

MMME2046 Dynamics and Control: Lecture 5

Planar Dynamics of Rigid Bodies

Mikhail Matveev

Mikhail.Matveev@Nottingham.ac.uk

C27, Advanced Manufacturing Building, Jubilee Campus

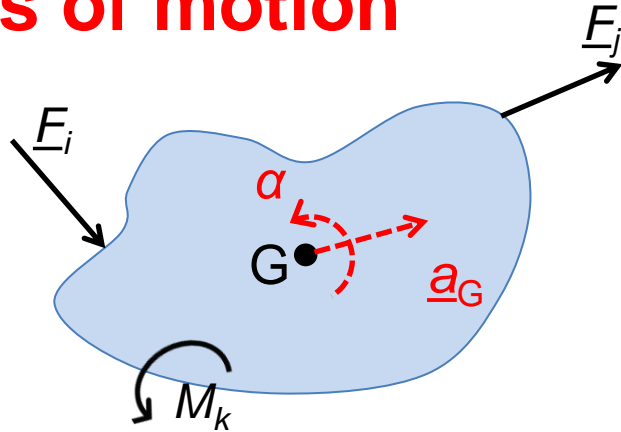
Handouts Chapter III

Lecture objectives

- Dynamics of a linkage mechanism
- Introduction to the coursework

Fundamental Laws of Rigid Body Motion

Equations of motion

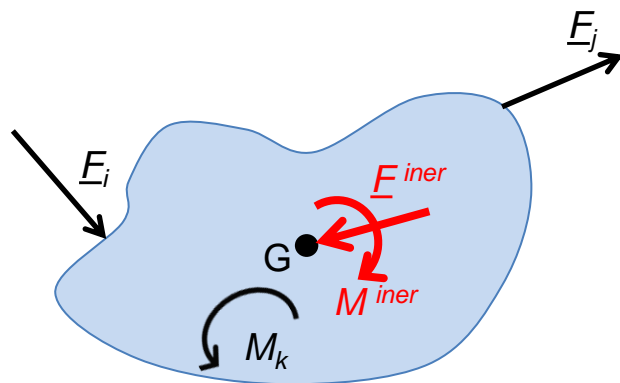


$$\rightarrow^+: \Sigma F_x = m_G a_{G,x}$$

$$\uparrow^+: \Sigma F_y = m_G a_{G,y}$$

$$\curvearrow^+: \Sigma M_G = J_G \alpha$$

D'Alembert's principle



$$\rightarrow^+: \Sigma F_x - F_x^{inertia} = \Sigma F_x - m_G a_{G,x} = 0$$

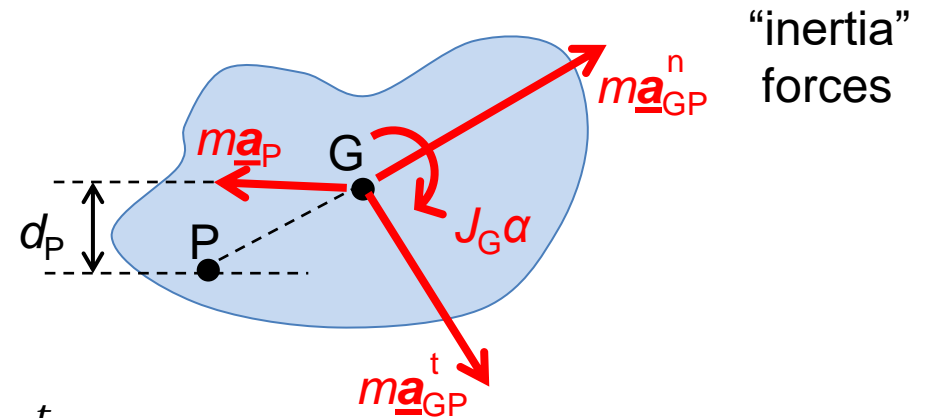
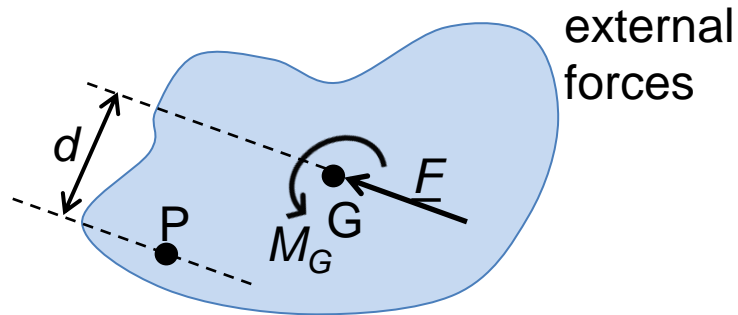
$$\uparrow^+: \Sigma F_y - F_y^{inertia} = \Sigma F_y - m_G a_{G,y} = 0$$

$$\curvearrow^+: \Sigma M_G - M^{inertia} = \Sigma M_G - J_G \alpha = 0$$

Fundamental Laws of Rigid Body Motion

D'Alembert's principle

Given: arbitrary point P with known \underline{a}_P

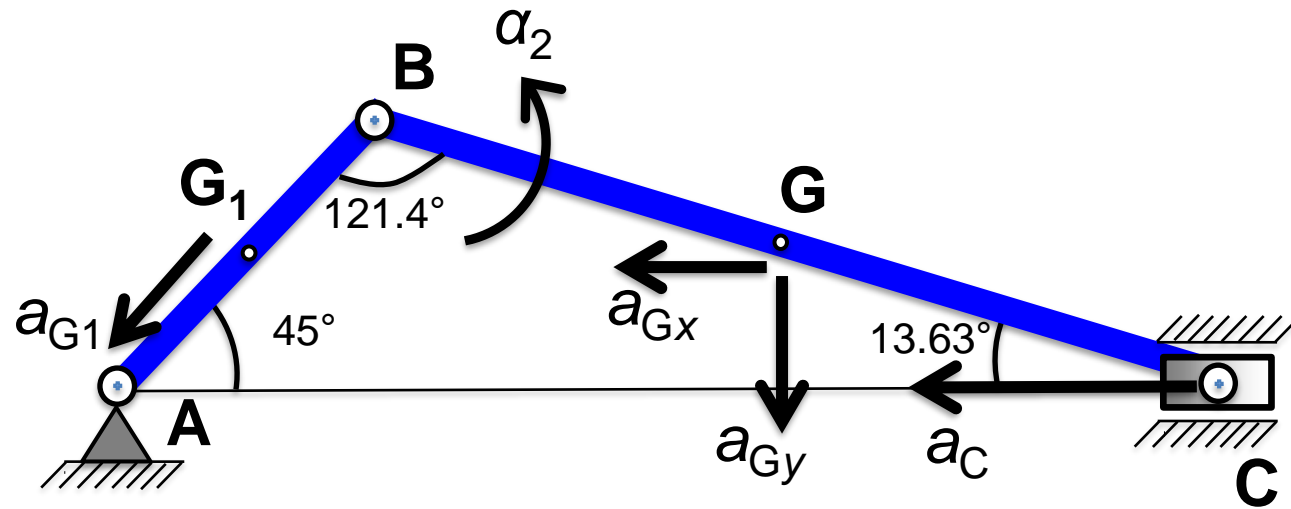


$$\underline{a}_G = \underline{a}_P + \underline{a}_{GP}^n + \underline{a}_{GP}^t$$

$$\curvearrowright^+: M'_P = M_G + Fd - J_G \alpha - mPG^2 \alpha + ma_P d_P = 0$$

$$M_P + ma_P d_P = J_P \alpha \quad (3)$$

Worked Example III.6: Slider-Crank mechanism



$$\omega_1 = 100 \text{ rad/s} = \text{const.}$$

$$BC = 240 \text{ mm}$$

$$\theta = 45^\circ$$

$$AB = 80 \text{ mm}$$

$$BG = 120 \text{ mm}$$

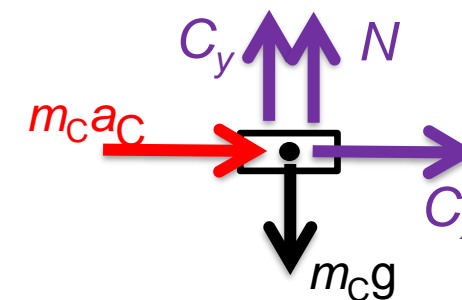
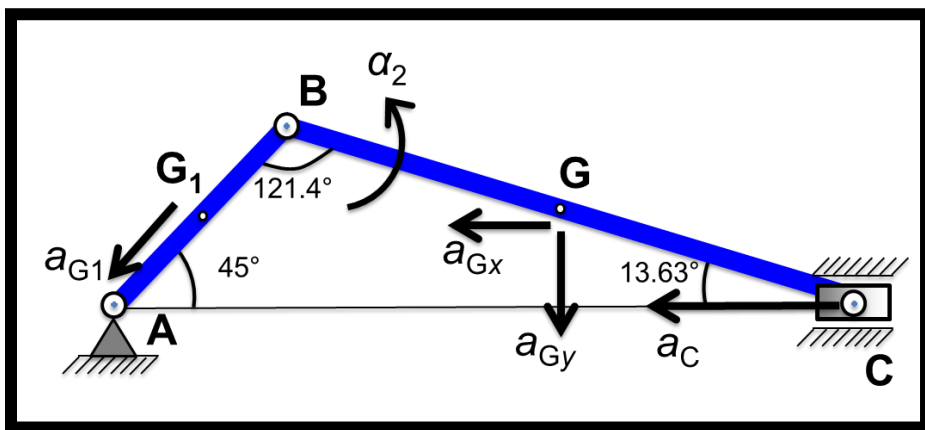
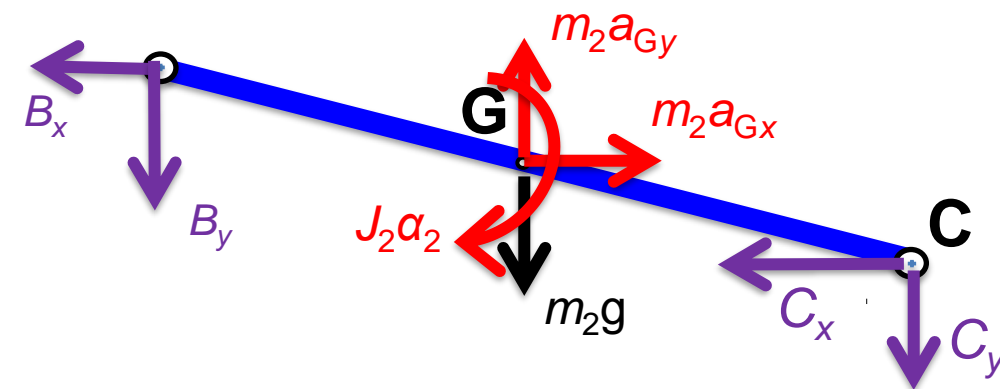
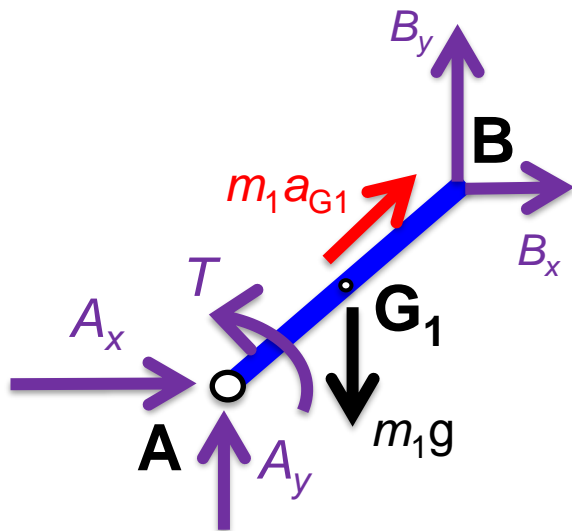
From Worked Example II.4 in Planar Kinematics of Rigid Bodies:

$$a_{G1} = 400 \text{ m/s}^2; \quad a_{Gx} = 579.7 \text{ m/s}^2; \quad a_{Gy} = 282.8 \text{ m/s}^2; \quad a_c = 573.7 \text{ m/s}^2; \quad \alpha = 2283 \text{ rad/s}^2$$

Assume frictionless cylinder and:

$$m_1 = 0.02 \text{ kg}, \quad m_2 = 0.06 \text{ kg} \quad \text{and} \quad m_c = 0.2 \text{ kg}$$

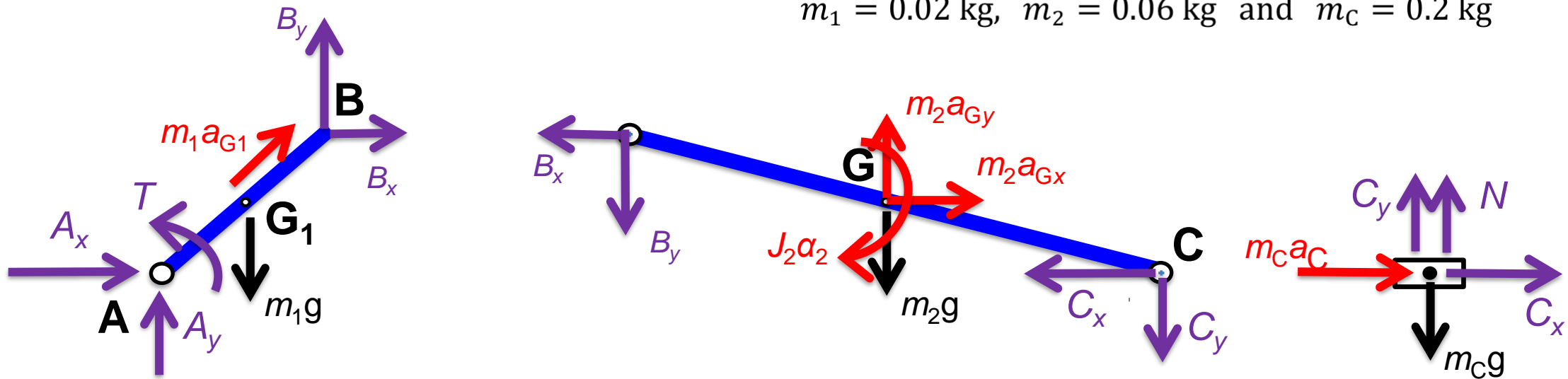
Worked Example III.6: Slider-Crank mechanism



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$$m_1 = 0.02 \text{ kg}, \quad m_2 = 0.06 \text{ kg} \quad \text{and} \quad m_c = 0.2 \text{ kg}$$



$$m_1g = 0.02 \times 9.8 = 0.196 \text{ N}$$

$$m_1a_{G1} = 0.02 \times 400 = 8 \text{ N}$$

$$m_2g = 0.06 \times 9.8 = 0.588 \text{ N}$$

$$m_2a_{Gx} = 0.06 \times 569.7 = 34.18 \text{ N}$$

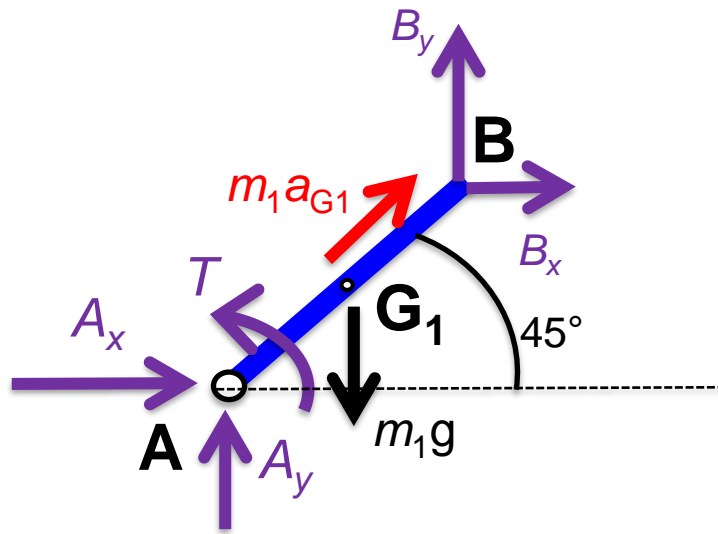
$$m_2a_{Gy} = 0.06 \times 282.8 = 16.97 \text{ N}$$

$$m_cg = 0.2 \times 9.8 = 1.96 \text{ N}$$

$$m_ca_c = 0.2 \times 573.7 = 114.7 \text{ N}$$

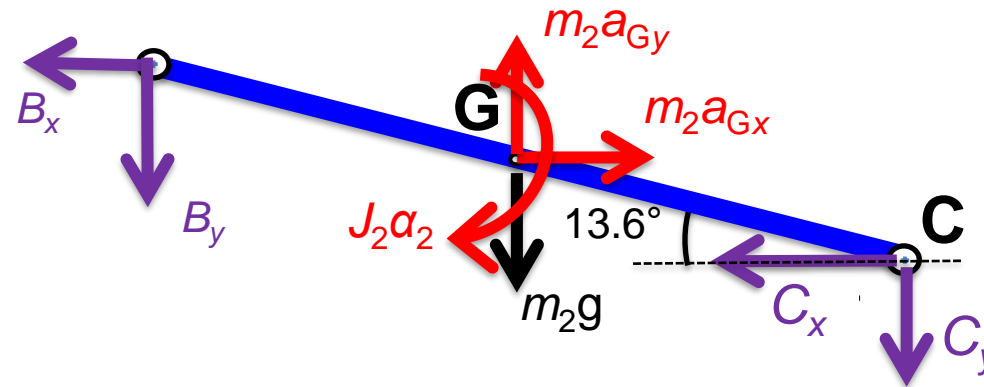
$$J_2\alpha_2 = \left(\frac{1}{12} m_2 BC^2 \right) \alpha_2 = \frac{1}{12} 0.06 \times 0.24^2 \times 2283 = 0.6575 \text{ Nm}$$

Worked Example III.6: Slider-Crank mechanism

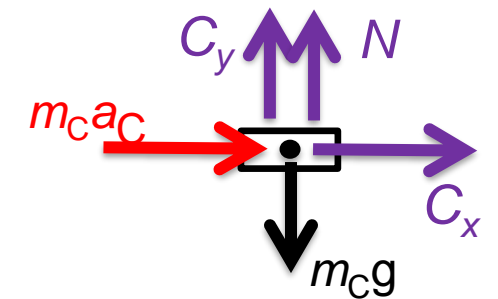


Rigid body: 3 E.O.M

Total 8 E.O.M for 8 unknowns



Rigid body: 3 E.O.M

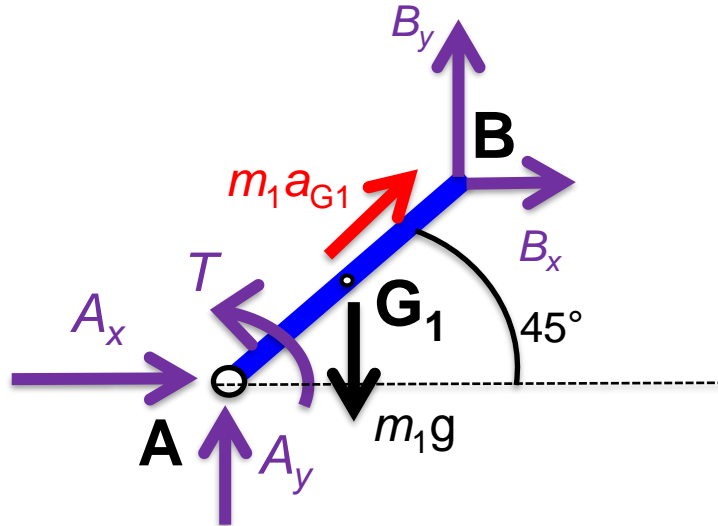


Particle: 2 E.O.M

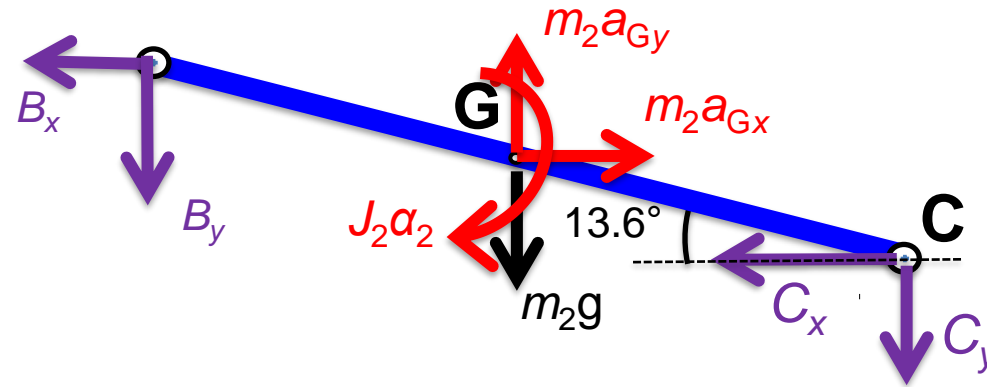
Worked Example III.6: Slider-Crank mechanism

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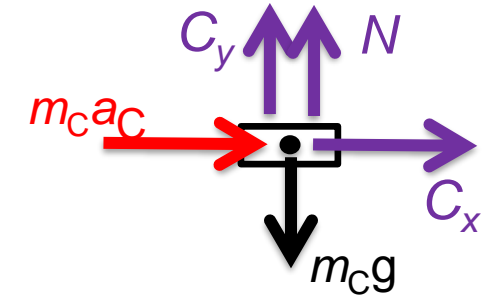
$$m_1 = 0.02 \text{ kg}, \quad m_2 = 0.06 \text{ kg} \quad \text{and} \quad m_c = 0.2 \text{ kg}$$



Rigid body: 3 E.O.M



Rigid body: 3 E.O.M



Particle: 2 E.O.M

$$\rightarrow^+ \Sigma F_x^C = 0:$$

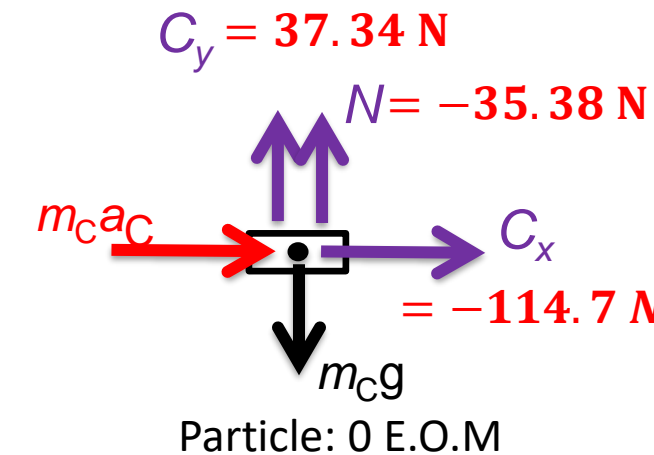
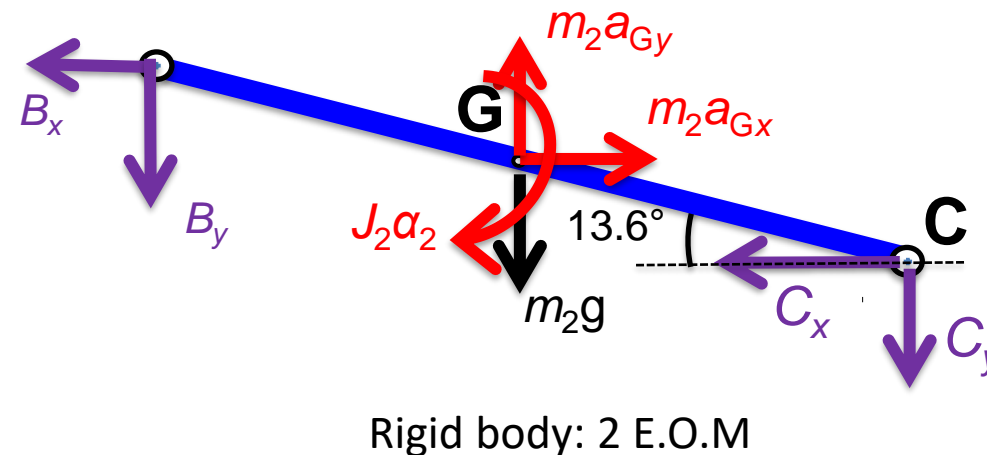
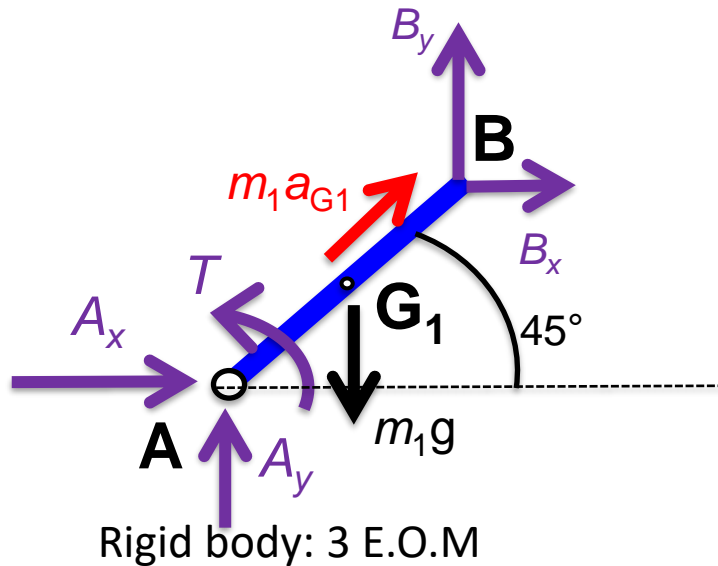
$$\curvearrow^+ \Sigma M_B^{BC} = 0:$$

$$\uparrow^+ \Sigma F_y^C = 0:$$

Worked Example III.6: Slider-Crank mechanism

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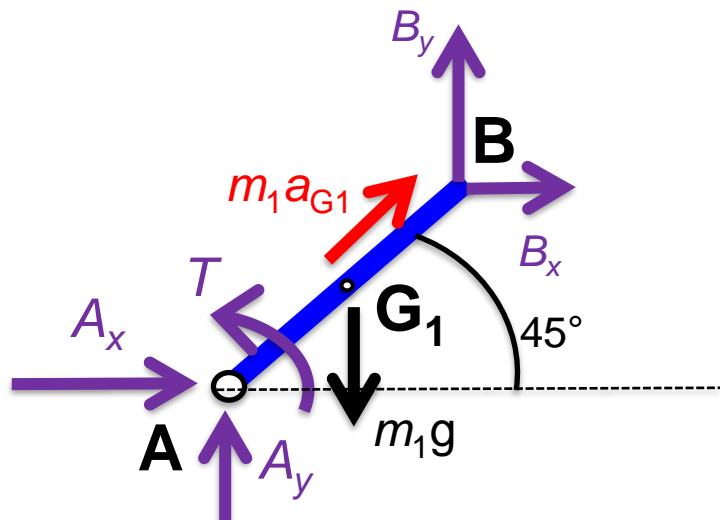
$$\rightarrow^+ \Sigma F_x^{(BC)} = 0:$$

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Worked Example III.6: Slider-Crank mechanism

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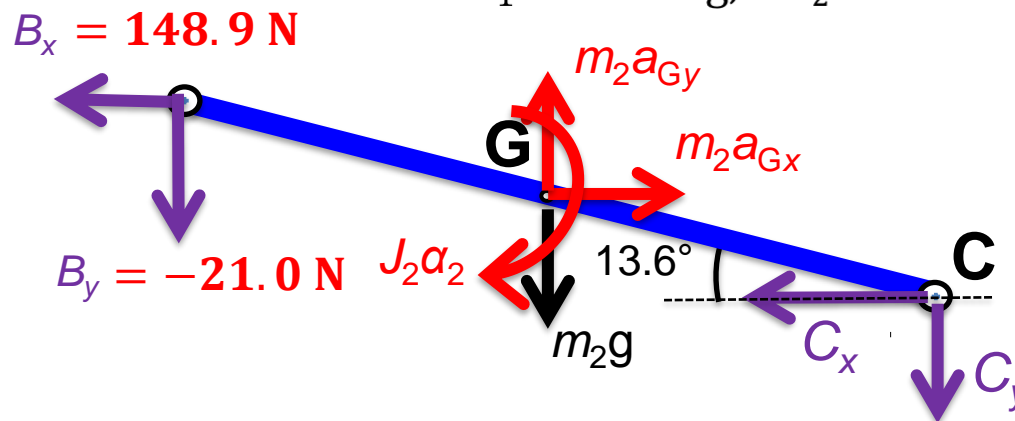


Rigid body: 3 E.O.M

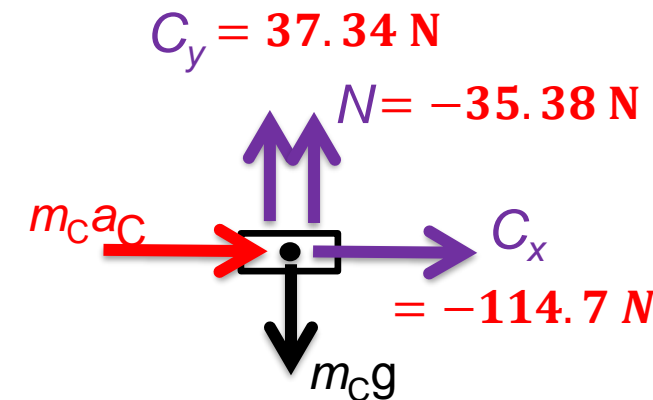
$$\rightarrow^+ \Sigma F_x^{(AB)} = 0:$$

$$\uparrow^+ \Sigma F_y^{(AB)} = 0:$$

$$\curvearrow^+ \Sigma M_A^{AB} = 0:$$

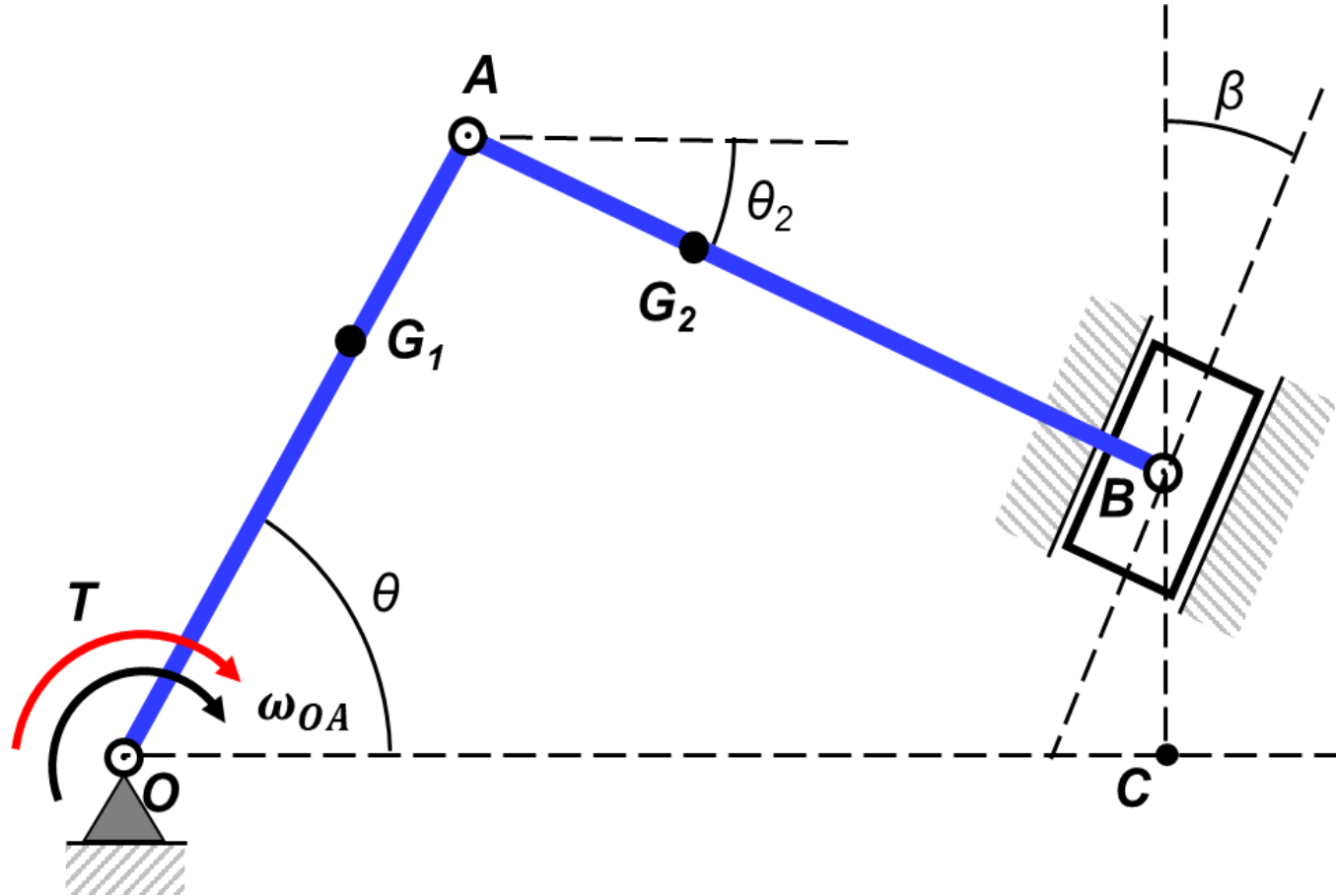


Rigid body: 0 E.O.M



Particle: 0 E.O.M

Machine Dynamics Case Study



Machine Dynamics Case Study

- Use your individual values
- Follow the submission process guidelines