electro-Mechanical Position Control System



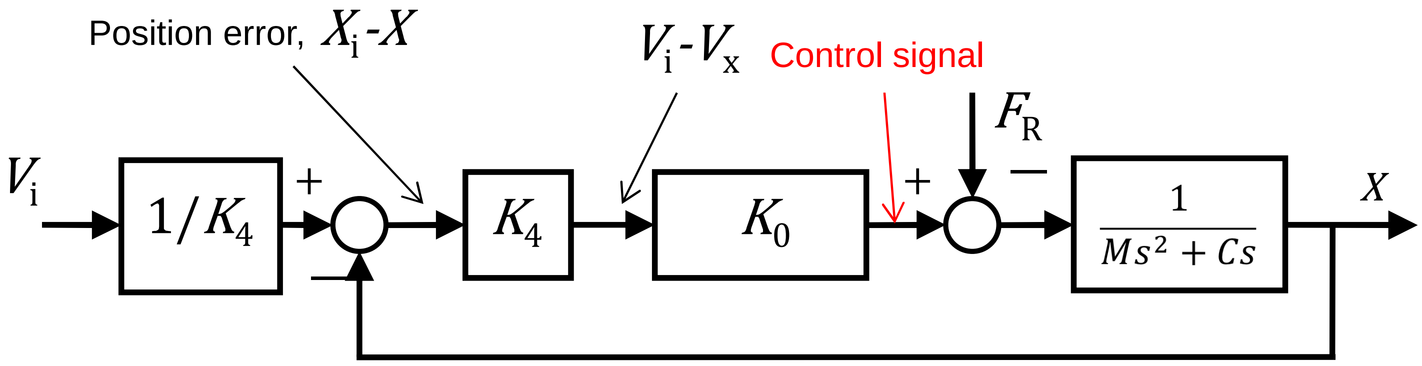
It was shown that the **transfer functions** may be written as

$$\frac{X(s)}{X_i(s)} = \frac{\omega_n^2}{s^2 + 2\gamma\omega_n s + \omega_n^2} \qquad \frac{X(s)}{F_R(s)} = \frac{-1}{M(s^2 + 2\gamma\omega_n s + \omega_n^2)}$$

2nd order system

Electro-Mechanical Position Control System

This is an example of a feedback system with **proportional** control.



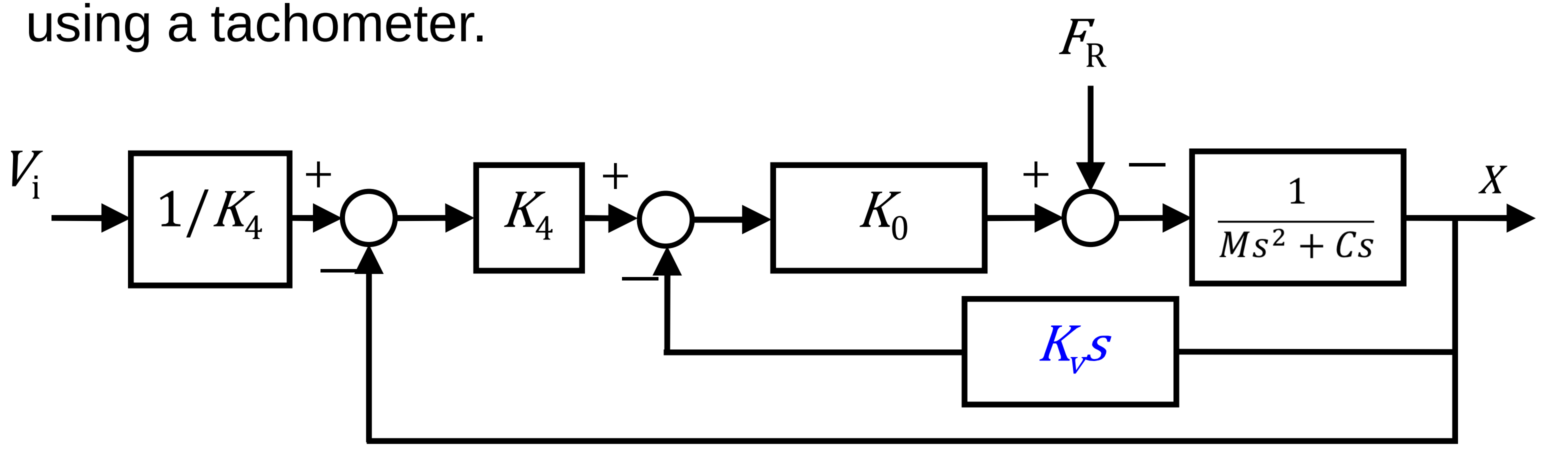
The control signal is a constant gain times the position error with $K_0 = K_1K_2K_3$ **(collecting the gains of servo-amplifier, servo-motor and lead screw)**.

Q: Are there any other forms of control that can improve transient and steady-state performance?

https://www.youtube.com/watch?v=7q4QDz1tcFw

a) Velocity Feedback

In addition to output feedback the **rate of change** of output is fed back. In a position control system, this element will be the velocity, measured using a tachometer.



The governing equation follows as:

$$[Ms^2 + (C + K_0 K_v)s + K_0 K_4]X(s) = K_0 K_4 X_i(s) - F_R(s)$$

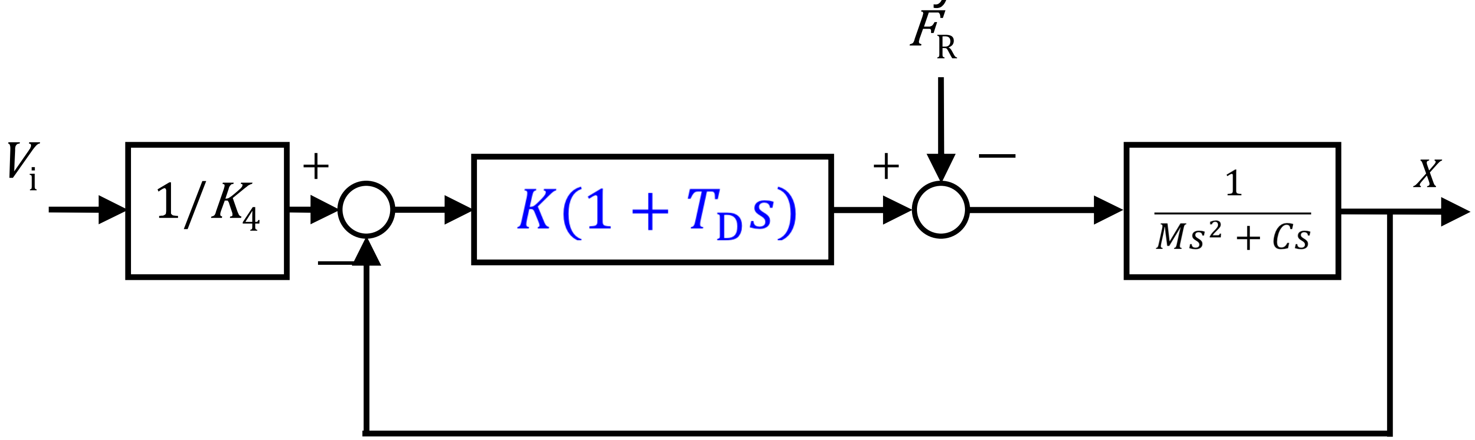
$$s^2 + 2\gamma\omega_n s + \omega_n^2 = 0$$ velocity feedback ➔ viscous damping

The Velocity lag ($F_R = 0$) under ramp remains

$$e_{ss} = \frac{[C + K_0 K_v]}{K_0 K_4}\Omega_x$$

b) Proportional and Derivative Control (P+D)

Proportional error is modified by adding to it a quantity proportional to the 1st derivative of error wrt time (i.e. the rate of change of error). Essentially then the D term is looking at the speed at which the actual results are approaching the desired. Often this is done electronically.



The governing equation follows as (check this):

$$[Ms^2+(C+KT_D)s+K]X(s)=K(1+T_D s)X_i(s)-F_R(s)$$

Notes:

i) Damping increased without increasing power consumption.

ii) $KT_D sX_i(s)$ term indicates that large overshoot is anticipated and the transient response can be improved.

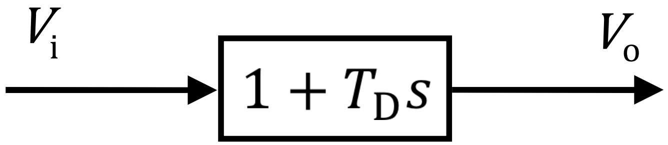
b) Proportional and Derivative Control (P+D)

Notes (continued):

iii) The steady-state error ($F_R = 0$) is independent of T_D (for a ramp check this):

$$e_{ss} = \frac{C}{K}\Omega_x$$

iv) Derivative action tends to amplify 'noise' in the system:



in time domain $$v_o(t) = v_i(t) + T_D \frac{dv_i(t)}{dt}$$

If the input signal is $$v_i(t) = V + v_n \sin(\omega t)$$

the output is $$v_o(t) = V + v_n \sin(\omega t) + T_D \omega v_n \cos(\omega t)$$

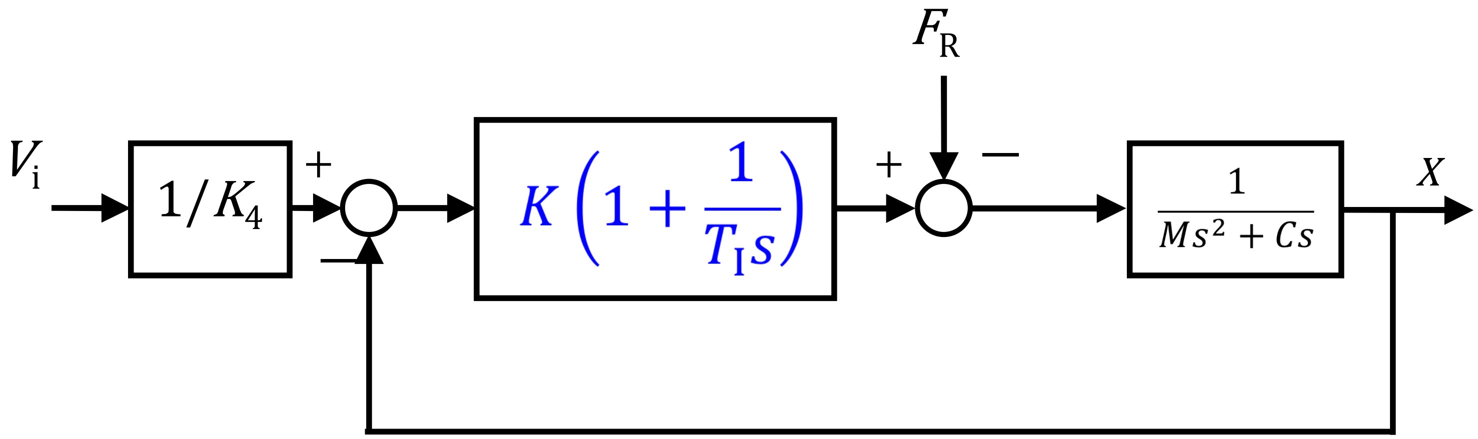
Now, if $\omega \gg 1/T_D$ noise is amplified $T_D \omega v_n \gg v_n$

Video Interlude

- What happens when a car stops suddenly:
  - https://www.youtube.com/watch?v=mnI-LiKCtuE
- Automated collision prevention:
  - https://www.youtube.com/watch?v=TJgUiZgX5rE
  - What does the sensor need to know?
    - Distance to object
    - Rate of change
    - Proportional – derivative control!

c) Proportional and Integral Control (P+I)

Proportional error is modified by adding an integral of error. This can also be carried out electronically.



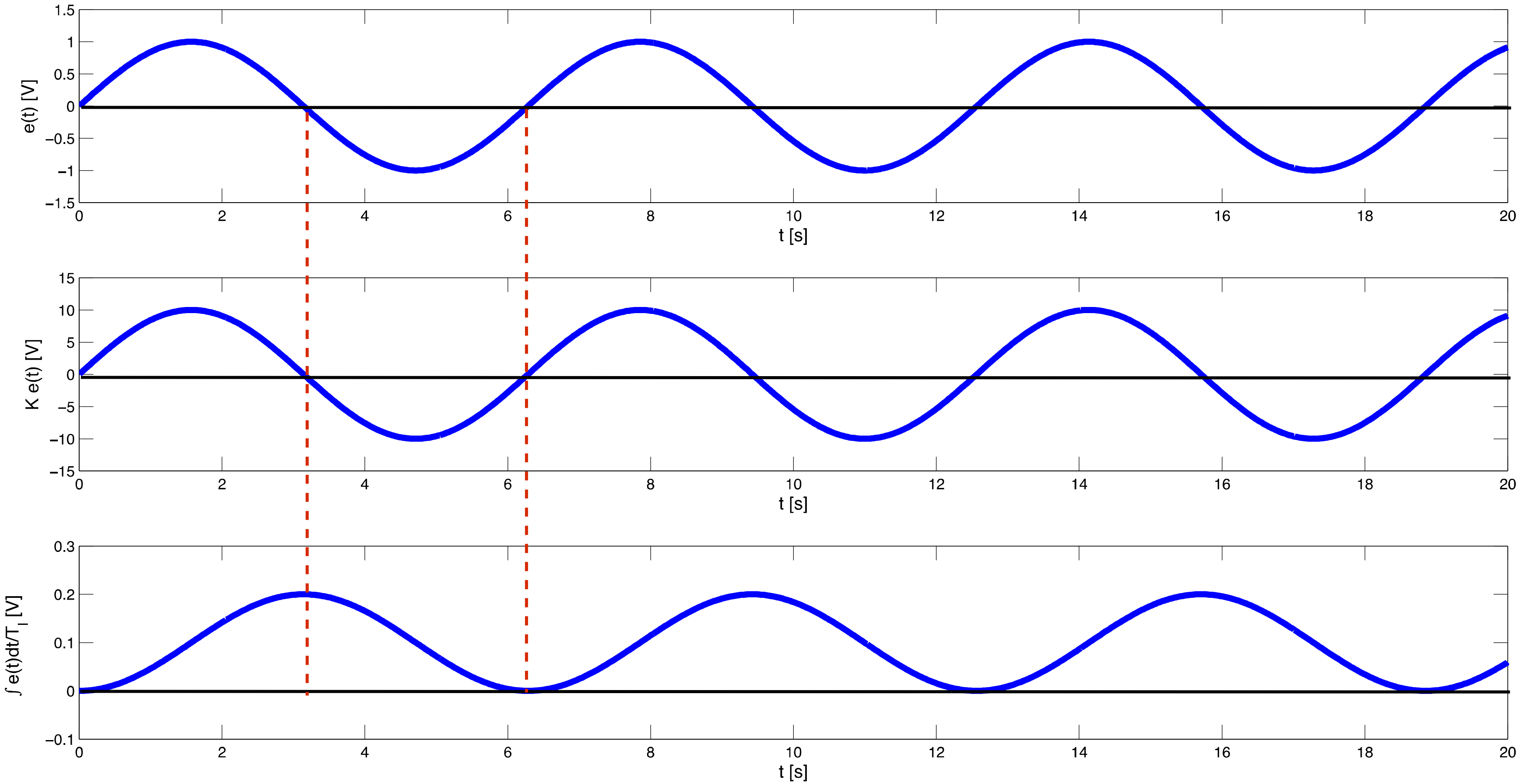
The governing equation follows as (check this):

$$\left(Ms^3 + Cs^2 + Ks + \frac{K}{T_I}\right)X(s) = \left(Ks + \frac{K}{T_I}\right)X_i(s) - sF_R(s)$$

The steady-state error ($F_R = 0$) under ramp is (check this):

$$e_{ss} = 0$$

c) Proportional and Integral Control (P+I)

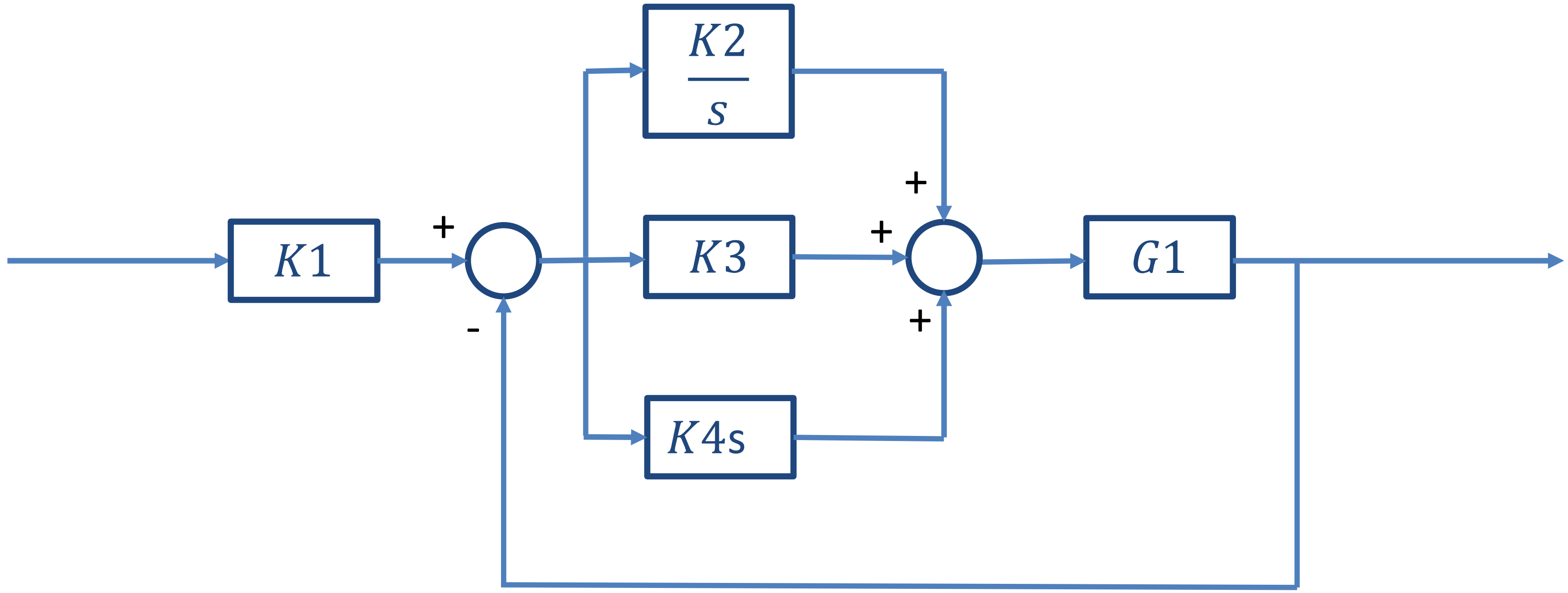


The **integral action** tends to **destabilise** the system: proportional action keeps track of sign changes, while integral action does not.

The effect is known as integral windup.

d) Proportional-Integral-Derivative Control (PID)

This the most common controller used in industry. To switch off an element, set gain to zero.



G1(s) is the open loop transfer function of the system

Tuning PID controllers is beyond this module's scope. What are the possible drawbacks of using PID control?

Remote control

- Simple example – remote operators for dockside container cranes
- https://www.youtube.com/watch?v=tEk2v4RyFh4
- How would you move to full automation?
  - Actuators stay the same
  - Sensors?
  - Software?

Transient response – Third and higher order systems

- Generalised transfer function for the system:

$$G(s) = \frac{Q(s)}{P(s)}$$

$$\mathrm{G}(s) = \frac{Q(s)}{(s-p_1)(s-p_s)\ldots(s-p_N)}$$

Transient Response – Higher order systems

- Values for which $Q(s)$ is zero are zeros of the transfer function
- Values for which $P(s)$ is zero (i.e. $G(s)$ becomes infinite) are the poles:
  - $p_1, p_2, \ldots, p_N$ for an N^th order system
  - These poles are either real (singular) or complex (pairs)

$$s = \sigma_r \text{ or } s = \sigma_c \pm \omega_c$$

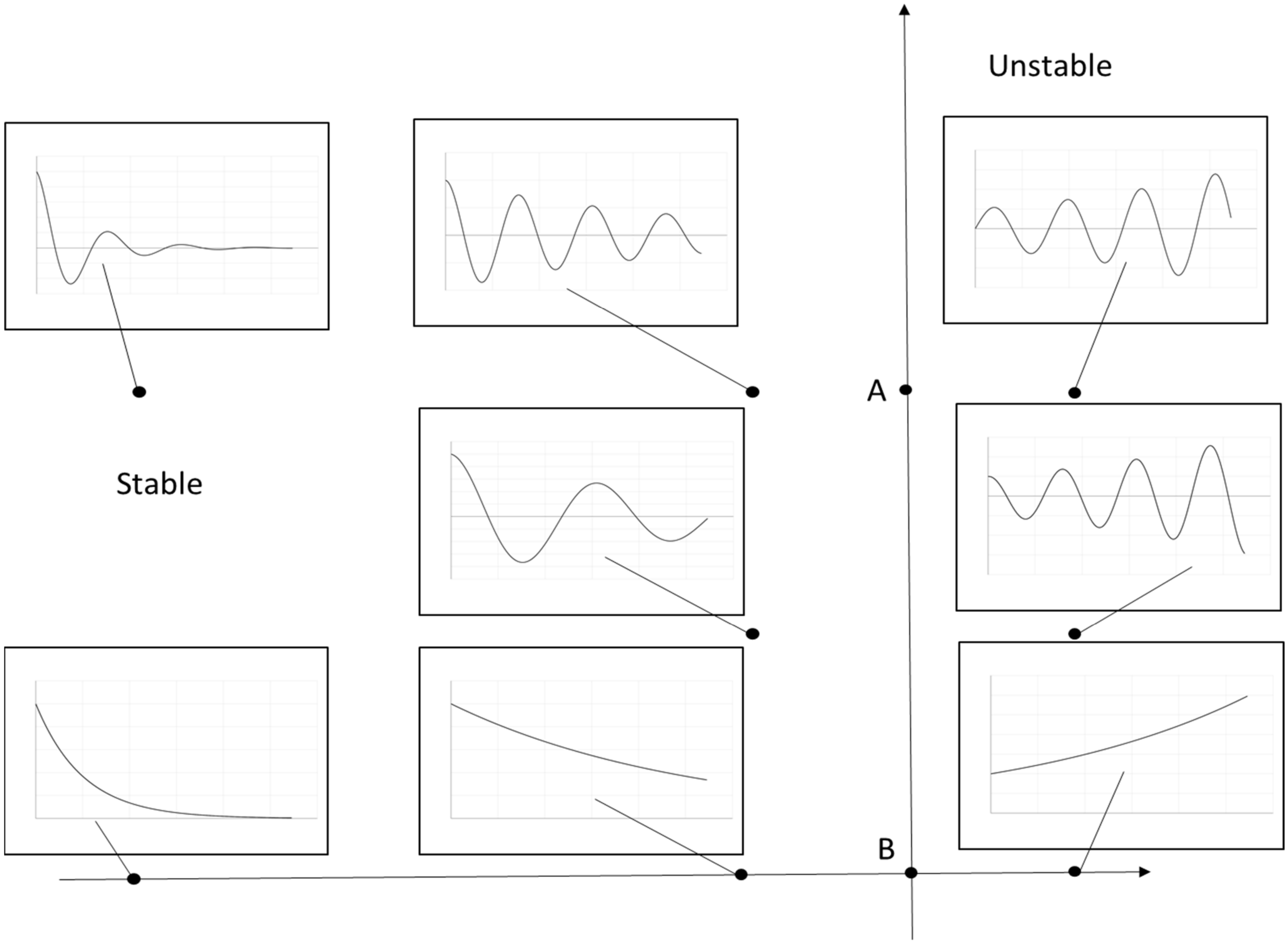
Transient Response – Higher order systems

If the input is a unit step: $X_i(s) = \frac{1}{s}$

Then:

$$X_0(s) = \frac{1}{s} + \sum_{r=1}^{N_R} \frac{A_r}{s - \sigma_r} + \sum_{c=1}^{N_c} \frac{A_r}{(s - \sigma_c)^2 + \omega^2}$$

$$x_0(t) = 1 + \sum_{r=1}^{N_R} B_r e^{\sigma_r t} + \sum_{c=1}^{N_c} B_c e^{\sigma_c t} sin(\omega t)$$


Unstable
A
Stable
B

Routh-Hurwitz Stability Criteria

$$P(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n = 0$$

Routh Hurwitz criteria for stability:

i) Necessary: All coefficients $a_0, a_1, a_2, \ldots, a_n$ are non-zero and have the same sign.

- i.e. if there is a change of sign in the denominator, the system will be unstable. No need to proceed to condition ii).
- However, it is possible for the system to be unstable without a change of sign …

Routh-Hurwitz Stability Criteria

$$P(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n = 0$$

Routh Hurwitz criteria for stability:

i) Necessary: All coefficients $a_0, a_1, a_2, \ldots, a_n$ are non-zero and have the same sign.

ii) Necessary and sufficient: if i) is satisfied, then the Hurwitz determinants $D_1, D_2, \ldots, D_n$ must be positive.

– This very quickly becomes laborious …

– Better to use a Routh Array

Routh-Hurwitz Stability Criteria (Routh Array)

s^n	a_0	a_2	a_4	a_6	…
s^{n-1}	a_1	a_3	a_5	a_7	…
s^{n-2}	b_1	b_2	b_3	…	…
s^{n-3}	c_1	c_2	c_3	…	…
…	…	…	…	…	…
s^0	…	…	…	…	…

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1} \qquad b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1} \qquad b_3 = \frac{a_1 a_6 - a_0 a_7}{a_1}$$

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1} \qquad c_2 = \frac{b_1 a_5 - a_1 b_3}{\mathrm{b}_1}$$

Routh-Hurwitz Stability Criteria

Using the Routh Array:

- If there is a change of sign in the ***first*** column, there is a root on the real, positive side of the s-plane. For every change of sign, there is another positive root.
- Thus, for the system to be stable, all values in the first column must be positive.
  - There is an issue if there is a zero in the first column, or there is a complete row of zeros so that the array cannot be completed.
  - Beyond the scope of MM2DYN!