MMME2046 Dynamics and Control: Lecture 2

Machine Dynamics: Planar Kinematics of Rigid Bodies

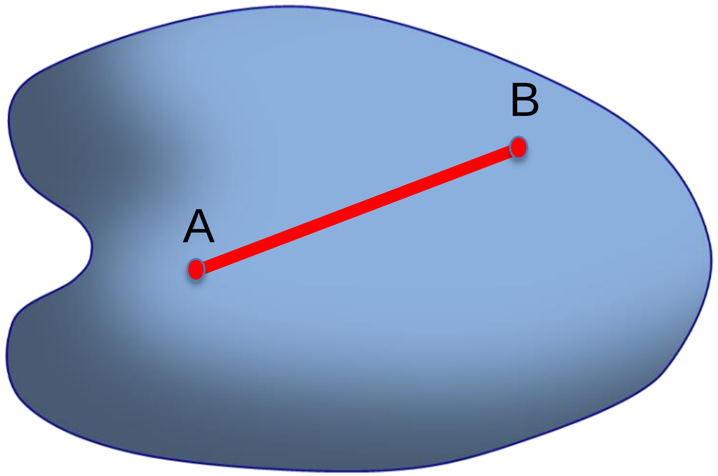
Mikhail Matveev
Mikhail.Matveev@Nottingham.ac.uk
C27, Advanced Manufacturing Building, Jubilee Campus

Handouts Chapter II.1-II.4

Lecture objectives

- Classify various types of rigid body motion
- Perform velocity and acceleration analysis on simple mechanisms

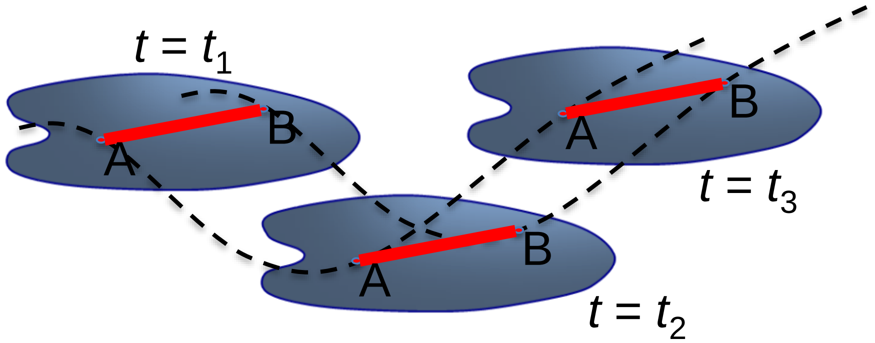
Rigid Body definition



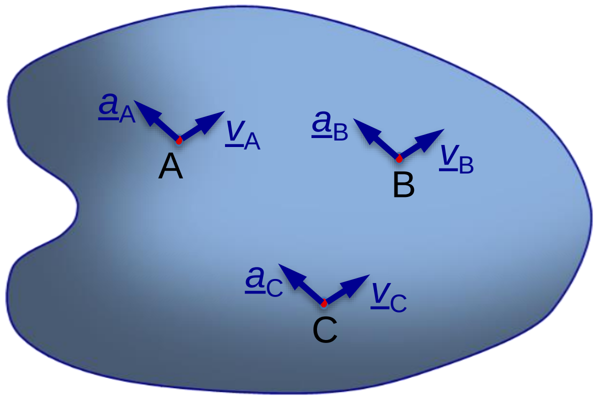
- System of particles
- Distances between particles remain unchanged
- Deformations are neglected

Particle − Rigid body − System of rigid bodies

Rigid Body motion: Pure translation



- Line segments maintain orientation
- Points move on “parallel” trajectories

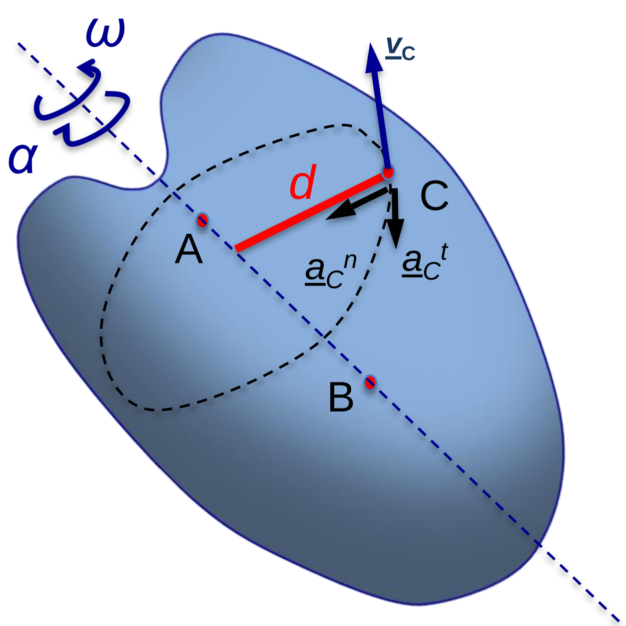


At any instant of time:

$$\underline{v}_A = \underline{v}_B = \underline{v}_C = \ldots$$

$$\underline{a}_A = \underline{a}_B = \underline{a}_C = \ldots$$

Rigid Body motion: Rotation about fixed axis



Kinematics of rigid body governed by:

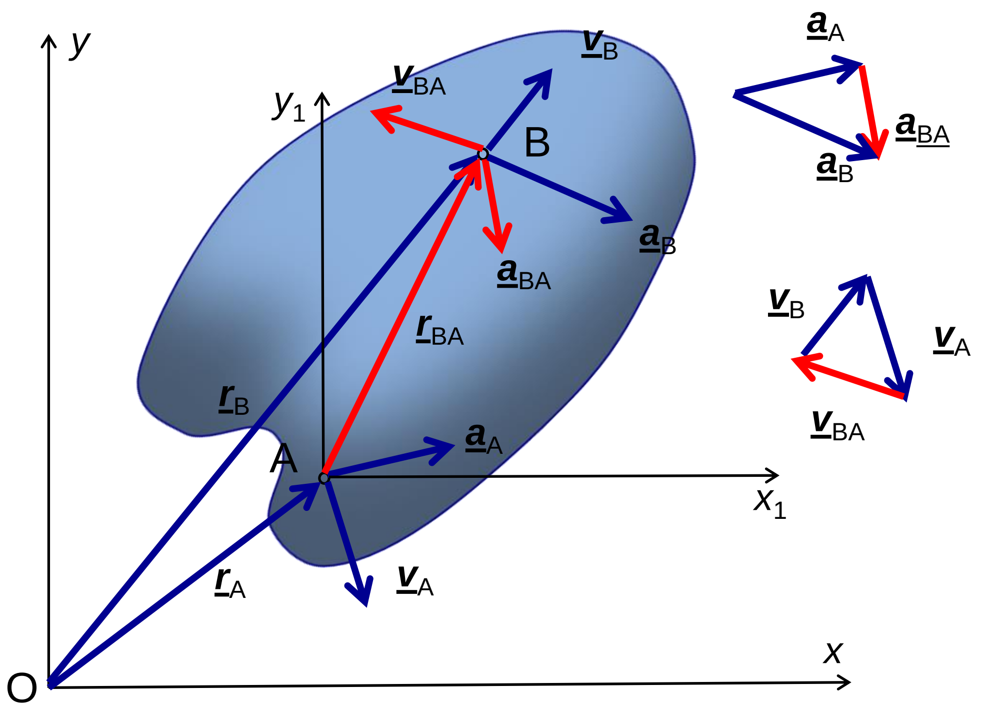
$\theta(t)$	angle of rotation
$\dot{\theta}(t) = \omega(t)$	angular velocity
$\ddot{\theta}(t) = \alpha(t)$	angular acceleration

Each point performs circular motion.
E.g. for point C:

$\mathrm{v}_c = \omega d$ velocity magnitude

$$\left.\begin{array}{l} \mathrm{a}_C^n = \omega^2\, d \\ \mathrm{a}_C^t = \alpha\, d \end{array}\right]$$ acceleration components

Relative motion



$$\underline{r}_B = \underline{r}_A + \underline{r}_{BA}$$

Fundamental equations for rigid bodies!

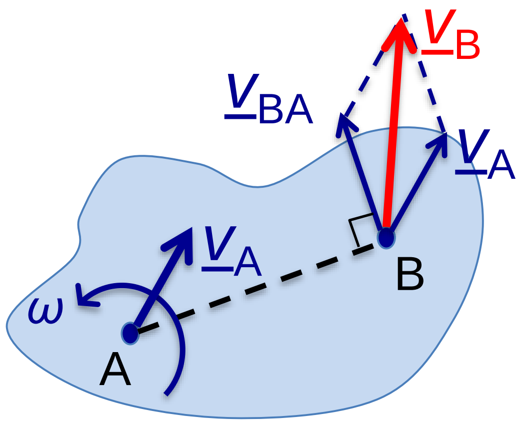
$$\underline{v}_B = \underline{v}_A + \underline{v}_{BA}$$

$$\underline{a}_B = \underline{a}_A + \underline{a}_{BA}$$

Note: changing order in “BA” completely changes the physical meaning!

Note: $\underline{r}_{BA}$ is read: ‘Position of B as seen by A’ (A is the reference point)

Velocity relations in planar motion



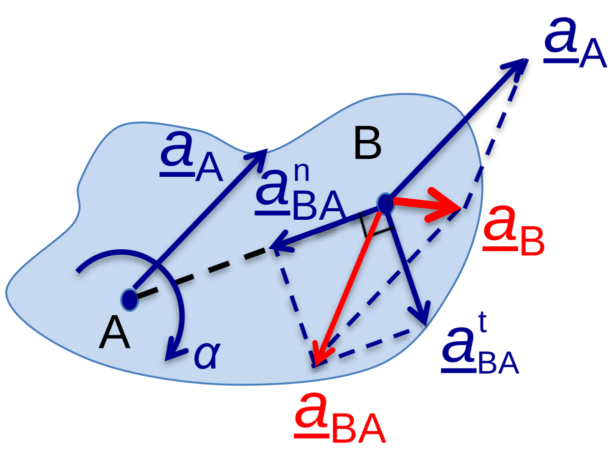
Given: velocity at A & angular velocity

Known: $\underline{v}_B = \underline{v}_A + \underline{v}_{BA}$ (1)

Relative motion at B is **circular** around A:

1) magnitude: $v_{BA} = \omega \, AB$
2) direction: perpendicular to AB
3) sense: governed by the angular velocity

Acceleration relations in planar motion



Given: acceleration at A, angular velocity & angular acceleration

Known: $\underline{a}_B = \underline{a}_A + \underline{a}_{BA} = \underline{a}_A + \underline{a}_{BA}{}^n + \underline{a}_{BA}{}^t$ (2)

Relative motion at B is **circular** around A:

1) magnitudes: $a^n_{BA} = \omega^2\, AB$ $\quad a^t_{BA} = \alpha\, AB$

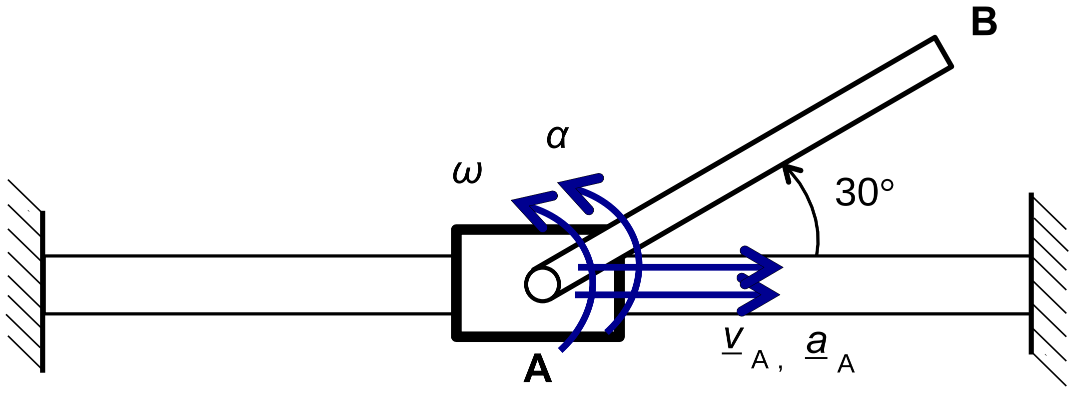
2) directions & senses:

- Normal component always has direction towards the reference point.
- Tangential component is perpendicular to AB with direction defined by α.

Example II.1: Slider mechanism

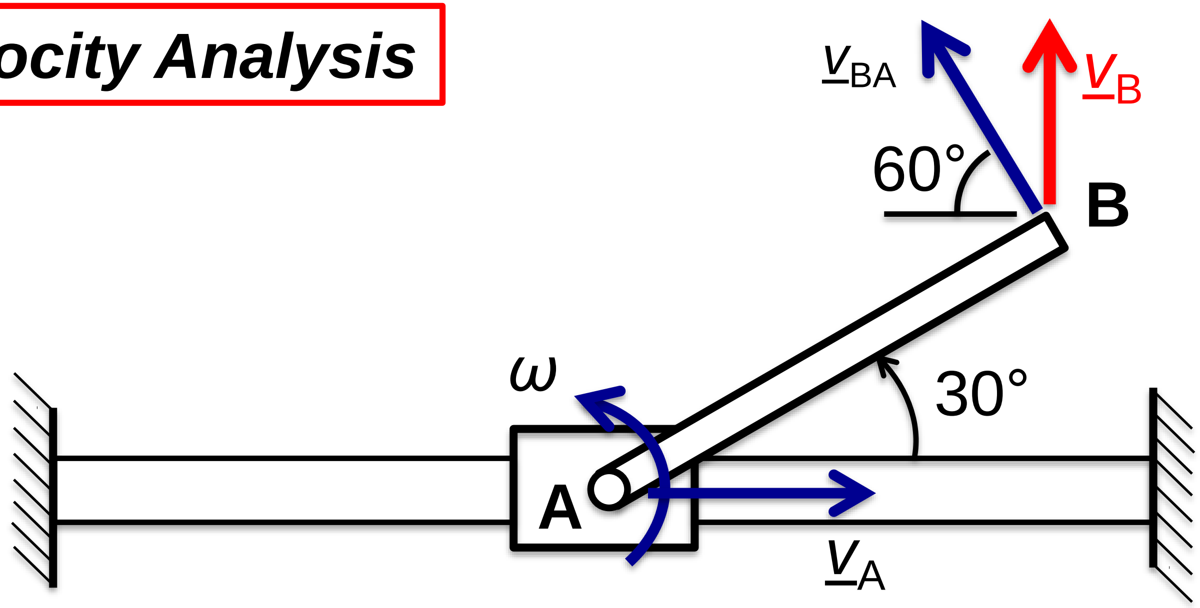
Figure shows part of a slider mechanism at a particular instant in time.
Collar A moves along a fixed horizontal track with **velocity *v* and acceleration a***.* Link AB has length 100 mm and **rotates with angular velocity $\boldsymbol{\omega}$ and angular acceleration $\boldsymbol{\alpha}$**. At the instant shown, link AB is at an angle of 30^o to the track, v=1 m/s, a=20 m/s^2, ω=20 rad/s and α=100 rad/s^2.

Determine the velocity and acceleration of point B at the instant shown.



Example II.1: Slider mechanism

Velocity Analysis



Velocity of B is calculated using

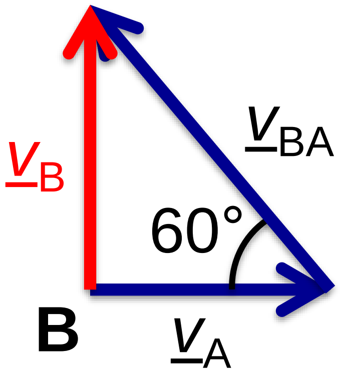
$$\underline{v}_B = \underline{v}_A + \underline{v}_{BA} \qquad (1)$$

$$AB = 0.1 \text{ m}$$

$$v_A = 1 \text{ m/s}$$

$$\omega = 20 \text{ rad/s}$$

$$v_{BA} = \omega\, AB = 20 \times 0.1 = 2 \text{ m/s}$$



Calculate velocity B by resolving equation (1) in horizontal and vertical directions:

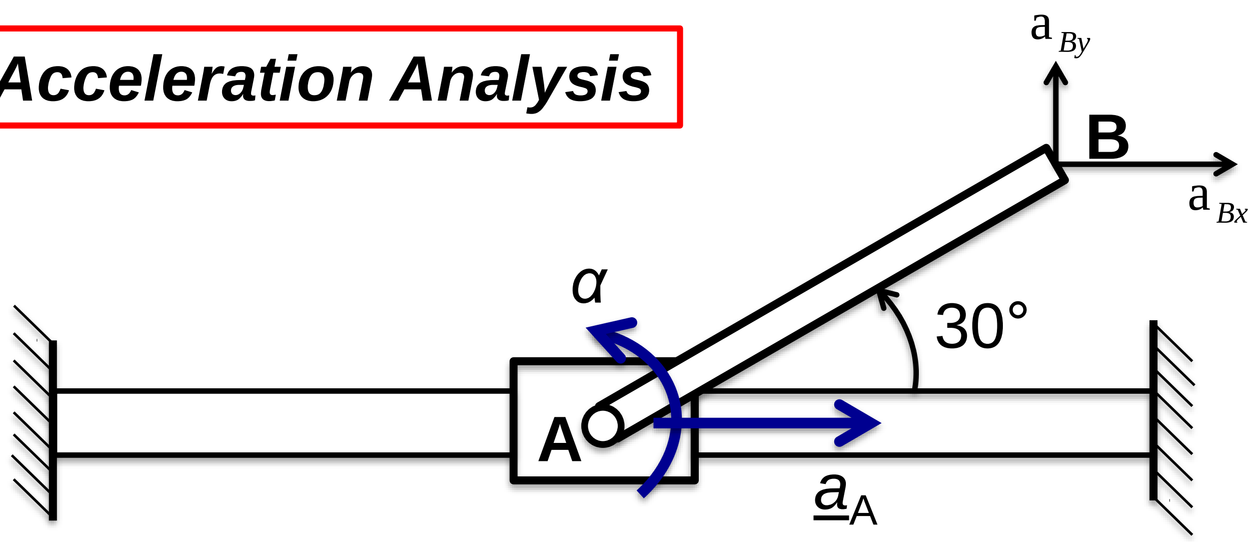
$$\rightarrow^{+}: \quad v_{Bx} = v_A - v_{BA}\cos 60° = 1 - 2\cos 60° = 0$$

$$\uparrow^{+}: \quad v_{By} = 0 + v_{BA}\sin 60° = 2\sin 60° = 1.732 \text{ m/s}$$

At the instant shown, the velocity of B is vertically upwards.

Example II.1: Slider mechanism

Acceleration Analysis



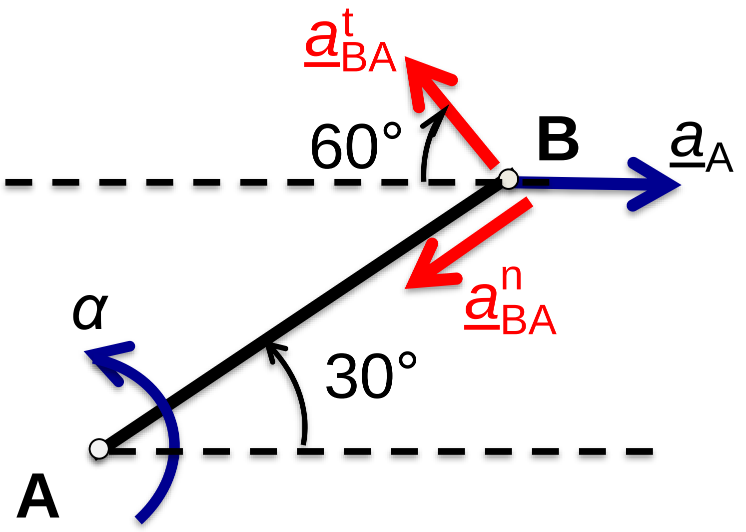
Acceleration of B is calculated using

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{BA}^n + \mathbf{a}_{BA}^t \qquad (2)$$

$AB = 0.1$ m $\omega = 20$ rad/s

$a_A = 20$ m/s^2 $\alpha = 100$ rad/s^2

$$a_{BA}^n = \omega^2 AB = 20^2 \times 0.1 = 40\ \mathrm{m/s^2}$$
$$a_{BA}^t = \alpha\ AB = 100 \times 0.1 = 10\ \mathrm{m/s^2}$$



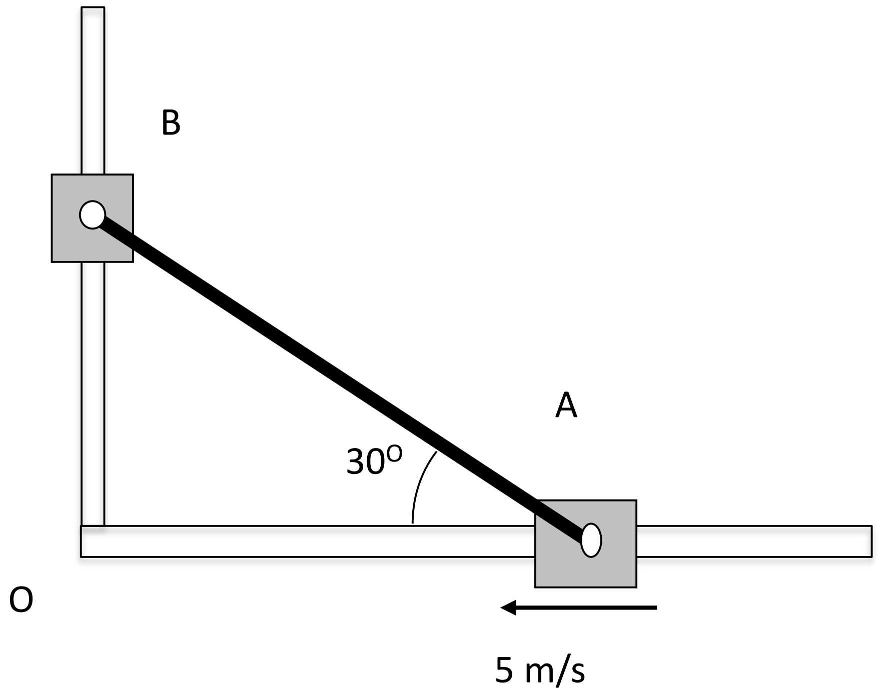
Calculate acceleration of B by resolving equation (2) in horizontal and vertical directions:

$$\rightarrow^+ \ \Sigma X: \quad a_{Bx} = a_A - a_{BA}^n \cos 30^o - a_{BA}^t \cos 60^o$$
$$= 20 - 40\cos 30^o - 10 \cos 60^o = -19.64\ \mathrm{m/s^2}$$

$$\uparrow^+ \Sigma Y: \quad a_{By} = 0 - a_{BA}^n \sin 30^o + a_{BA}^t \sin 60^o = -11.34\ \mathrm{m/s^2}$$

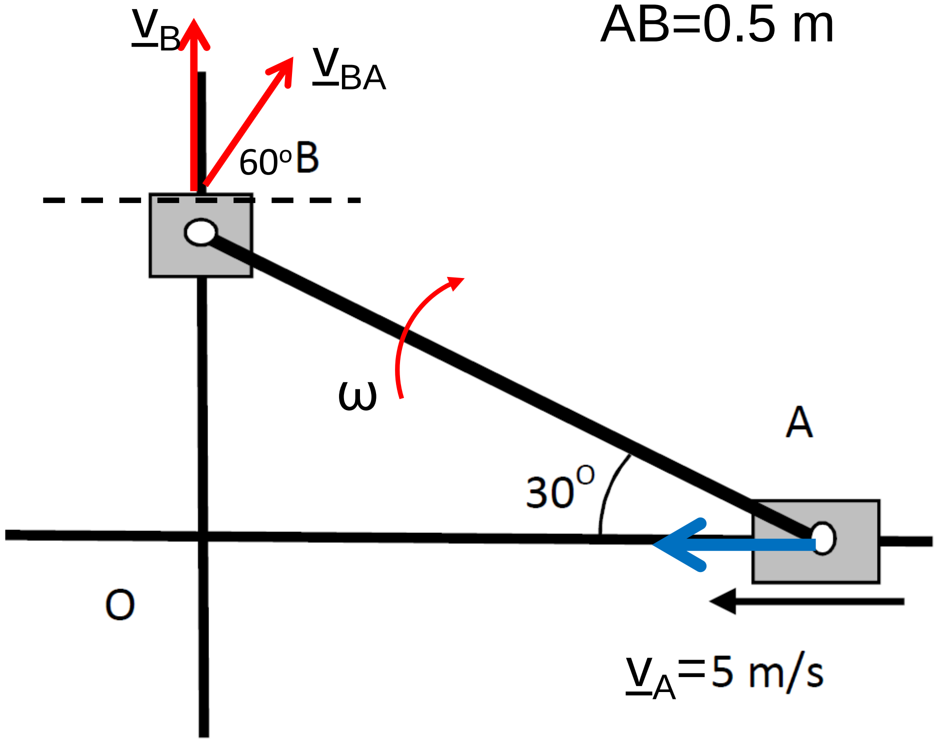
Example II.2: Rigid link

The ends A and B of a rigid link (*AB*=0.5 m) move along fixed horizontal and vertical guides. In the position shown, A is moving towards O with a constant velocity of 5 m/s. Calculate the velocity and acceleration of B and the angular velocity and angular acceleration of AB.



Example II.2: Rigid link

Velocity Analysis



Velocity of B is calculated using

$$\underline{\mathbf{v}}_B = \underline{\mathbf{v}}_A + \underline{\mathbf{v}}_{BA} \qquad (1^*)$$

$$v_{BA} = \omega\, AB$$

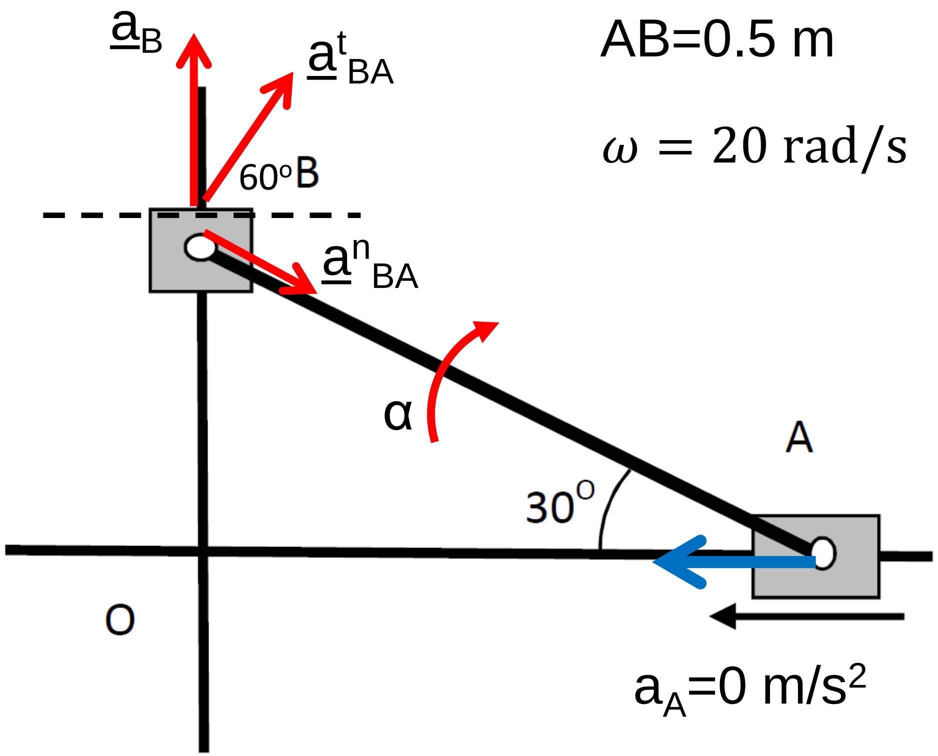
$$\rightarrow^{+}: \quad 0 = -v_A + v_{BA}cos60° = -v_A + \omega ABcos60°$$

$$\omega = \frac{v_A}{ABcos60°} = \frac{5}{0.5 \times 0.5} = 20 \text{ rad/s}$$

$$\uparrow^{+}: \quad v_B = 0 + v_{BA}sin60° = 20 \times 0.5sin60° = \\ = 8.66 \text{ m/s}$$

Example II.2: Rigid link

Acceleration Analysis



AB=0.5 m

$\omega = 20$ rad/s

Acceleration of B is calculated using

$$\underline{\mathbf{a}}_B = \underline{\mathbf{a}}_A + \underline{\mathbf{a}}^n_{BA} + \underline{\mathbf{a}}^t_{BA}$$

$$a^n_{BA} = \omega^2 AB = 200 \text{ m/s}^2$$
$$a^t_{BA} = \alpha\, AB$$

$$\rightarrow^+: \quad 0 = 0 + a^n_{BA} \cos 30° + \alpha AB \cos 60°$$

$$\alpha = -\frac{a^n_{BA} \cos 30°}{AB \cos 60°} = -692.8 \text{ rad/s}^2$$

$$\uparrow^+: \quad a_B = 0 - a^n_{BA} \sin 30° + \alpha AB \sin 60° = -400 \text{ m/s}^2$$

Lecture objectives

- Classify various types of rigid body motion
- Perform velocity and acceleration analysis on simple mechanisms

Next lecture

- Perform velocity and acceleration analysis on more complex linkage mechanisms

- Instantaneous centre of rotation and use of it for velocity analysis