Elastic-Plastic Deformations

Lecture 1 – Introduction & Elastic-Plastic Material Behaviour Models

Department of Mechanical, Materials & Manufacturing Engineering

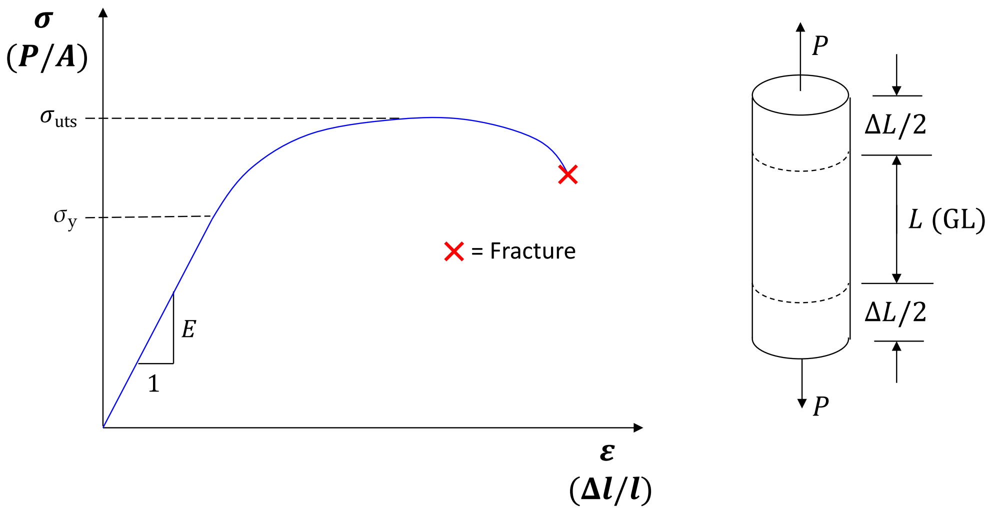
MMME2053 – Mechanics of Solids

University of Nottingham
UK | CHINA | MALAYSIA

Elastic-Plastic Deformations

Introduction

When materials are subjected to an increasing load (or stress), the strain response is often such that there is a linear (elastic) region in the stress-strain plot followed by a non-linear (plastic) region, as shown schematically below.



The ability to predict this material behaviour is extremely important, within many applications, in order to determine maximum allowable loads, that can be applied to components. These allowable loads are usually based on both the displacement this load causes as well as the remaining (residual) deformation upon unloading.

Several mathematical models can be used to estimate this material behaviour.

Elastic-Plastic Deformations

Learning Outcomes

1. Know the shapes of uniaxial stress-strain curves and the elastic-perfectly-plastic approximation (knowledge);
2. Know the kinematic and isotropic material behaviour models used to represent cyclic loading behaviour (knowledge);
3. Understand elastic-plastic bending of beams (comprehension) and be able to use equilibrium, compatibility and σ-ε behaviour to solve these types of problems for deformation and stress state (application);
4. Understand elastic-plastic torsion of shafts (comprehension) and be able to use equilibrium, compatibility and τ-γ behaviour to solve these types of problems for deformation and stress state (application);
5. Be able to determine residual deformations and residual stresses in beams under bending and shafts under torsion (application).

Elastic-Plastic Deformations

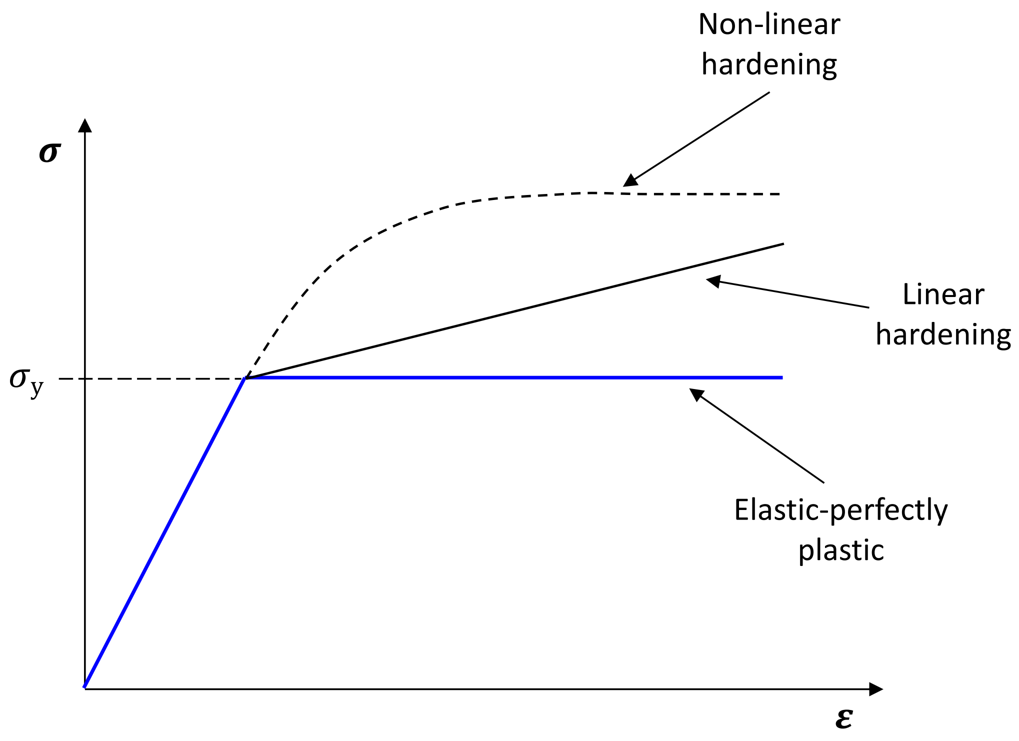
Learning Outcomes

1. Know the shapes of uniaxial stress-strain curves and the elastic-perfectly-plastic approximation (knowledge);
2. Know the kinematic and isotropic material behaviour models used to represent cyclic loading behaviour (knowledge);
3. Understand elastic-plastic bending of beams (comprehension) and be able to use equilibrium, compatibility and σ-ε behaviour to solve these types of problems for deformation and stress state (application);
4. Understand elastic-plastic torsion of shafts (comprehension) and be able to use equilibrium, compatibility and τ-γ behaviour to solve these types of problems for deformation and stress state (application);
5. Be able to determine residual deformations and residual stresses in beams under bending and shafts under torsion (application).

Elastic-Plastic Material Behaviour Models

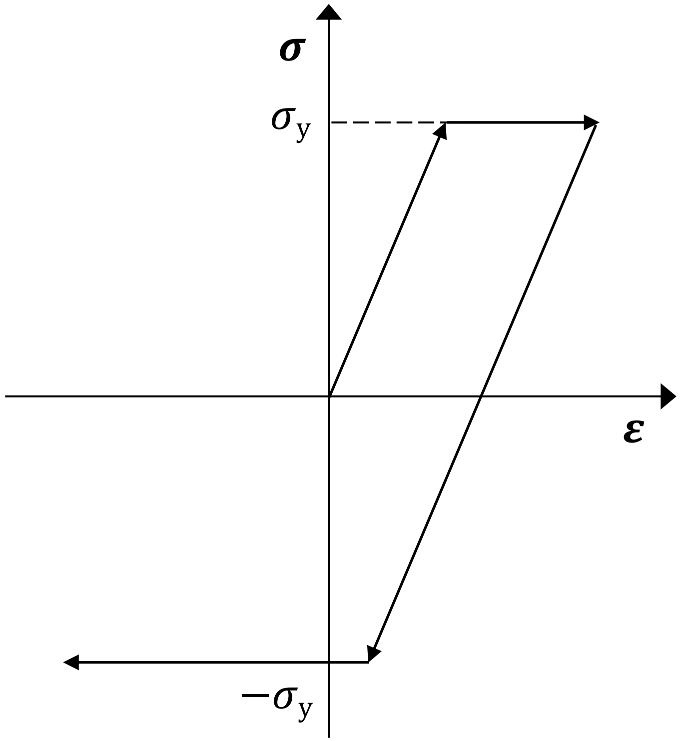
Elastic Perfectly-Plastic (EPP)

In this case, there is assumed to be no material hardening upon yield. I.e., once the yield stress, σ_y, is reached, further straining causes no further increase in stress, as shown by the blue curve below.

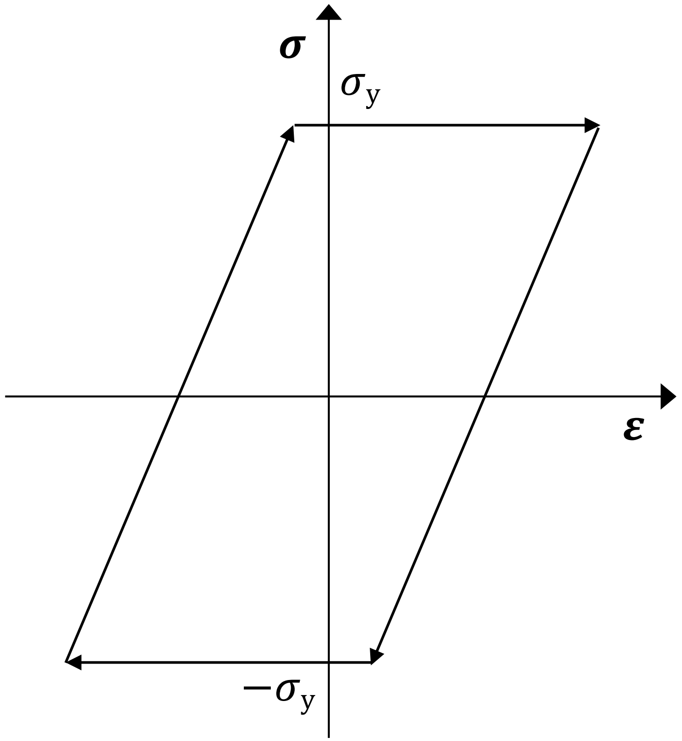


This behaviour is also applicable in compression.

I.e., if the loading is reversed, the behaviour shown below is observed, where it can be seen that the stress magnitude increases in compression until the compressive yield stress, $-\sigma_{\mathrm{y}}$, is reached, after which no further change to the stress response occurs with increasing strain magnitude.

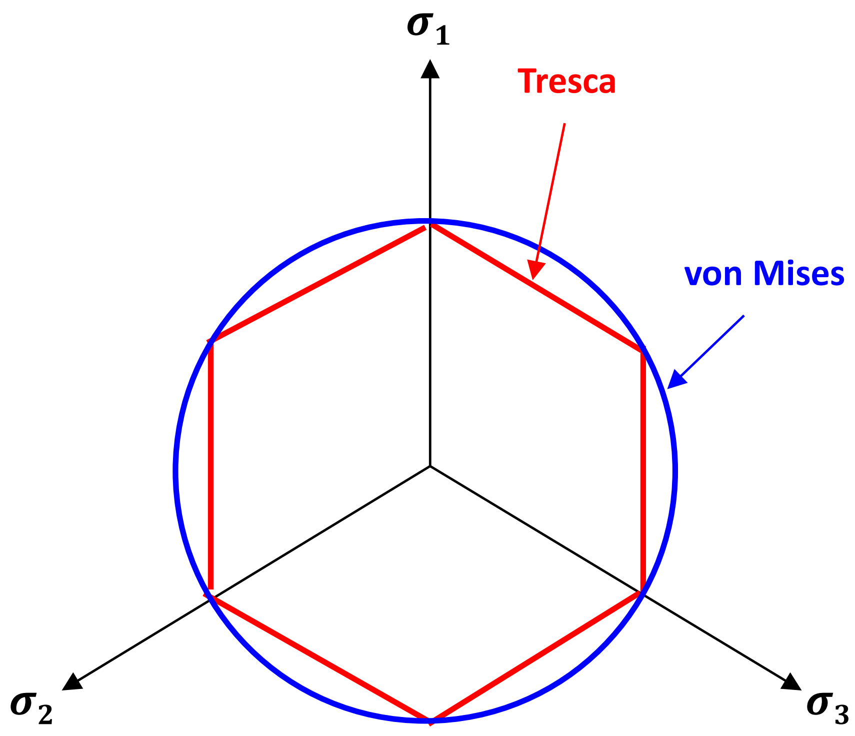


If the loading is then cycled between tension and compression, the material will continue to behave in the same way (regardless of any previous plastic deformation) resulting in the hysteresis loop shown below.



EPP, is a good material model for mild steel, for example, which demonstrates moderate plasticity.

For such material behaviour, as loading conditions cause yielding (plasticity), there is no change to the yield surface, shown below (in blue for the von Mises yield criterion and in red for the Tresca yield criterion), in the principal stress-space.



Elastic-Plastic Deformations

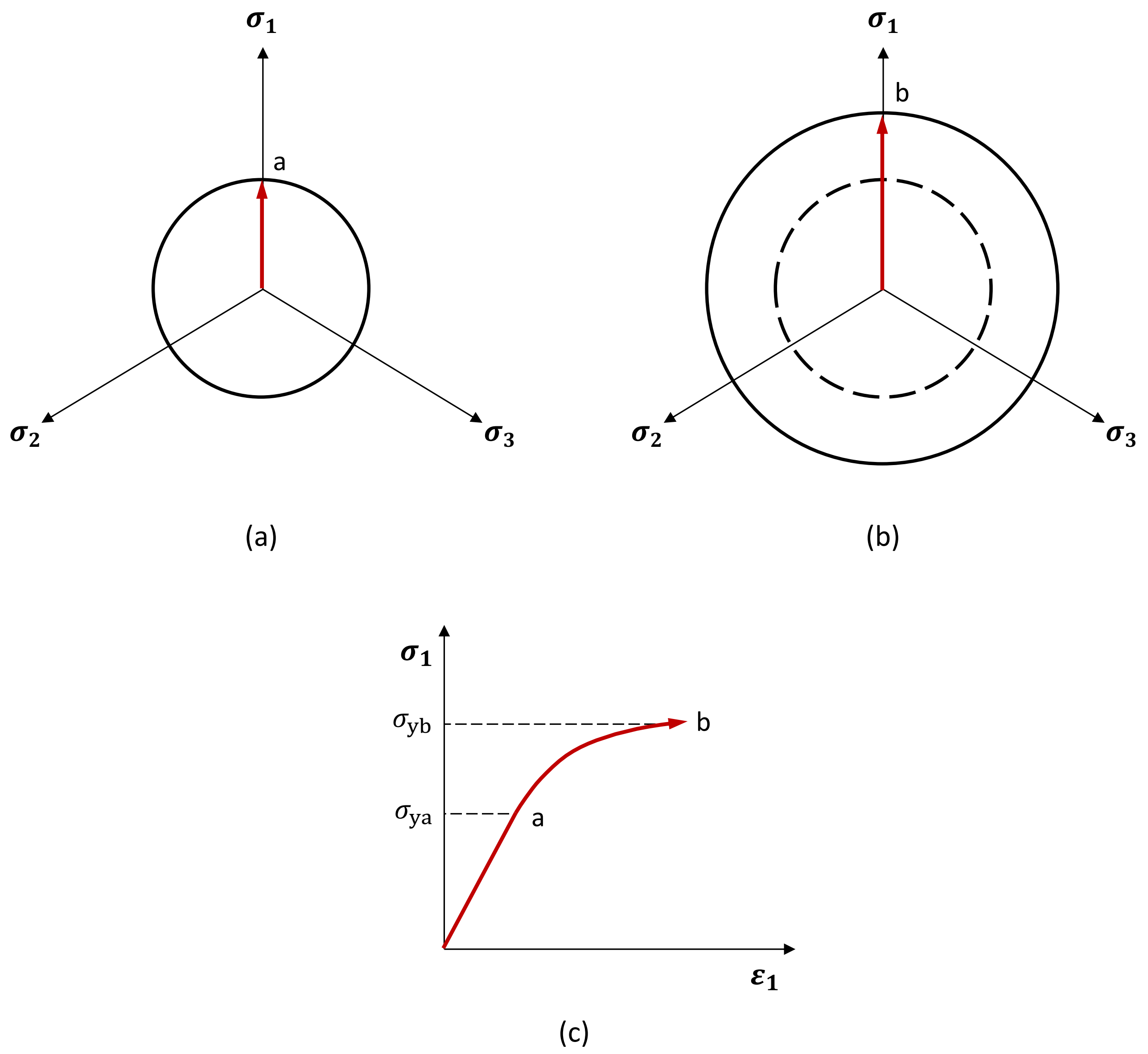
Learning Outcomes

1. Know the shapes of uniaxial stress-strain curves and the elastic-perfectly-plastic approximation (knowledge);
2. Know the kinematic and isotropic material behaviour models used to represent cyclic loading behaviour (knowledge);
3. Understand elastic-plastic bending of beams (comprehension) and be able to use equilibrium, compatibility and σ-ε behaviour to solve these types of problems for deformation and stress state (application);
4. Understand elastic-plastic torsion of shafts (comprehension) and be able to use equilibrium, compatibility and τ-γ behaviour to solve these types of problems for deformation and stress state (application);
5. Be able to determine residual deformations and residual stresses in beams under bending and shafts under torsion (application).

Isotropic Hardening

For materials which harden, as shown in tension, for both linear and non-linear cases, on slide 5, this hardening behaviour can also be observed as changes to the yield surface.

For the case of isotropic hardening behaviour, when the loading state, shown by the red arrow in part (a) of the Figure opposite, reaches the point of causing yielding (plasticity), i.e., point a, the yield surface will begin to grow. As the loading state is further increased to point b, as shown in parts (b) and (c) of the Figure, the yield surface remains centred at the same position, but its radius grows in all directions.

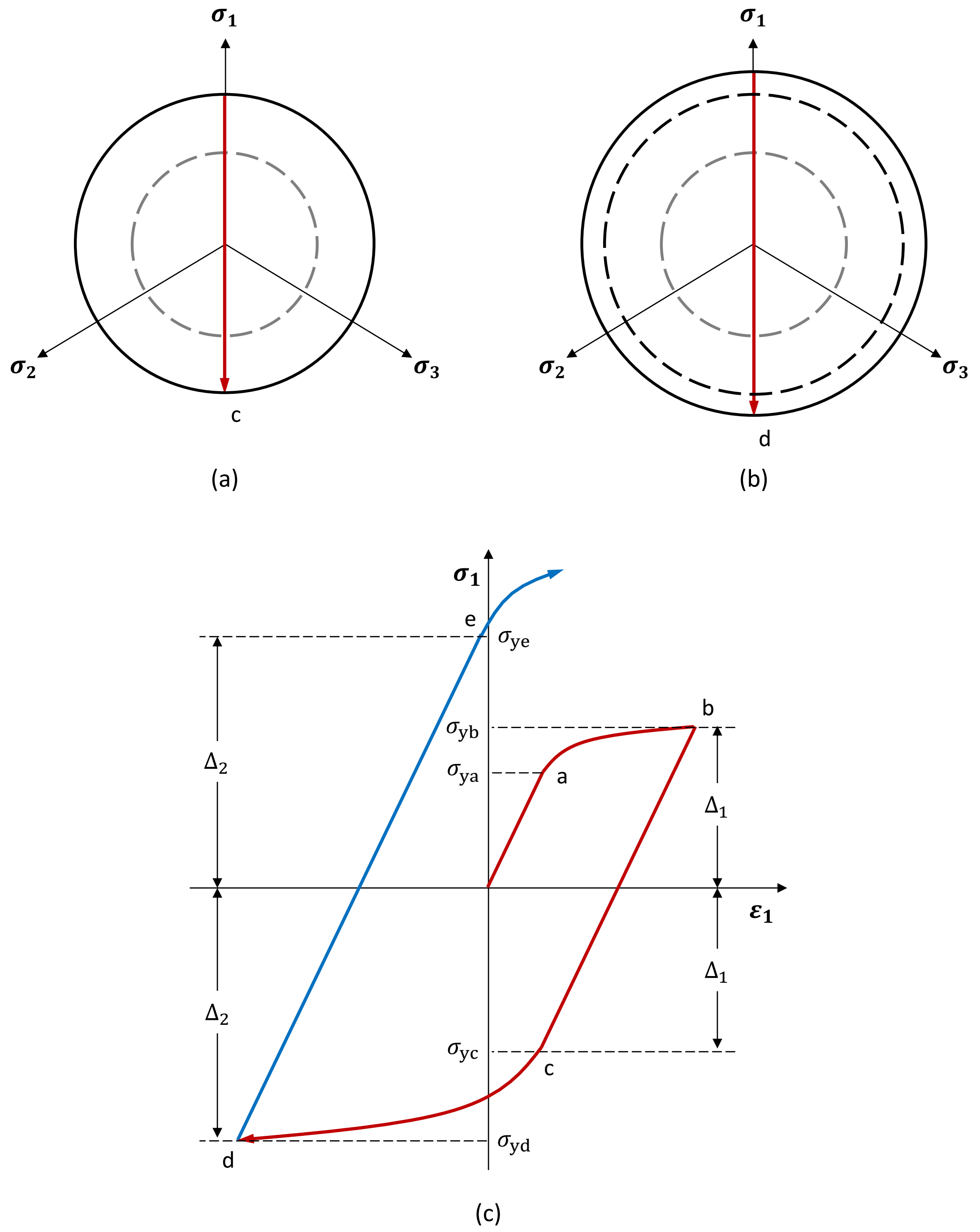


If the loading is then reversed, as shown in part (a) of the Figure opposite, further hardening does not occur until the magnitude of the reserved loading is such that the edge of the increased yield surface, point c, is reached. This can also be represented on the equivalent stress-strain curve, as shown in part (c) of the Figure.

The position of compressive yield is at a larger load magnitude than if loaded in this direction originally, due to the growth of the yield surface (isotropic hardening) during the prior tensile loading.

As the compressive load magnitude is further increased to point d, the yield surface again remains centred at the same position, but its radius grows further in all directions.

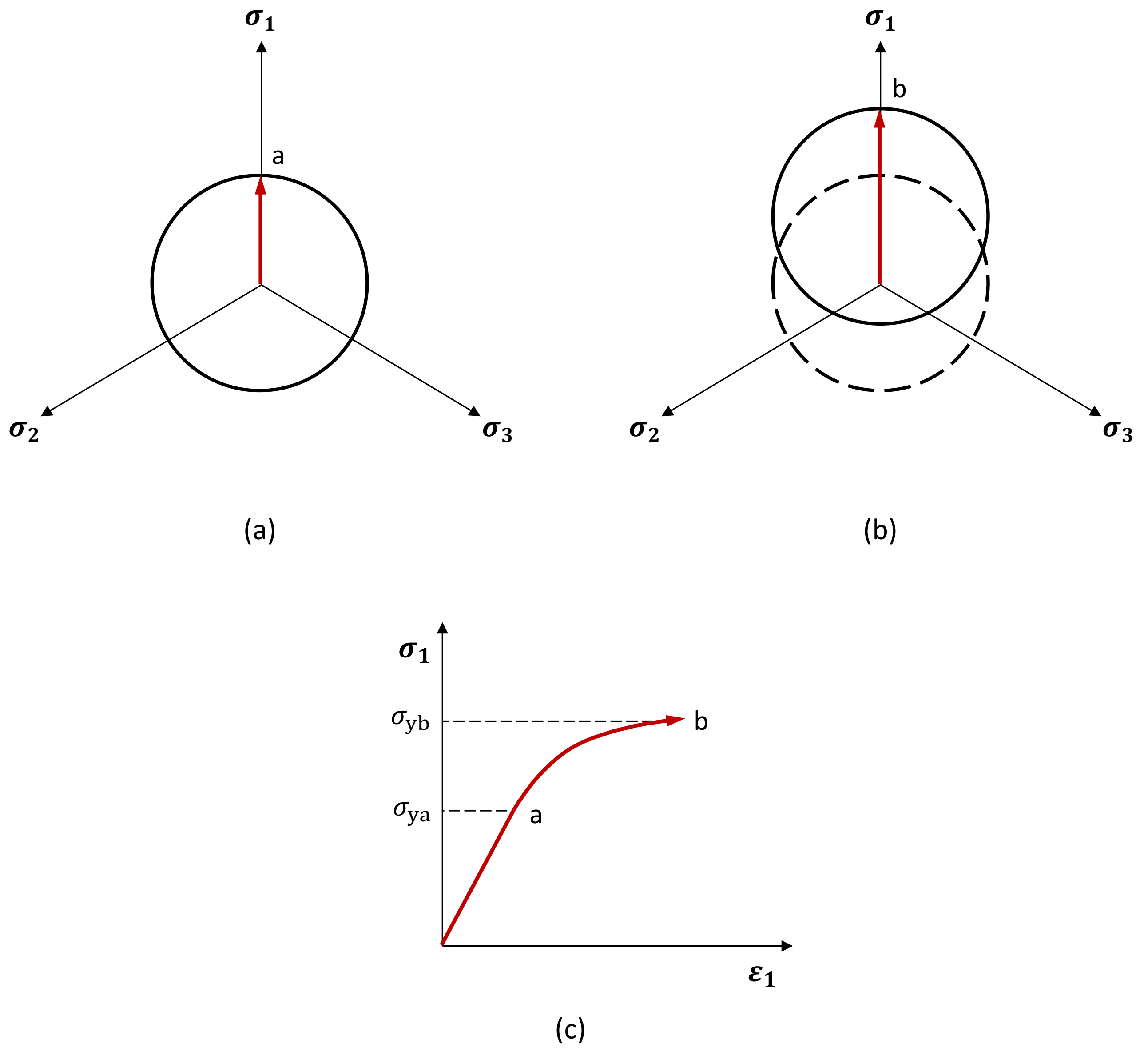
The blue loading curve shows how the material would behave under a further tensile loading and shows that again, further hardening does not occur until the magnitude of the loading is such that the edge of the increased yield surface, point e, is reached.



Kinematic Hardening

In the case of kinematic hardening behaviour, when the loading state reaches the point of causing yielding within the material, i.e., point a in part (a) of the Figure opposite, the yield surface begins moves in the direction of the loading.

As the load is further increased to point b, as shown in parts (b) and (c) of the Figure 8, the yield surface remains the same size (diameter of $2\sigma_{ya}$) but moves in the direction of the loading.

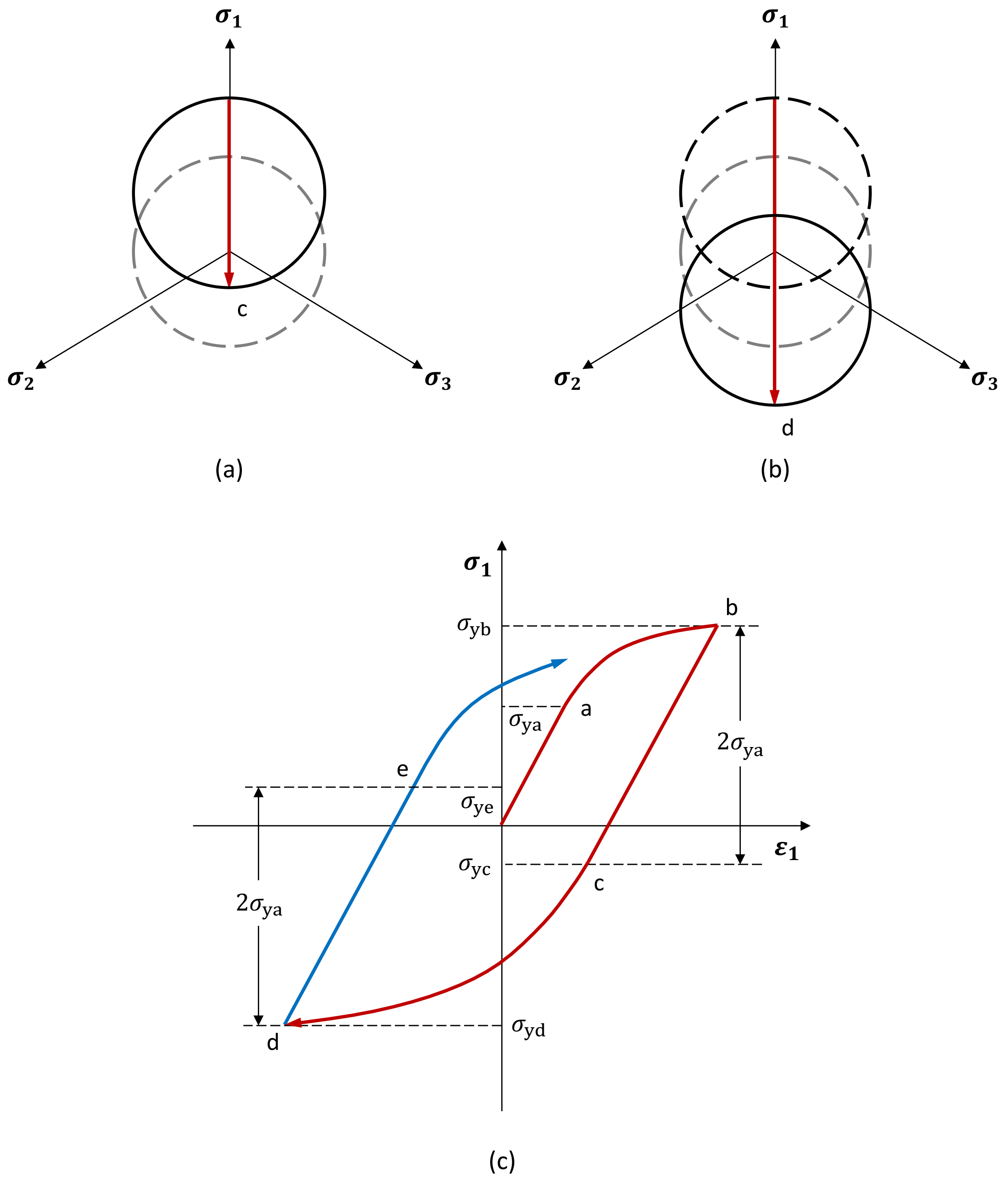


If the loading is then reversed, as shown in part (a) of the Figure opposite, further hardening will occur at position c. This can also be represented on the equivalent stress-strain curve, as shown in part (c) of the Figure.

The position of compressive yield is now at a lower load magnitude than if loaded in this direction originally, due to the movement of the yield surface (kinematic hardening) during the prior tensile loading.

As the compressive load magnitude is further increased to point d, as shown in parts (b) and (c) of the Figure 9, the yield surface remains the same size, but its centre again moves in the direction of the loading.

The blue loading curve shows how the material would behave under a further tensile loading and shows that again, further hardening occurs at a lower stress magnitude, point e, than in the previous tensile and compressive loadings, due to the movement of the yield surface (kinematic hardening) during the prior compressive loading.



Elastic-Plastic Deformations

Learning Outcomes

1. Know the shapes of uniaxial stress-strain curves and the elastic-perfectly-plastic approximation (knowledge);

2. Know the kinematic and isotropic material behaviour models used to represent cyclic loading behaviour (knowledge);

3. Understand elastic-plastic bending of beams (comprehension) and be able to use equilibrium, compatibility and σ-ε behaviour to solve these types of problems for deformation and stress state (application);

4. Understand elastic-plastic torsion of shafts (comprehension) and be able to use equilibrium, compatibility and τ-γ behaviour to solve these types of problems for deformation and stress state (application);

5. Be able to determine residual deformations and residual stresses in beams under bending and shafts under torsion (application).