$\tilde{R}$ J/kmol K Universal gas constant =8.3145 x 10^3 J/kmol K
T K, absolute temperature (0°C = 273K)

Perfect Gas

Perfect gas equation
$pV = mRT$ or $pv = RT$ or $p = \rho RT$

Specific gas constant
$R = \tilde{R}/\tilde{m}$ where $\tilde{R}$= 8.3145 x 10^3 J/kmol K

Relationship between R, c_p, and c_v

$\frac{c_p}{c_v} = \gamma$ and $c_p - c_v = R$

Enthalpy

Enthalpy definition: H = U + pV, or h = u + pv

Change in enthalpy
$H_2 - H_1 = mc_p (T_2 - T_1)$

Change in internal energy
$U_2 - U_1 = mc_v (T_2 - T_1)$

Entropy

Definition $dS = \left(\frac{dQ}{T}\right)_{rev}$

Change in entropy (definition: $S_2 - S_1 = \int_1^2 \left(\frac{dQ}{T}\right)_{rev}$)

$$S_2 - S_1 = m R \ln\left(\frac{v_2}{v_1}\right) + m c_v \ln\left(\frac{T_2}{T_1}\right)$$

$$S_2 - S_1 = m c_p \ln\left(\frac{T_2}{T_1}\right) - m R \ln\left(\frac{p_2}{p_1}\right)$$

Relationship between p, v, and T for polytropic processes
(pv^n = constant) for a perfect gas

$$\frac{p_2}{p_1} = \left(\frac{v_2}{v_1}\right)^{-n}, \quad \frac{v_1}{v_2} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{n-1}}, \quad \frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}}$$

NB. For a reversible adiabatic (isentropic) process the polytropic index $n=\gamma$

Closed-Systems/Non-Flow Processes

First law, for a cycle $W_{net} + Q_{net} = 0$

First law, for a process $W + Q = U_2 - U_1$

Work transfer for reversible processes:

$$W = \int_{x_1}^{x_2} F dx = -\int_{V_1}^{V_2} p dV = -m\int_{v_1}^{v_2} p dv$$ general case

$W = -mp (v_2 - v_1)$ constant pressure

$W = 0$ constant volume

$W = -m RT \ln\left(\frac{v_2}{v_1}\right)$ isothermal, perfect gas

$W = -m (p_2 v_2 - p_1 v_1)/(1-n)$ polytropic, pV^n = constant

NB. For a reversible adiabatic (isentropic) process the polytropic index $n=\gamma$

Second Law/Heat Engines

Definition of efficiency $\eta = \dfrac{\text{network done}}{\text{heatsupplied}}$

$$= \frac{|W|}{|Q_1|} = 1 - \frac{|Q_2|}{|Q_1|}$$

Carnot efficiency $\eta_{carnot} = 1 - \dfrac{T_2}{T_1}$

Open Systems/Flow Processes

Steady flow energy equation (SFEE) specific energy form

$$q + w_s = \left[u_2 + \frac{p_2}{\rho_2} + gz_2 + \frac{v_2^2}{2}\right] - \left[u_1 + \frac{p_1}{\rho_1} + gz_1 + \frac{v_1^2}{2}\right]$$

Steady flow energy equation (SFEE) power form

$$\dot{Q} + \dot{W} = \dot{m}\left[(h_2 - h_1) + (gz_2 - gz_1) + \left(\frac{v_2^2}{2} - \frac{v_1^2}{2}\right)\right]$$

Work transfer for <u>reversible</u> processes with negligible changes in kinetic and potential energy

$\dot{W} = \dot{m}\int_1^2 v\,dp$ general case

$\dot{W} = 0$ constant pressure

$\dot{W} = \dot{m}v(p_2 - p_1)$ constant specific volume/density

$\dot{W} = \dot{m}RTln\left(\frac{p_2}{p_1}\right)$ isothermal, perfect gas

$\dot{W} = \dot{m}\frac{n}{n-1}(p_2v_2 - p_1v_1)$ polytropic, pv^n = constant

NB. For a reversible adiabatic (isentropic) process the polytropic index $n=\gamma$

Fluids Mechanics

Power = force × velocity; Pressure $p = \dfrac{F}{A}$; Density $\rho = \dfrac{m}{V}$

Fluid Statics

Variation of pressure with elevation

$$\Delta p = -\rho g \Delta z = \rho g \Delta h$$

differential manometer $\Delta p = p_1 - p_2 = (\rho_m - \rho_w) g \Delta z$

inclined tube manometer $p_1 - p_2 = \rho_p g L\left(\dfrac{A_2}{A_1} + \sin\theta\right)$

Hydrostatic force on a submerged element

$$\delta F_{net} = \rho g h\, \delta A$$

Moment due to hydrostatic force on a submerged element:

$$\delta M_O = (\delta F_{net}) y = (\rho g h\, \delta A) y$$

Archimedes $F_B = W$;

Buoyancy Force = $\rho V_{sub} g$

Fluid dynamics

Shear stress $\tau = \mu \dfrac{dv}{dy}$

Continuity $\dot{m}_1 = \dot{m}_2; \quad \rho_1 A_1 v_1 = \rho_2 A_2 v_2$

Reynolds Number (pipe flow) $Re = \frac{\rho v d}{\mu}$

Bernoulli equation (pressure form)

$$p + \rho g z + \frac{1}{2}\rho v^2 = constant$$

Bernoulli equation (head form)

$$\frac{p}{\rho g} + z + \frac{v^2}{2g} = constant$$

$\frac{p}{\rho g} = H_p =$ the pressure head

$z =$ elevation head

$\frac{v^2}{2g} = H_v =$ velocity head

Venturimeter equation: $\dot{V}_{real} = c_d \dot{V}_{ideal} = c_d A_2 \sqrt{\frac{2g(\Delta H_{pz})}{\left(1-\left(\frac{A_2}{A_1}\right)^2\right)}}$

$\dot{V}_{real} = c_d \dot{V}_{ideal}$

Orifice plate equation

$$\dot{V}_{real} = c_d A_o \sqrt{\frac{2g(\Delta H_{pz})}{\left(1-\left(\frac{A_o}{A_1}\right)^2\right)}}$$

Pitot-static probe equation $v = \sqrt{\frac{2}{\rho}(p^+ - p)}$

SFEE, no heat transfer (head form)

$$\frac{w_s}{g} = H_{T2} - H_{T1} + H_f$$

Extended Bernoulli (special case of SFEE)

$$H_{T1} - H_{T2} = H_f$$

Head lost due to friction in pipe flow $H_f = \frac{4fl}{d}\frac{v^2}{2g}$

Hydraulic diameter for non-circular pipes and ducts:

$$d_h = \frac{4(\text{flow area})}{\text{wetted perimeter}}$$

Head loss due to friction: $H_f = K\left(\frac{v^2}{2g}\right)$

Power dissipated due to friction $\dot{W} = \dot{m}\,(gH_f)$

Pump equation (special case of SFEE)

$$w_s = \frac{p_2 - p_1}{\rho} + gH_f$$

pump efficiency $\eta_{HP} = \frac{w_{s,i}(ideal)}{w_s(actual)}$

<u>Linear Momentum</u>

Linear momentum equation (general form)

$$F_{x,total} = \dot{m}(v_{x,out} - v_{x,in})$$

F_x includes all forces acting on a control volume including structural forces, pressure forces and gravitational forces.

Volume of sphere $V = \frac{4}{3}\pi r^3$

Plane (flat) Walls

Conduction:

Heat flow

$$\dot{Q} = -kA\frac{(T_2 - T_1)}{\Delta x}$$

Thermal resistance $\frac{\Delta x}{kA}$ K/W

Heat flow per unit area

$$\dot{Q}'' = \frac{\dot{Q}}{A} = -k\frac{(T_2 - T_1)}{\Delta x}$$

Thermal resistance per unit area $\frac{\Delta x}{k}$ Km²/W

Convection at a Flat Wall Solid Boundary

Heat flow

$$\dot{Q} = h\ A\,(T_{surface} - T_{fluid})$$

Thermal resistance $\frac{1}{hA}$ K/W

Heat flow per unit area

$$\dot{Q}'' = h\,(T_{surface} - T_{fluid})$$

Thermal resistance per unit area $\frac{1}{h}$ Km²/W

Cylindrical Walls (pipes)

(Area of a cylinder is $2\pi rL$)

Conduction:

Heat flow

$$\dot{Q} = -\frac{(T_2 - T_1)}{\left(\frac{\ln(r_2/r_1)}{2\pi Lk}\right)}$$

Thermal resistance $\frac{\ln\left(r_2/r_1\right)}{2\pi Lk}$ K/W

Heat flow per unit length

$$\dot{Q}' = \frac{\dot{Q}}{L} = -\frac{(T_2 - T_1)}{\left(\frac{\ln(r_2/r_1)}{2\pi\ k}\right)}$$

Thermal resistance per unit length $\frac{\ln\left(r_2/r_1\right)}{2\pi k}$ Km/W

Convection at a cylindrical boundary:

Heat Flow

$$\dot{Q} = h\ 2\pi rL\,(T_{surface} - T_{fluid})$$

Thermal resistance: $\frac{1}{h2\pi rL}$ K/W

Heat Flow per unit length

$$\dot{Q}' = \frac{\dot{Q}}{L} = h\ 2\pi r\,(T_{surface} - T_{fluid})$$

Thermal resistance per unit length $\frac{1}{h2\pi r}$ Km/W