MTHS2007 **Advanced Mathematics for Mechanical Engineers: exam feedback 2021-22**

As a general comment, you might find it useful to compare this feedback to the previous year's feedback and note how similar the documents are. There is a lesson in there somewhere.
Lots of answers to all questions gave very little working/explanation/justification. If there is an error in calculations then this makes it very hard to give part-marks for method, since the method is not clear from what is written on the script. Remember that the exam is not just for you to solve the problems given: it is for you to use those problems to demonstrate that you understand what was taught in the module - and showing your working, giving explanations, etc is an important part of demonstrating understanding.
*Where I point to similar examples being done in Minilectures below, if you instead chose to follow the live Friday lectures, you will find those examples in the lecture of the corresponding week.*

1. This is the question that students did most well in, perhaps not surprising as it was the first topic covered. A problem that was especially prevalent this year here was mixing up the independent variable, for example writing functions of $t$ instead of $x$ in part (a) or writing x instead of $t$ in part (b). Sometimes this is just a slip of the pen, so wasn't penalised hard, but for some students it was done more seriously, leading in part (b) for example to differentiating with respect to the wrong variable when eliminating one of the dependent variables. Finding the particular solution in part (a) was actually pretty straightforward this year, but many students jumped to the conclusion that it must be a challenging case and mistakenly made a substitution of the form $axe^{3x}$ instead of $ae^{3x}$. *Solving ODES with exponential RHSs was covered in Minilecture 2C and a similar example was also done in Example Class 1. See also Quiz 2C. Examples with systems of equations like in part (b) were done in Minilecture 3B, Quiz 3B and the additional problem provided for Problem Class 1.*

2. A common error in part (a) was to simply give the graph of $\cos x$, without replacing with $|\cos x| = -\cos x$ where this was negative, or labeling where the graph hit the $x$-axis incorrectly - getting this right was important for part (b). In part (b), it was common to get a factor of 2 wrong in the shortest period, which was strongly correlated with issues in part (a). Most students went wrong somewhere along the way in part (c) but did enough correctly to get most of the marks, once initial errors were accounted for (with "marks carried forward"). Common errors included getting the half-period in front of the integral for $a_n$ wrong or writing the limits of the integral incorrectly. Some students failed to identify and use the symmetry of the function. While full marks were still available in principle, in practice students taking this route typically got bogged down and got other things wrong. *Note that problems plotting rectified and otherwise chopped-up trig functions appeared in Quiz 3C . Evaluating Fourier coefficients of such functions appeared in Quiz 3E - and in the Coursework!*

   Part (c) was just a short test of knowing that the Fourier series should converge to the simple value of the function when it is continuous, but a lot of students didn't attempt it or tried to do complicated calculations involving the expressions found for Fourier coefficients. *This topic was covered in Minilecture 4B and analogous problems appeared in Quiz 4B.*

   Part (d) was often not attempted. The key was to recognise that the problem was solved by inserting some particular value of $x$ into the Fourier series. Students who did generally did well. *Similar problems using values of a Fourier series with a particular x were done in Example Class 2 and were tested in Quiz 4B - and a* ***very*** *similar example was done in part 3(c) of the Coursework!*

3. In part (a), a lot of students correctly identified that *something* discontinuity-like happened in the graph at $t = 1$ because of the $H(t-1)$ in some way, but it was common to get the detailed sketch wrong. *Note that similar examples were given in Minilecture 6A and in Examples Class 3. Problems like it were also done in QUIZ 6A.*

   A lot of students wrote the Laplace transforms of $t$ and 1 separately and then multiplied by an exponential, but remember that the *Shifting* Theorem uses the Laplace transform in which $t-1$ is replaced by $t$. *Note that similar calculations were done as part of the worked examples of in Minilectures 6C and 6D, and in Examples Class 3.*

   Most of the marks were available for part (c), even for students who did not do parts (a) and (b), with "marks carried forward". Some students lost a small amount of credit for not stating clearly how the Laplace transform of the RHS entered in the solution. Others confused the Laplace transform $\bar{y}$ of the solution with the Laplace transform $\bar{f}$ of the RHS - doing this often led to a significant losses later, even with "marks carried forward".

   In part (d), it was common to make partial fraction substutions of the wrong form, and to mishandle the parts involving $e^{-s}$ and the Second shifting theorem. *Note that similar worked examples involving discontinuous RHSs (and Heaviside step functions) were given in Minilectures 6C and 6D, and in Examples Class 3. They were also the topic of QUIZ 6C.*

4. Overall this question was not done very well. Many responses made very little progress; many made good progress through (a), (b) and a little bit of (c); only a very few made it all the way through. I gave many marks for good attempts at later parts where earlier parts contained an error - it is worth persisting!

   (a) was generally done well by those who attempted it, but a very common error was to manipulate the "separated PDE" $X''(x)T(t) - T''(t)X(x) = X(x)T(t)$ to $X''(x)/X(x) - T''(t)/T(t) = 0$ where there should be a 1 on the RHS, not 0. *Problems with similar relabelling of contants came up in QUIZ 7D and Example Class 4 (where the "spring-loaded wave equattion example is identical except for the appearance of parameters) and it was otherwise similar to the calculations in minilecture 8B and the worked example in Problem Class 4.*

   (b) Most attempts at this question had the right basic idea but missed some important details. For example, using the same constants in the solutions for $X(x)$ and $T(t)$, or finding solutions for all real values of lambda when you had been told to only consider positive lambda, or solving the ODEs for $X$ and for $T$ in very different ways when they are of exactly the same form. *Note that an identical ODE and boundary conditions were solved in Minilecture 8A, where the simple wave equation is solved. Even if incorrect expressions had previously been found for $\lambda'$, partial credit was available for solving the rest of the problem in a similar manner.*

   (c) Same general comment as for (b): most attempts had the right idea but missed important details, like solving $\sin(kL) = 0$ to get $kL = n\pi$ but not specifying which values of n this applies for, or using the boundary conditions to simplify $X(x)$ but not putting this together with $T(t)$ to actually answer the question which asks for the solution $\varphi(x,t) = X(x)T(t)$, and - very commonly - writing a solution that depends on $n$ but not summing over all values of $n$. *Note that an identical ODE and boundary conditions were solved in Minilecture 8A, where the simple wave equation is solved.*

(d) Most attempts tried to use the initial conditions sensibly, but often got stuck with the resulting algebra and/or with earlier errors making this part difficult/impossible. *This is very similar to the worked examples in Minilecture 8A and especially Example Class 4, except easier as the initial conditions are already given in the form of a Fourier series. It was also tested in QUIZ 8C.*

5. Part (a) was generally done quite well, though significant minorities did (i) assuming that the events were independent and/or used incorrect formulae for (ii) and/or (iii). *Similar problems were done in Minilecture 9A, Examples Class 5 and were tested in QUIZ 9A.* Part (b) was generally done very well, though for (ii) there were quite a lot of misinterpretations of "at least 2" and for (iii) a significant minority tried to work with their answer to (i) in an incorrect way (for example by multiplying or dividing it by 2; instead of squaring it). *Similar problems were done in Minilecture 9C, Examples Class 5 and were tested in QUIZ 9C.*

   Part (c) was generally done quite poorly, the most common errors were calculating the variance of the wall height incorrectly; not dealing with the different units given in the question correctly; calculating a probability with no working so I could not give part-marks for working for incorrect answers. *Similar problems were done in Minilecture 10C, and were tested in QUIZ 10C.*

   Most solutions for part (d) tried to use the correct formula in the right way, but there were lots of errors in using variance instead of standard deviation; and in not calculating the critical z value correctly. *Similar problems were done in Minilectures 11A, 11B and 11C, and were tested in QUIZ 11C.*