University of Nottingham

School of Mathematical Sciences

MTHS2007 Advanced Mathematics and Statistics for Mechanical Engineers

Fourier Series Problem Sheet 2

1. If $f(x+2\ell) = f(x)$ for all x and

$$f(x) = \begin{cases} -1 & \text{for} \quad -\ell < x < 0, \\ 1 & \text{for} \quad 0 \le x \le \ell, \end{cases}$$

sketch the function $f(x)$ in the range $-2\ell < x < 2\ell$ and show that

$$f(x) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin\left((2n-1)\frac{\pi x}{\ell}\right), \quad -\ell < x < \ell.$$

2. The function $f(x)$ is defined by

$$f(x) = \begin{cases} -x & \text{for} \quad -\ell \le x < 0 \\ x & \text{for} \quad 0 \le x < \ell \end{cases}$$

and $f(x+2\ell) = f(x)$.

Sketch $f(x)$ in $-2\ell < x < 2\ell$ and find its Fourier series.

To what does the series converge when (a) $x = 0$ (b) $x = \ell$?

By choosing an appropriate value for x in the Fourier series for $f(x)$, show that

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}.$$

3. The function $f(x)$ is defined by

$$f(x) = \begin{cases} x & \text{for} \quad 0 \le x < \frac{1}{3}\ell \\ \frac{1}{2}(\ell - x) & \text{for} \quad \frac{1}{3}\ell \le x < \ell \end{cases}$$

$$f(-x) = -f(x) \quad \text{and} \quad f(x+2\ell) = f(x).$$

Sketch $f(x)$ in $-2\ell < x < 2\ell$ and find its Fourier series.

4. The function $g(x)$ is defined by

$$g(x) = \begin{cases} x & \text{for} \quad 0 \leq x < \frac{1}{3}\ell \\ \frac{1}{2}(\ell - x) & \text{for} \quad \frac{1}{3}\ell \leq x < \ell \end{cases}$$

$$g(-x) = g(x) \quad \text{and} \quad g(x + 2\ell) = g(x).$$

Sketch $g(x)$ in $-2\ell < x < 2\ell$ and find its Fourier series.

5. Consider the function f defined by

$$f(x) = x \cos x$$

for $-\pi < x \leq \pi$, and $f(x + 2\pi) = f(x)$ for all x (ie the function is 2π-periodic).

(a) To what value will the Fourier series converge at $x = \pi$?

(b) Find the Fourier series for the function f.
[Hint: You may use the fact that $\sin((n+1)x) = \sin(nx)\cos x + \cos(nx)\sin x$*, and a similar formula for* $\sin((n-1)x)$*. Remember how we derived the formulas for* a_n *and* b_n*.]*

(c) Confirm that the series you found in part (b) converges to the value you have found in part (a) at $x = \pi$.