1 Fatigue and Fracture

Learning Summary

1. Know the various stages leading to fatigue failure (knowledge);
2. Know the basis of the total life and of the damage tolerant approaches to estimating the number of cycles to failure (knowledge);
3. Be able to include the effects of mean and alternating stress on cycles to failure using the Gerber, modified Goodman and Soderberg methods (application);
4. Be able to include the effect of a stress concentration on fatigue life (application);
5. Be able to apply the S-N design procedure for fatigue life (application);
6. Know the meaning of linear-elastic fracture mechanics (LEFM) (knowledge);
7. Know what the three crack tip loading modes are (knowledge);
8. Be able to use the energy and stress intensity factor (Westergaard crack tip stress field) approaches to LEFM (application);
9. Know the meaning of small-scale yielding and fracture toughness (knowledge);
10. Understand the Paris equation for fatigue crack growth and the effects of the mean and alternating components of the stress intensity factor (knowledge/comprehension).

1.1 Fatigue

1.1.1 Introduction

Fatigue failure of components and structures results from cyclic (or repeated) loading and from the associated cyclic stresses and strains, as opposed to failure due to monotonic or static stresses or strains, such as buckling or plastic collapse due to excessive plastic deformation yielding. The topic of fatigue is extremely important in mechanical engineering, since machines have moving parts, which in turn give rise to stresses and strains which may vary with time, typically in a repetitive fashion. For example, the axle of a car which will transmit a time-varying torque, that changes from zero to some finite value when the car is put into gear and driven (and back to zero again when the car is taken out of gear).

An important design consideration, with respect to fatigue, is the fact that fatigue failure can occur at stresses which are well below the ultimate tensile strength of the material and often below the yield strength.

1.1.2 Basic phenomena

The failure mechanism for an initially un-cracked component with a smooth (polished) surface can be split into three parts, namely crack initiation, crack propagation and final fracture, as follows:

(i) Stage I crack growth: The micro-structural phenomenon which causes the initiation of a fatigue cracks is the development of persistent slip bands at the surfaces of the specimen. These persistent slip bands are the result of dislocations moving along crystallographic planes leading to both slip band intrusions and extrusions on the surface. These act as excellent stress concentrations and can thus lead to crack initiation. Crystallographic slip is primarily controlled by shear stresses rather than normal stresses so that fatigue cracks initially tend to grow in a plane of maximum shear stress range. This stage leads to short cracks, usually only of the order of a few grains.

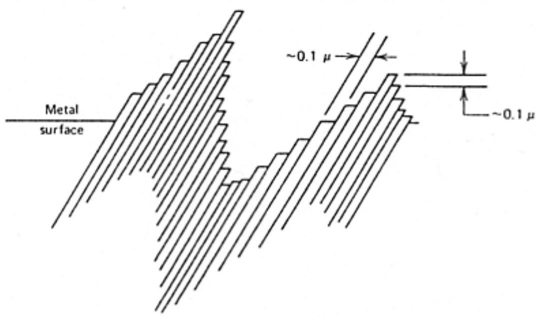


Figure 1.1: Persistent slip bands in ductile metals subjected to cyclic stress

(ii) Stage II crack growth: As cycling continues, the fatigue cracks tend to coalesce and grow along planes of maximum tensile stress range.

(iii) Final fracture; this occurs when the crack reaches a critical length and results in either ductile tearing (plastic collapse) at one extreme, or cleavage (brittle fracture) at the other extreme.

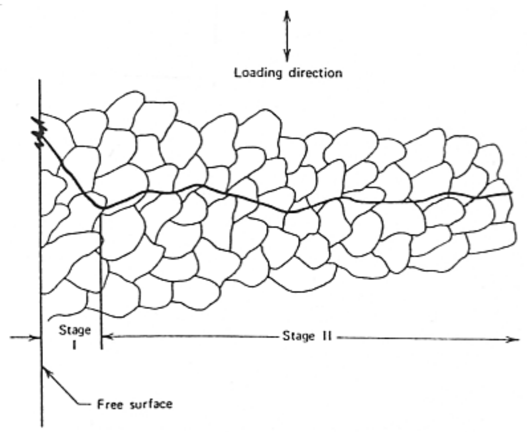


Figure 1.2: Schematic of stages I and II transcrystalline microscopic fatigue crack growth.

1.1.3 Fatigue Life Analysis

In order to allow for fatigue in the analysis and design of components, a number of different approaches are adopted; two of these approaches are described here. The more traditional approach is what is now referred to as the *total life approach* (see section 1.1.4), based on laboratory tests, which are carried out under either stress- or strain-controlled loading conditions on idealised specimens. These tests furnish the number of loading cycles to the initiation of a 'measurable' crack as a function of applied stress or strain parameters. The 'measurability' is dictated by the resolution accuracy of the crack detection method employed. A typical 'measurable' crack is about 0.75 mm to 1 mm. The challenge of fatigue design is then to relate these test results to actual component lives under real loading conditions. The second approach is known as the *damage tolerant approach* (see section 1.2.5). This approach is based on the inclusion of fatigue as a crack growth process, taking account of the fact that all components have inherent flaws or cracks. The development of fracture mechanics techniques to predict crack growth has facilitated this approach as a competing technique to the total life approach. Both of the approaches have advantages and disadvantages; the former has more appeal to design engineers while the latter is more often used by material scientists and researchers. Nonetheless, even in routine design, the damage tolerant approach is gaining popularity.

1.1.4 Total life approach

The total life approach is based on the results of stress- and strain-controlled cyclic testing of laboratory test specimens of material, in order to obtain the numbers of cycles to failure as a function of the applied alternating stress, for example. Figure 1.3 shows a rotating bending test machine set-up. This is a constant load amplitude machine since the load doesn't change even with crack growth. The specimens usually have finely polished surfaces to minimise surface roughness effects, which would particularly affect Stage I growth. In this approach, no distinction is made between crack initiation and propagation. Stress concentration effects can be studied by machining in grooves, notches or holes.

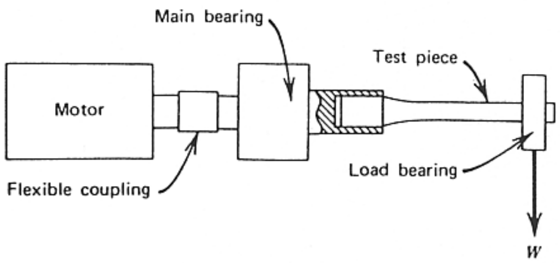


Figure 1.3: Rotating bending moment test apparatus for fully-reversed fatigue loading.

Traditionally, most fatigue testing was based on fully-reversed (i.e. zero mean stress, $S_m = 0$), stress-controlled conditions and the fatigue design data was presented in the form of *S-N* curves (see Figure 1.6), which are either semi-log or log-log plots of alternating stress, S_a, against the measured number of cycles to failure, N, where failure is defined as fracture. Some of the important stress parameters for cyclic loading are shown in Figure 1.4.

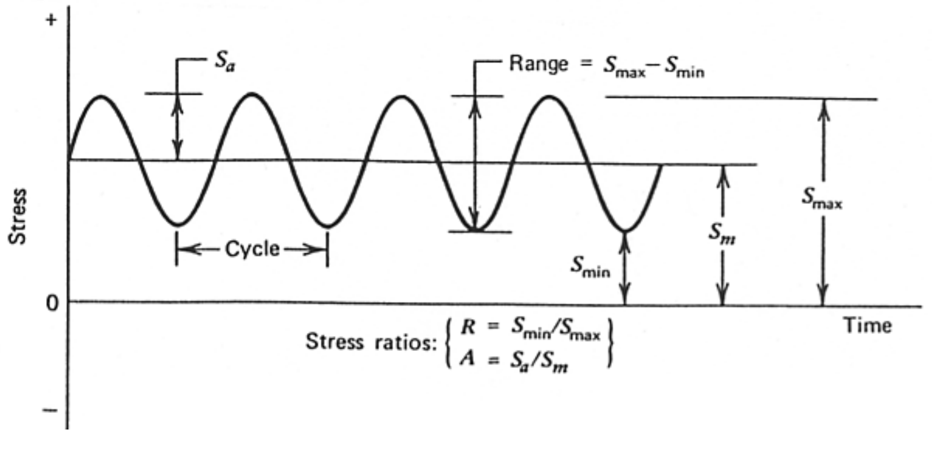


Figure 1.4: Notation used to describe constant load fatigue test cycles.

Figure 1.5 contains schematic representations of two typical *S-N* curves obtained from load (or stress)-controlled tests on smooth specimens. Figure 1.5(a) shows a continuously sloping curve, while Figure 1.5(b) shows a discontinuity or "knee" in the curve. A "knee" is only found in a few materials (notably low strength steels) between 10^6 and 10^7 cycles under non-corrosive conditions. The curves are normally drawn through the median life value (of the scatter in *N*) and thus represents 50 percent expected failure. The *fatigue life*, *N*, is the number of cycles of stress (or strain) range of a specified character that a given specimen sustains before failure of a specified nature occurs. *Fatigue strength* is a hypothetical value of stress range at failure for exactly *N* cycles as obtained from an *S*-*N* curve. The *fatigue limit* (sometimes called the *endurance limit*) is the limiting value of the median fatigue strength as *N* becomes very large, e.g. $>10^8$ cycles.

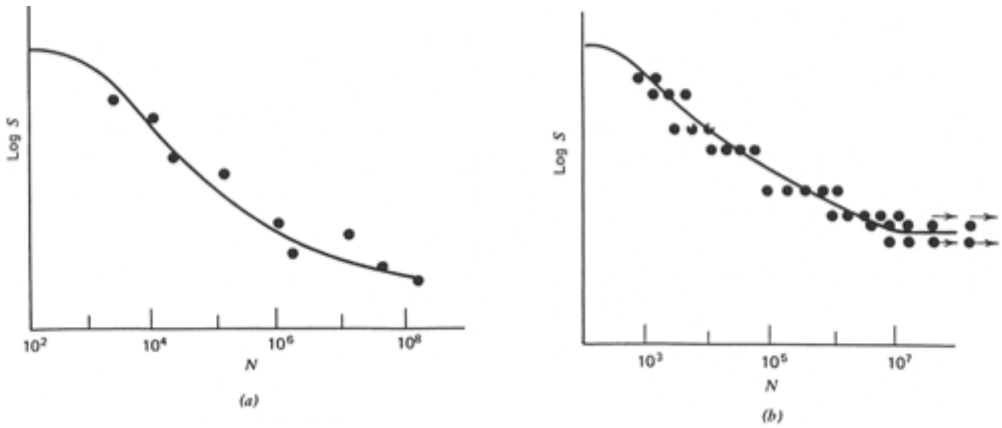


Figure 1.5: Typical S-N diagrams.

1.1.5 Effect of mean stress

The alternating stress, S_a, and the mean stress, S_m, are defined in Figure 1.4. Early investigators of fatigue assumed that only the alternating stress affected the fatigue life of a cyclically-loaded component. However, it has since been established that the mean stress has a significant effect on fatigue behaviour, as shown in Figure 1.6. It can be seen that tensile mean stresses are detrimental while compressive mean stresses are beneficial in comparison to zero mean stresses.

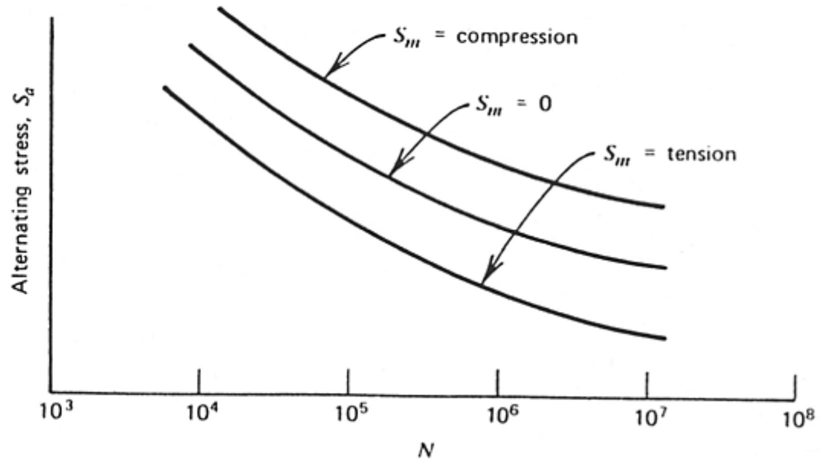


Figure 1.6: The effect of mean stress on fatigue life.

The effect of mean stress is commonly represented as a plot of S_a versus S_m for a given fatigue life. Attempts have been made to develop this relationship into general relations. Three of these common relations between allowable alternating stress for a given life as a function of mean stress, are shown in Figure 1.7.

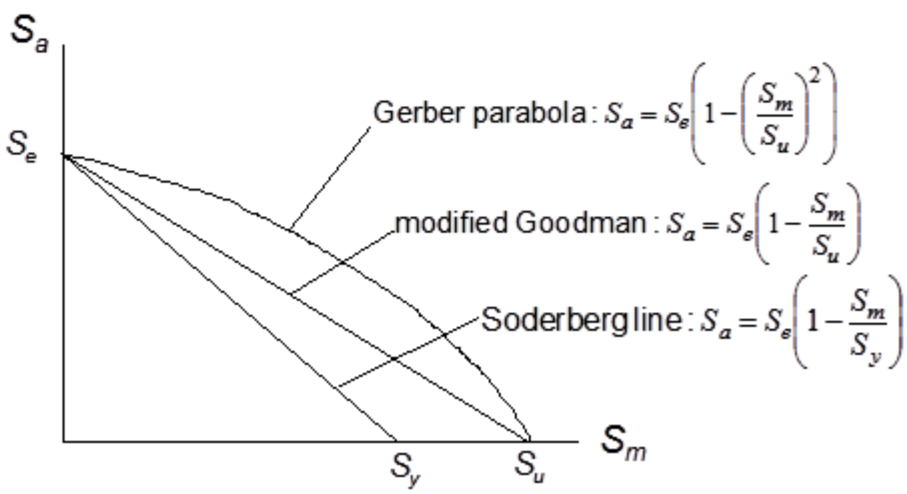


Figure 1.7: Gerber, modified Goodman and Soderberg relationships between S_a and S_m.

The modified Goodman line assumes a linear relationship between the allowable S_a and the corresponding mean stress S_m, where the slope and intercepts are defined by the fatigue strength, S_e, and the material UTS, S_u, respectively. The Gerber parabola employs the same end-points but, in this case, the relation is defined by a parabola. Finally, the Soderberg line again assumes a linear relation, but this time the mean stress axis end-point is taken as the yield stress, S_y. The modified Goodman line, for example, can be extended into the compressive mean stress region to give increasing allowable alternating stress with increasing compressive mean stress, but this is normally taken to be horizontal for design purposes and for conservatism.

1.1.6 Effect of stress concentrations

Ever since the first occurrences of fatigue failure, it has been recognised that such failures are most commonly associated with notch-type features in components. It is impossible to avoid notches in engineering structures, although the effects of such notches can be reduced through appropriate design. The stress concentration associated with notch-type features leads typically to local plastic strain which eventually leads to fatigue cracking. Consequently, the estimation of stress concentration factors associated with various types of notches and geometrical discontinuities has received a lot of attention. This is typically expressed in terms of an elastic stress concentration factor (SCF), K_t, which is simply the relationship between the maximum local stress and an appropriate nominal stress, as follows:

$$K_t = \frac{\sigma_{\max}^{el}}{\sigma_{nom}}$$

It was once thought that the fatigue strength of a notched component could be predicted as the strength of a smooth component divided by the SCF. However, this is not the case. The reduction is, in fact, often less than K_t and is defined by the *fatigue notch factor,* K_f, which is defined as the ratio of the smooth fatigue strength to the notched fatigue strength as follows:

$$K_f = \frac{S_{a,smooth}}{S_{a,notch}}$$

However, this fatigue notch factor is also found to vary with both alternating and mean stress level and thus with number of cycles to failure. Figure 1.8 shows the effect of a notch, with an SCF of 3.4, on the fatigue behaviour of a wrought aluminium alloy, where the smooth lines are for the smooth specimen and the dotted lines are for the notched specimen.

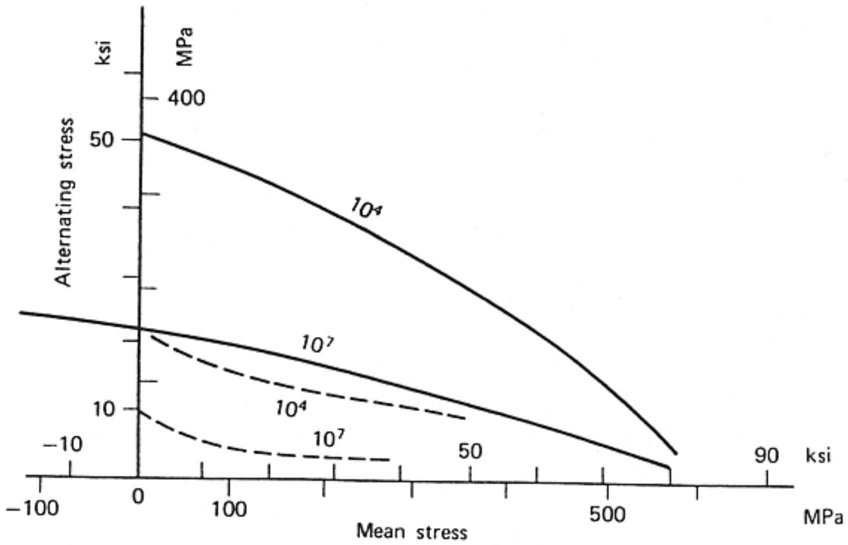


Figure 1.8: Constant life diagrams for a wrought aluminium alloy for both smooth and notched specimens (SCF = 3.4).

Table 1.1 shows how the fatigue notch factor changes with mean stress level and fatigue life. Clearly, the fatigue notch factor increases from 3.2 to 5.7 from 10^4 cycles to 10^7 cycles at 172 MPa mean stress, but remains unchanged between these lives at 2.3 for zero mean stress.

Table 1.1: Fatigue notch factor change with mean stress and fatigue life

Mean stress	10^4 cycles	10^7 cycles
0 MPa	51/22 = 2.3	22/9 = 2.3
172 MPa	42/13 = 3.2	17/3 = 5.7

1.1.7 *S-N* Design Procedure for Fatigue

Constant life diagrams plotted as S_a versus S_m, also called Goodman diagrams, as shown in Figure 1.9, can be used in design to give safe estimates of fatigue life and loads.

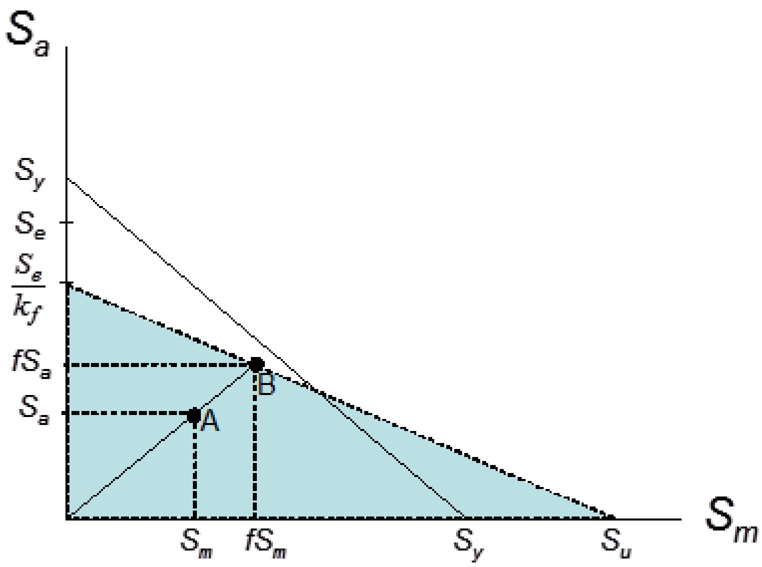


Figure 1.9: Goodman diagram.

(i) The Goodman line connects the endurance limit, S_e (or long life fatigue strength), to the U.T.S., S_u

(ii) The fatigue strength for zero mean stress is reduced by the fatigue notch factor, K_f. The stress concentration factor, K_t is used if K_f is not known.

(iii) For static loading of a ductile component with a stress concentration, failure still occurs when the mean stress is equal to the U.T.S. Failure at intermediate values of mean stress is assumed to be given by the dotted line.

(iv) In order to avoid yield of the whole cross-section of the component, the maximum nominal stress must be less than the yield stress, S_y , i.e. $S_m + S_a < S_y$
This relationship gives the yield line joining S_y to S_y.

(v) The region of the diagram nearest to the origin is the 'safe' region (can also be extended to include compressive yield).

(vi) A component is assessed by plotting the point corresponding to the nominal alternating stress, S_a, and the nominal mean stress, S_m, i.e. not the maximum values associated with the notch. The factor of safety is determined from the position of the point relative to the safe/fail boundary.
i.e. factor of safety F = OB/OA

from similar triangles
$$\frac{S_a}{\left(\frac{S_u}{F} - S_m\right)} = \frac{S_e}{k_f S_u}$$

$$\frac{1}{F} = \frac{S_a k_f}{S_e} + \frac{S_m}{S_u}$$

A procedure similar to that described above for long life can also be used to design for a specified number of cycles. In this case the endurance limit and the fatigue notch factor are replaced by the fatigue strength and the fatigue notch factor for the specified number of cycles.

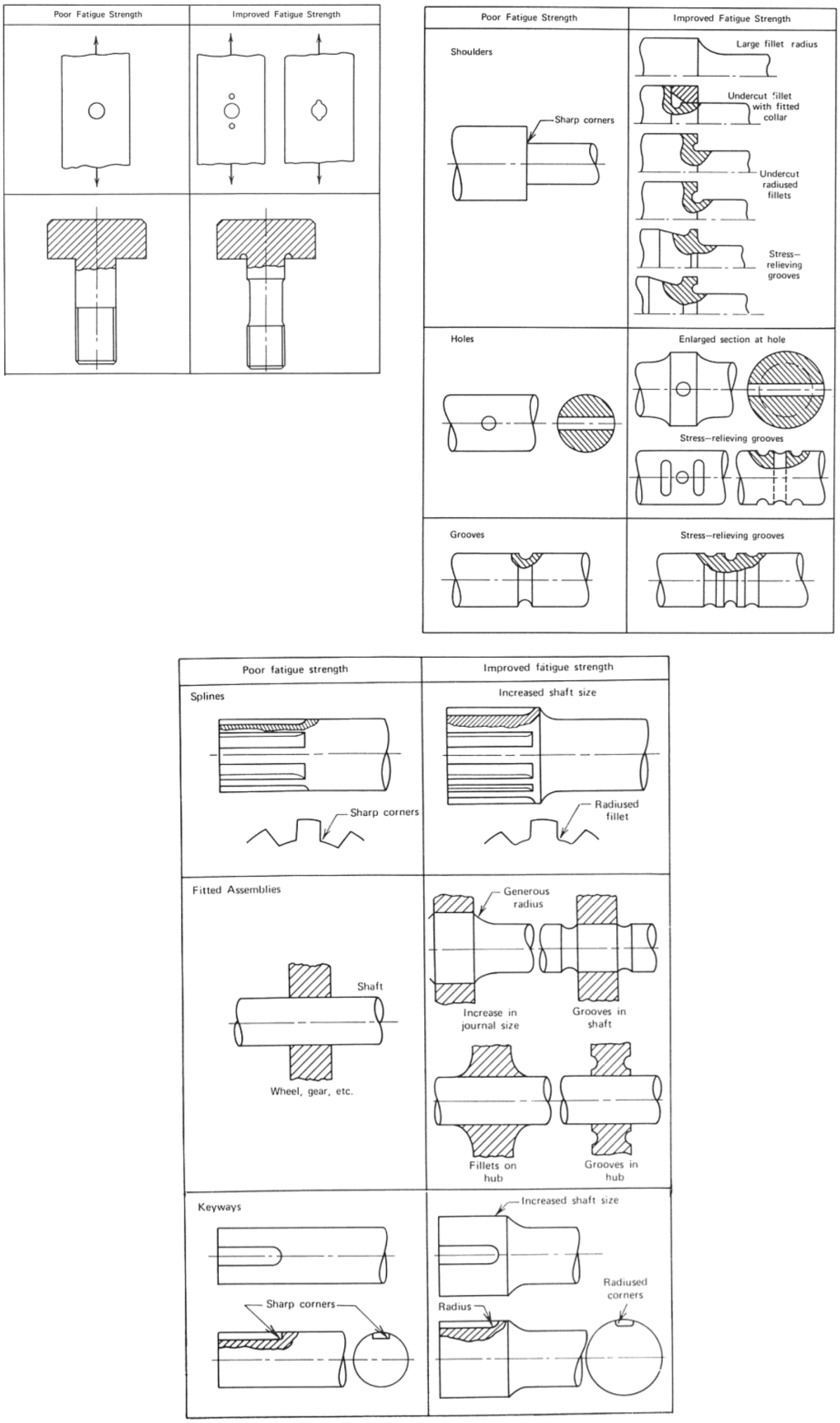


Figure 1.10: Examples of geometries/components with poor and improved fatigue strength.

1.2 Linear Elastic Fracture Mechanics (LEFM)

1.2.1 Introduction

Consider the stress concentration factor for an elliptical hole in a large, linear-elastic plate subjected to a remote, uniaxial stress.

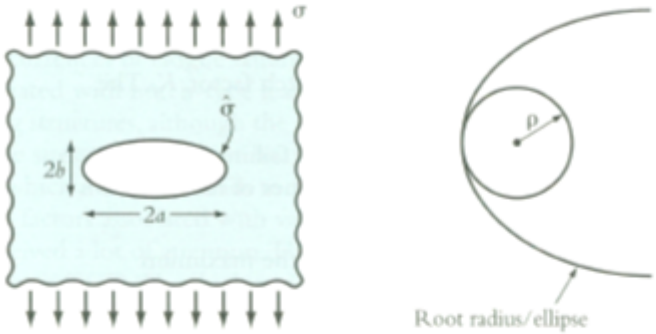


Figure 1.11: Elliptical hole in an infinite plate root radius ellipse

It can be shown that the stress concentration factor is as follows:

$$K_t = \frac{\sigma_{\max}^{el}}{\sigma} = 1 + \frac{2a}{b}$$

Thus, as $b \to 0$, the elliptical hole degenerates to a crack, and $\frac{a}{b} \to \infty$, so that the notch stress also goes to infinity (i.e. becomes singular), $K_t \to \infty$, provided the material behaviour remains linear elastic.

The root radius for an ellipse is given by

$$\rho = \frac{b^2}{a} \qquad \therefore\ b = \sqrt{a\rho}$$

so that

$$\frac{\hat{\sigma}}{\sigma} = 1 + 2\sqrt{\frac{a}{\rho}}$$

and again, as the notch tip radius goes to zero, i.e. $\rho \to 0$, the notch tip stress again goes to infinity,

$$\frac{\hat{\sigma}}{\sigma} \to 2\sqrt{\frac{a}{\rho}} \to \infty$$

The singular (infinite) state of stress at a crack tip is one of the fundamental and most important aspects of fracture mechanics.

1.2.2 Basis of the energy approach to fracture mechanics

Griffith (1921) studied the brittle fracture of glass and adopted an energy approach to solve the problem. He reasoned that unstable crack propagation occurs only if an increment of crack growth, d*a*, results in more strain energy being released than is absorbed by the creation of the new crack surfaces. This can be re-expressed as the change in strain energy *U*, due to crack extension, being greater than the energy absorbed by the creation of the new crack surfaces. Thus, if we designate the surface energy per unit area of the crack γ_s, then the surface energy associated with a crack of length 2*a* in a body of thickness *B* (as shown in the Figure 1.12) is given by:

$$W_s = 4aB\gamma_s$$

Detailed stress analysis of an elliptical hole in an infinite elastic plate has established that the strain energy in such a body is

$$W_p = -\frac{\pi a^2 \sigma^2 B}{E'}$$

where σ is the remote stress (away from the hole) and where, for plane strain and plane stress, respectively,

$$E' = \frac{E}{1-\nu^2} \quad \text{and } E' = E$$

The total system energy is thus

$$W = -\frac{\pi a^2 \sigma^2 B}{E'} + 4aB\gamma_s$$

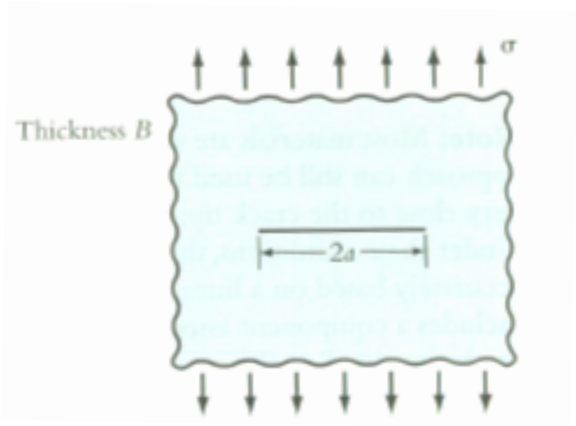


Figure 1.12: Crack in Infinite Plate

According to Griffith, the critical condition for the onset of crack growth is

$$\frac{dU}{dA} = -\frac{\pi a \sigma^2}{E'} + 2\gamma_s = 0$$

Therefore:

$$\frac{\pi a \sigma^2}{E'} = 2\gamma_s$$

where A = 2aB is the crack area and dA denotes an incremental increase in crack area. The total surface area of the two crack surfaces is 2A. This relationship is conventionally re-expressed as

$$G = G_c$$

where G is called the *strain energy release rate*, the *crack tip driving force* or the *crack extension force*. G_c is a material property, which is known as the *critical strain energy release rate*, the *toughness* or the *critical crack extension force*. A high value of G_c means that it is difficult to cause unstable crack growth in the material whereas a low value means it is easy to make a crack grow unstably. Thus, copper, for example, has a value of $G_c \approx 10^6$ Jm^{-2}, whereas glass has a value of $G_c \approx 10$ Jm^{-2}. The following relationships for plane stress and plane strain, respectively, follow from the above:

$$G = \frac{\pi a \sigma^2}{E} \text{ (plane stress)}$$

$$G = \frac{(1-\nu^2)}{E}\pi a \sigma^2 \text{ (plane strain)}$$

Note that plane stress and plane strain are two contrasting two-dimensional assumptions which permit simplification of three-dimensional problems to two-dimensional ones. Plane stress corresponds physically to thin plate type situations while plane strain corresponds to thick plate type situations. Plane strain testing of fracture leads to lower values of G_c, so that the material property value of G_c for design purposes is taken as the plane strain value and is designated as G_{Ic}.

The critical stress, which causes a crack to propagate in an unstable fashion, giving fracture, is governed by the following relationships

$$\sigma\sqrt{\pi a} = \sqrt{EG_c} \text{ (plane stress)}$$

$$\sigma\sqrt{\pi a} = \sqrt{\frac{EG_c}{1-\nu^2}} \text{ (plane strain)}$$

Since the term on the right-hand side of these equations is a material constant and since the term on the left hand side is so common, it is usually abbreviated to the symbol, K, which is referred to as the *stress intensity factor* and the equations can be re-expressed as:

$$K = K_c$$

where K_c is called the *critical stress intensity factor* or the *fracture toughness*. Thus

$$K_c = \sqrt{EG_c}$$

In summary,

$K = \sigma\sqrt{\pi a}$ is called the *stress intensity factor*

K_c is called the fracture *toughness* of *critical stress intensity factor*

G_c is called the *toughness* or the *critical strain energy release rate*.

Note

Most materials are not linear elastic up to failure. However, the energy approach can still be used if the plastic strain is restricted to a region very close to the crack tip; this is referred to as *small scale yielding*. Under these conditions, the energy release rate can still be reasonably accurately based on a linear elastic analysis. Also, G_c or G_{Ic} now includes a component associated with plastic deformation of the crack tip as well as the creation of the surfaces. So far, we have only considered the so-called Mode I loading case. There are actually three different loading modes considered in fracture mechanics, as shown in Figure 1.13.

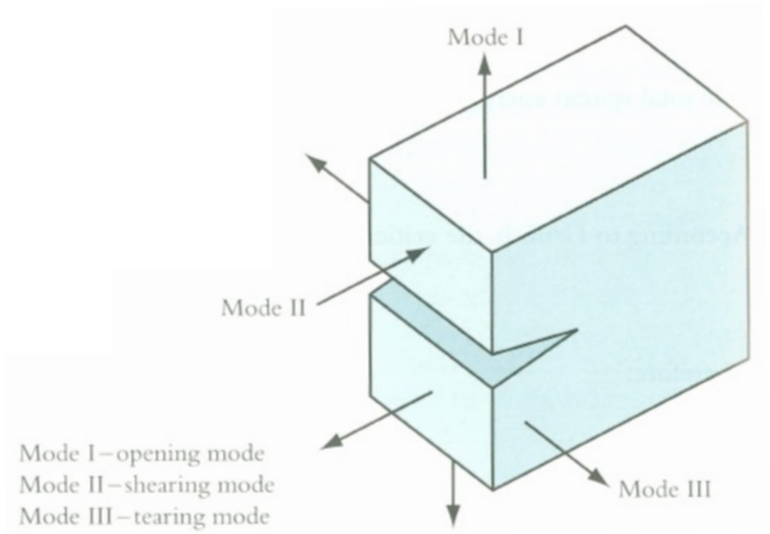


Figure 1.13: Crack tip loading modes

In general, the energy release rate under mixed-mode loading is given by

$$G_{total} = G_I + G_{II} + G_{III}$$

1.2.3 Elastic crack tip stress fields

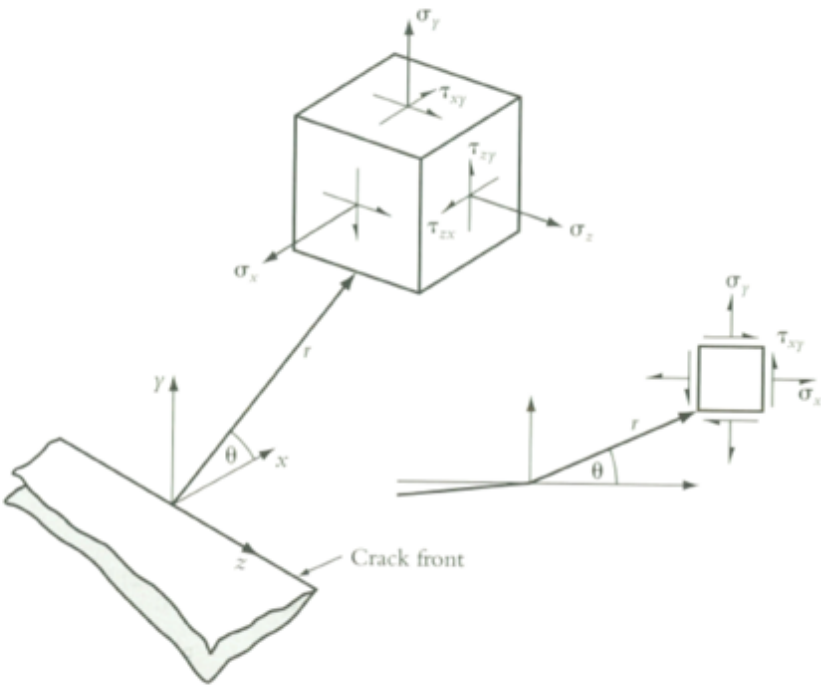


Figure 1.14. Crack tip stress fields

Westergaard (1939) established the following equations for the elastic stress field in the vicinity of a crack tip:

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right)\left[1 - \sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right] + \text{non} - \text{singular terms}$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right)\left[1 + \sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right] + \text{non} - \text{singular terms}$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right) + \text{non} - \text{singular terms}$$

$$\tau_{zx} = \tau_{zy} = 0;\ \sigma_z = 0\ \text{(plane stress)};\ \sigma_z = \nu(\sigma_x + \sigma_y)\ \text{(plane strain)}$$

K_I is the Mode-I stress-intensity factor (units N/m$^{3/2}$) which defines the magnitude of the elastic stress field in the vicinity of the crack tip. Similar expressions exist, in terms of K_{II} and K_{III}, for the Mode II and III loading situations. For mixed-mode loading, the stress fields can be added together directly. It can be seen that K_I, K_{II} and K_{III} characterize the entire stress field (and hence the strain fields) in the vicinity of the crack tip. It therefore seems reasonable to assume that, for Mode I loading for example, failure will occur when K_I reaches a critical value K_c (K_{Ic} under plane strain conditions).

The energy approach and the stress intensity approach are equivalent. Generally, for plane stress:

$$G_{total} = G_I + G_{II} + G_{III} = \frac{1}{E}(K_I^2 + K_{II}^2 + K_{III}^2)$$

and for plane strain:

$\frac{1}{E}$ is replaced by $\frac{(1-\nu^2)}{E}$.

Generally, for geometries with finite boundaries, the following expression is employed for stress intensity factor

$$K_I = Y\sigma\sqrt{\pi a}$$

and similarly for K_{II} and K_{III}, where Y is a function of the crack and component dimensions.

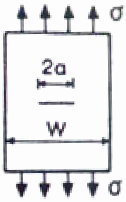
Table 1.2: Typical Fracture Toughness Values

Material	$\sigma_y \left(MN/m^2\right)$	$K_{Ic}\left(MN/m^{3/2}\right)$
Mild Steel	220	140 to 200
Pressure Vessel Steel (HY130)	1700	170
Aluminium Alloys	100 to 600	45 to 23
Cast Iron	200 to 1000	20 to 6

Solutions for Y can be found in the literature for a wide range of geometries and loadings, e.g.

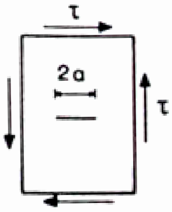
1. H Tada, P Paris and G Irwin, "The stress analysis of cracks handbook", DEL Research Corporation, Hellertown, Pennsylvania, 1973.
2. G P Rooke and D J Cartwright, "Compendium of stress intensity factors", HMSO, 1975.
3. Y Murakami (Editor), "Stress-intensity factors handbook", Pergamon Press, Oxford 1987, (2 volumes).

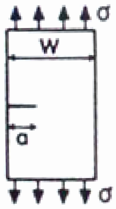
1.2.4 The effect of finite boundaries on expressions for stress intensity factors



$$K_I = \sigma\sqrt{\pi a}\left(\sec\frac{\pi a}{W}\right)^{1/2}$$

$$K_{II} = \tau\sqrt{\pi a}\left(\text{small } \frac{a}{W}\right)$$



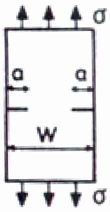


$$K_I = 1.12\sigma\sqrt{\pi a}\left(\text{small } \frac{a}{W}\right)$$

or $K_I = Y\sigma\sqrt{a}$

with $Y = 1.99 - 0.41\frac{a}{W} + 18.7\left(\frac{a}{W}\right)^2 - 38.48\left(\frac{a}{W}\right)^3 + 53.85\left(\frac{a}{W}\right)^4$

$(1.99 = 1.12\sqrt{\pi})$

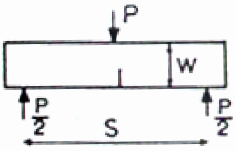


$$K_I = 1.12\sigma\sqrt{\pi a}\left(\text{small } \frac{a}{W}\right)$$

or $K_I = Y\sigma\sqrt{a}$

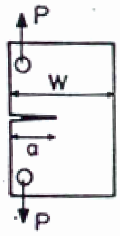
with $Y = 1.99 + 0.76\frac{a}{W} - 8.48\left(\frac{a}{W}\right)^2 + 27.36\left(\frac{a}{W}\right)^3$

$(1.99 = 1.12\sqrt{\pi})$



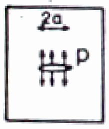
Thickness B

$$K_I = \frac{PS}{BW^{3/2}}\left[2.9\left(\frac{a}{W}\right)^{1/2} - 4.6\left(\frac{a}{W}\right)^{3/2} + 21.8\left(\frac{a}{W}\right)^{5/2} - 37.6\left(\frac{a}{W}\right)^{7/2} + 38.7\left(\frac{a}{W}\right)^{9/2}\right]$$



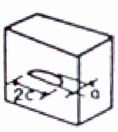
Thickness B

$$K_I = \frac{P}{BW^{1/2}}\left[29.6\left(\frac{a}{W}\right)^{1/2} - 185.5\left(\frac{a}{W}\right)^{3/2} + 655.7\left(\frac{a}{W}\right)^{5/2} - 1017\left(\frac{a}{W}\right)^{7/2} + 63.9\left(\frac{a}{W}\right)^{9/2}\right]$$



p per unit thickness

$K_I = p\sqrt{\pi a}$



$$K_{I_{max}} = 1.12\frac{\sigma}{\Phi}\sqrt{\pi a}$$

$$K_{I_{min}} = 1.12\frac{\sigma}{\Phi}\sqrt{\pi a^2/c}$$

$$\Phi = \int_0^{\pi/2}\left[1 - \frac{c^2 - a^2}{c^2}\sin^2\varphi\right]d\varphi$$

$$\Phi \approx \frac{3\pi}{8} + \frac{\pi}{8}\frac{a^2}{c^2}$$

Figure 1.15. Stress intensity factors for finite bodies.

Example

A large high carbon steel plate with a thumbnail crack, shown in Figure 1.16, for which

$$K_{\max} = 1.2\sigma\sqrt{\pi a}$$

has a fracture toughness of 72MN/m$^{3/2}$ and σ_y = 1450 MPa.

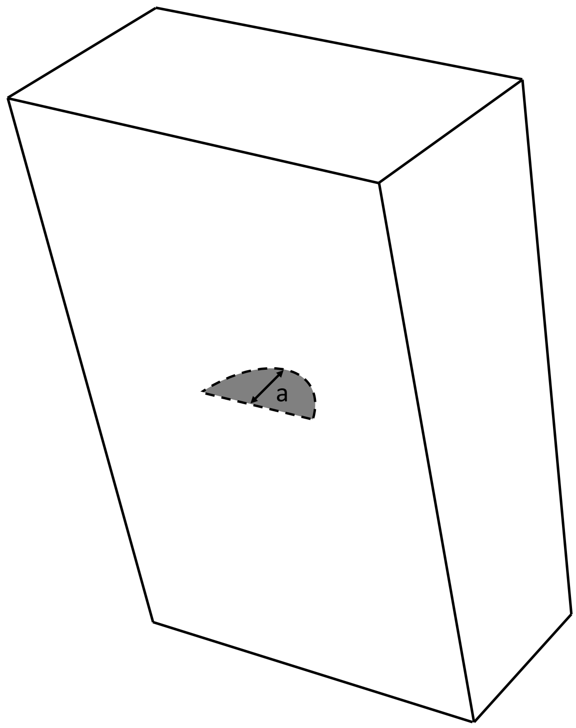


Figure 1.16: Thumbnail crack geometry.

If $\sigma = \frac{2}{3}\sigma_y$, determine the critical initial crack size assuming linear elastic material.

Solution

At fracture, with

$$\sigma = \frac{2}{3}\sigma_y = \frac{2}{3} \times 1450 = 966.67MPa$$

and

$$K_{IC} = 72\frac{MN}{m^{3/2}}$$

Then (from $K_{\max} = 1.2\sigma\sqrt{\pi a}$),

$$72\frac{MN}{m^{3/2}} = 1.2 \times 966.67\frac{MN}{m^2} \times \sqrt{\pi \times a_{crit}}\, m^{1/2}$$

Therefore,

$$a_{crit} = \frac{1}{\pi}\left(\frac{72}{1.2\times 966.67}\right)^2 = 1.226 \times 10^{-3}m = 1.226mm$$

If the material was mild steel, with $\sigma_y = 210MPa\ and\ K_{IC} = 200\frac{MN}{m^{3/2}}$, then a_{crit} = 451mm, i.e., it is much more likely to be detected during inspection!

1.2.5 Fatigue crack growth

It has been shown by Paris and co-workers (1961) that, for a wide range of conditions, there is a logarithmic linear relationship between crack growth rate and the stress intensity factor range during cyclic loading of cracked components. Although this proposition had difficulty being accepted initially, it has become the basis of the damage tolerant approach to fatigue life estimation and is widely used both in industry and in research. Essentially, it means that crack growth can be modelled and estimated based on knowledge of crack and component geometry, loading conditions and using experimentally-measured crack growth data to furnish material constants. This section describes the basics of this approach.

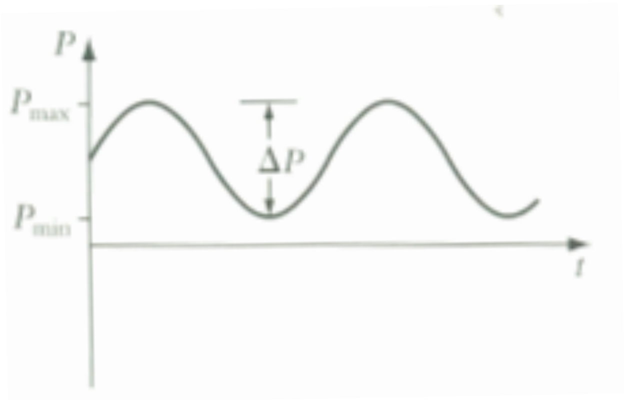


Figure 1.17: Variation of P (load) with t (time)

Considering a load cycle as shown in Figure 1.17 which gives rise to a load range acting on a cracked body:

$$\Delta P = P_{\max} - P_{\min}$$

The load range and crack geometry gives rises to a cyclic variation in stress intensity factor, which is given by:

$$\Delta K = K_{\max} - K_{\min}$$

Even though the stress intensity factor may be less than the critical stress intensity factor for unstable crack growth, stable crack growth may occur if the stress intensity range, ΔK, is greater than an empirically-determined material property called the *threshold stress intensity factor range*, designated ΔK_{th}. In addition, Paris showed that the subsequent crack growth can be represented by an empirical relationship as follows:

$$\frac{da}{dN} = C\,(\Delta K)^m$$

where C and m are empirically-determined material constants. This relationship is known as the Paris equation. Fatigue crack growth data is often plotted as the logarithm of crack growth per load cycle, da/dN, and the logarithm of stress intensity factor range. There are three stages, as shown in Figure 1.18. Below ΔK_{th}., no observable crack growth occurs; region II shows an essentially linear relationship between log(d*a*/d*N*) and log(ΔK), where *m* is the slope of the curve and *C* is the vertical axis intercept; in region III, rapid crack growth occurs and little life is involved. Region III is primarily controlled by K_c or K_{Ic}.

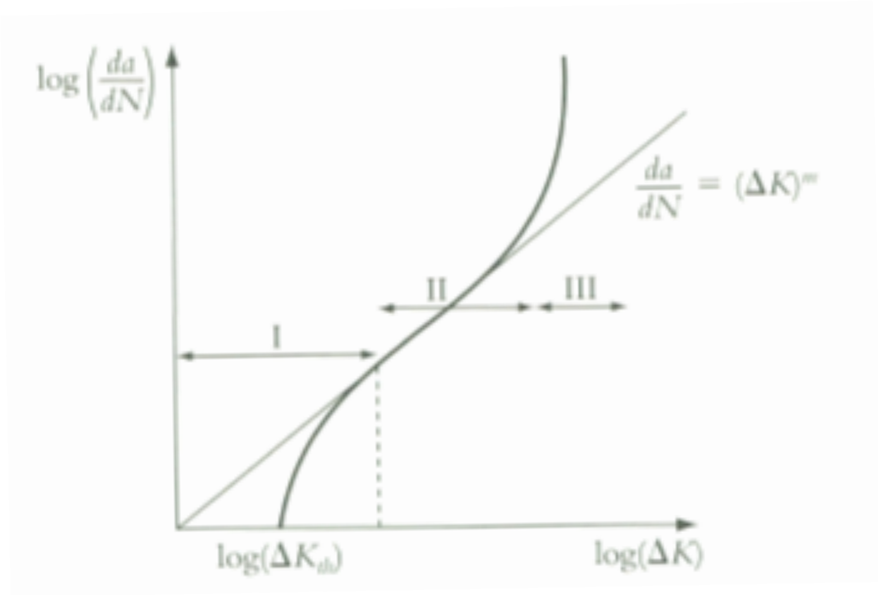


Figure 1.18: Typical (schematic) variation of log (da/dN) with log (ΔK)

The linear regime (Region II) is the region in which engineering components which fail by fatigue propagation occupy most of their life. Knowing the stress intensity factor expression for a given component and loading, the fatigue crack growth life of the component can be obtained by integrating the Paris Equation between the limits of initial crack size and final crack size.

For most materials, the constant *C* is found to be dependent on *R* where *R* is a measure of the mean stress defined as:

$$R = \frac{K_{\min}}{K_{\max}}$$

as shown below in Figure 1.19.

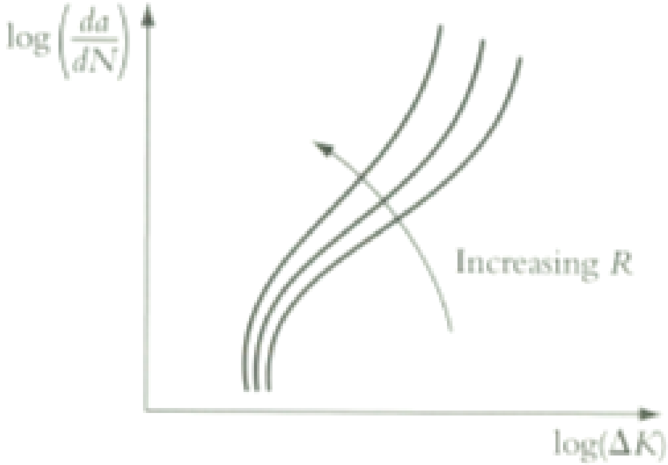


Figure 1.19: Effect of R on fatigue crack growth

Some typical fracture mechanics values for a range of materials are shown in Table 1.3.

Table 1.3: Typical values for ΔK_{th}, m and ΔK

Material	ΔK_{th} ($MN/m^{3/2}$)	m	ΔK ($MN/m^{3/2}$) for da/dN = 10^{-6} mm/cycle
Mild Steel	4 to 7	3.3	6.2
316 stainless steel	4 to 6	3.1	6.3
Aluminium	1 to 2	2.9	2.9
Copper	1 to 3	3.9	4.3
Brass	2 to 4	4.0	4.3 to 66.3
Nickel	4 to 8	4.0	8.8

From L.P. Pook, *J. Strain Analysis*, 1978, pp 114-135.

(N.B. The ΔK_{th} and ΔK (for da/dN = 10^{-6} mm/cycle) values depend on the R-value.)