University of Nottingham
Department of Mechanical Engineering

MM2MS2 - Mechanics of Solids 2
Exercise Sheet 1 - Combined Loading Solutions

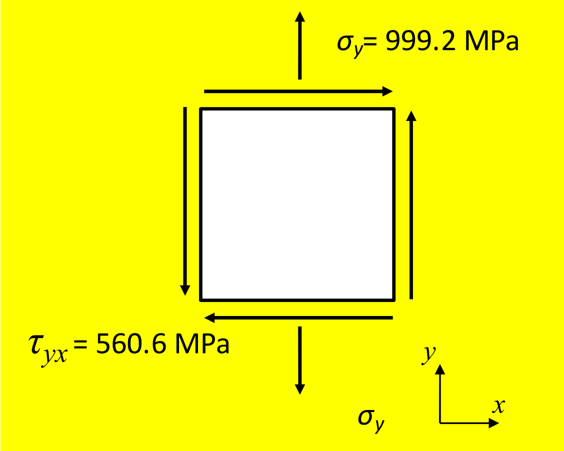
1. In an experiment involving the behaviour of a thin wire of 0.25mm diameter, a mass of 5 kg is suspended from the wire and a torque of 1.72 mNm is applied. Calculate the in-plane principal stresses and the maximum shear stress for this case.
[*Ans:* σ_1 = 1250.5 MPa, σ_2 = -251.3 MPa, τ_{max} = 751 MPa]

The applied loads cause two stresses to be acting on a plane stress element on the surface of the wire. An axial stress, which can be calculated with $\sigma_y = \frac{F}{A}$ and a torsional shear stress from the applied torque which can be determined by $\tau_{yx} = \frac{TR}{J}$ where $J = \frac{\pi D^4}{32}$.

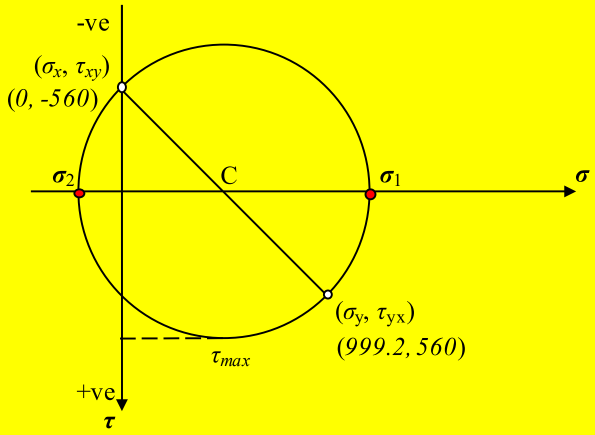
The mass leads to an axial force of $F = mg = 5 \times 9.81 = 49.05$ N, which in turn leads to an axial stress of $\sigma_y = \frac{F}{A} = \frac{F}{\pi r^2} = \frac{49.05}{\pi \times (0.125 \times 10^{-3})^2}$ = 999.2 MPa

$\tau_{yx} = \frac{TR}{J} = \frac{32 \times 1.72 \times 10^{-3} \times 0.125 \times 10^{-3}}{\pi \times (0.25 \times 10^{-3})^4}$ = 560 MPa

Therefore, the plane stress element on the surface of the wire looks like:



The Mohr's circle for this problem therefore looks like:



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So one principal stress will be positive, the other negative.

Recalling the equations for Mohr's circle, the centre is given by, $C = \frac{\sigma_x + \sigma_y}{2}$ while the radius is $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{yx}^2}$.

The in-plane principal stresses can then be determined as $\sigma_1 = C + R$ and $\sigma_2 = C - R$ while the maximum in-plane shear stress $\tau_{max} = R$.

For this case, $C = \frac{999.2}{2}$ = 499.6 MPa, $R = \sqrt{\left(\frac{-999.2}{2}\right)^2 + 560.6^2}$ = 751 MPa

Giving:

$\sigma_1 = C + R$ = ***1250.5 MPa***,

$\sigma_2 = C - R$ = ***-251.3 MPa***,

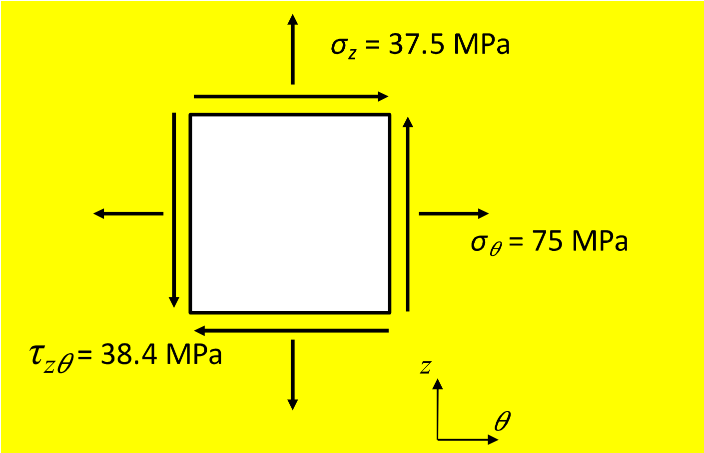
$\tau_{max} = R$ = ***751 MPa***

2. A thin-walled cylindrical tank is subjected to an internal pressure of 300 kPa and a torsional moment of 15kNm. The outer radius of the tank is 250 mm and the wall thickness is 1 mm. Calculate
i) the in-plane principal stresses and the maximum in-plane shear stress
ii) the overall maximum shear stress for the stress system
[Ans: i) σ_1 = 98.95 MPa, σ_2 = 13.55 MPa, τ_{max} = 42.7 MPa;
ii) τ_{max} = 49.48 MPa]

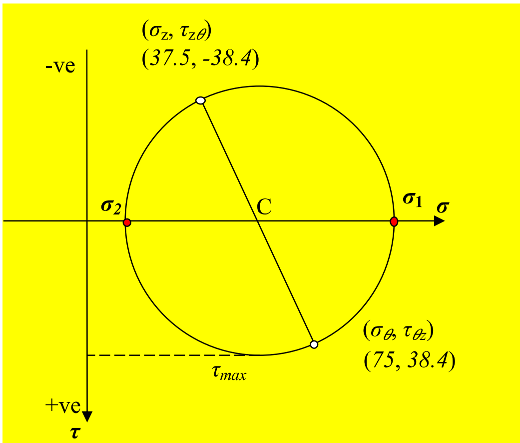
For a thin-walled cylinder, the internal pressure leads to a hoop and axial stress which can be determined using $\sigma_\theta = \frac{pR}{t}$ and $\sigma_z = \frac{pR}{2t}$ respectively, we can make the assumption that $\sigma_r = 0$. The torque results in a torsional shear stress, $\tau_{z\theta} = \frac{TR}{J}$, where $J = \frac{\pi}{32}(D_o^4 - D_i^4)$

Calculating these values, $\sigma_\theta = \frac{300\times10^3\times0.25}{1\times10^{-3}}$ = 75 x 10^6 Pa = 75 MPa, $\sigma_z = \frac{300\times10^3\times0.25}{2\times1\times10^{-3}}$ = 37.5 x 10^6 Pa = 37.5 MPa, $\tau_{yx} = \frac{TR}{J} = \frac{32\times15\times10^3\times0.25}{\pi\times(0.5^4-0.498^4)}$ = 38.4 x 10^6 Pa = 38.4 MPa

The plane stress element on the surface of the cylinder looks like:



Which gives a Mohr's circle for this plane of:



(not to scale)

Allowing us to calculate the values of the in-plane principal stresses and maximum shear stress.

For this case, $C = \frac{75+3\ .5}{2}$ = 56.25 MPa,

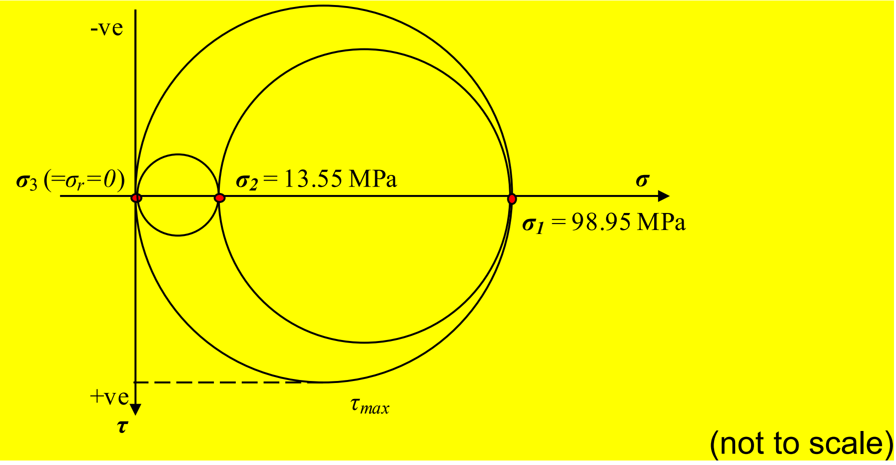
$R = \sqrt{\left(\frac{75-37.5}{2}\right)^2 + 38.4^2} = \sqrt{18.75^2 + 38.4^2}$ = 42.7 MPa

Giving the results for the in-plane values as:

$\sigma_1 = C + R$ = ***98.95 MPa***,

$\sigma_2 = C - R =$ ***13.55 MPa***,
$\tau_{max} = R =$ ***42.7 MPa***

To determine the overall maximum shear stress for the stress system, it is important to consider the third principal stress, σ_3, which in this case is $\sigma_r = 0$. We can then draw the Mohr's circle including all of the three planes as below:



(not to scale)

And the maximum shear stress can be calculated by:

$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{98.95 - 0}{2}$ = ***49.48 MPa***

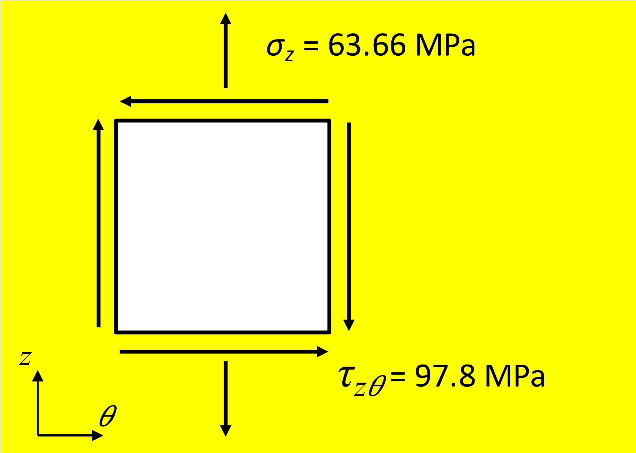
3. A helicopter rotor shaft, 50mm in diameter, transmits a torque of 2.4 kNm and an upward tensile lifting force of 125 kN. Determine the maximum tensile stress, maximum compressive stress and maximum shear stress in the shaft.
[Ans: σ_1 = 134.6 MPa, σ_2 = -71 MPa, τ_{max} = 102.8 MPa]

The applied loads cause two stresses to be acting on a plane stress element on the surface of the wire. An axial stress, which can be calculated with $\sigma_y = \frac{F}{A}$ and a torsional shear stress from the applied torque which can be determined by $\tau_{yx} = \frac{TR}{J}$ where $J = \frac{\pi D^4}{32}$.

The axial force of 125 kN leads to an axial stress of $\sigma_y = \frac{F}{A} = \frac{F}{\pi r^2} = \frac{125000}{\pi \times (50 \times 10^{-3})^2}$ = 63.66 MPa

$\tau_{yx} = \frac{TR}{J} = \frac{32 \times 2400 \times 25 \times 10^{-3}}{\pi \times (50 \times 10^{-3})^4}$ = 97.8 MPa

This gives us the following stress-state on a plane stress element on the shaft surface:



For this case, $C = \frac{0+63.66}{2} =$ 31.8 MPa,

$R = \sqrt{\left(\frac{0-63.66}{2}\right)^2 + 97.8^2} = \sqrt{-31.8^2 + 97.8^2}$ = 102.8 MPa

Giving the results for the in-plane values as:

$\sigma_1 = C + R =$ **134.6 MPa**,

$\sigma_2 = C - R =$ **-71 MPa**,

$\tau_{max} = R =$ **102.8 MPa**

4. A generator shaft of hollow circular cross-section is subjected to a torque of 25 kNm and a compressive load of 900 kN. The outer and inner diameters of the shaft are 200 mm and 160 mm respectively. Determine the in-plane principal stresses and maximum shear stress.
[Ans: σ_1 = 8.3 MPa, σ_2 = -87.9 MPa, τ_{max} = 48.1 MPa]

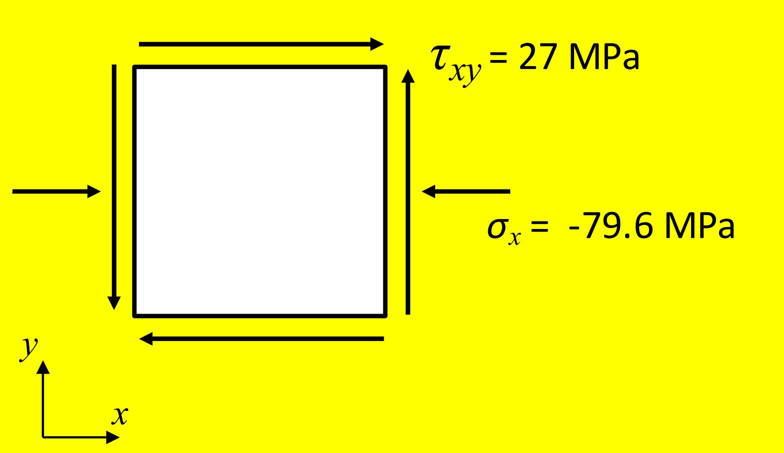
The applied loads cause two stresses to be acting on a plane stress element on the surface of the wire. An axial stress, which can be calculated with $\sigma_y = \frac{F}{A}$ and a torsional shear stress from the applied torque which can be determined by $\tau_{yx} = \frac{TR}{J}$ where $J = \frac{\pi}{32}(D_o^4 - D_i^4)$

The axial force of -900 kN leads to an axial stress of $\sigma_y = \frac{F}{A} = \frac{F}{\pi(r_o^2 - r_i^2)} = \frac{-900000}{\pi\times(0.1^2-0.08^2)}$ = -79.6 MPa

$\tau_{yx} = \frac{TR}{J} = \frac{32\times25000\times0.1}{\pi\times(0.2^4-0.16^4)}$ = 27 MPa

MM2MS2 - Mechanics of Solids 2
Exercise Sheet 1 - Combined Loading Solutions

This gives us the following stress-state on a plane stress element on the shaft surface:



For this case, $C = \frac{-79.6}{2}$ = -39.8 MPa,

$R = \sqrt{\left(\frac{-79.6}{2}\right)^2 + 27^2} = \sqrt{-39.8^2 + 27^2}$ = 48.1 MPa

Giving the results for the in-plane values as:

$\sigma_1 = C + R$ = ***8.3 MPa***,

$\sigma_2 = C - R$ = ***-87.9 MPa***,

$\tau_{max} = R$ = ***48.1 MPa***

5. For the purpose of analysis, a segment of a crankshaft in a vehicle is presented as shown in Figure Q5. The load P = 1 kN, and the dimensions are b1 = 80 mm, b2 = 120 mm and b3 = 40mm. The diameter of the shaft is d = 20 mm. Determine the maximum tensile, compressive and shear stresses at point A, located on the surface of the shaft at the z-axis.
   [Ans: σ_1 = 31.6 MPa, σ_2 = -184.6 MPa, τ_{max} = 108.1 MPa]

MM2MS2 - Mechanics of Solids 2
Exercise Sheet 1 - Combined Loading Solutions

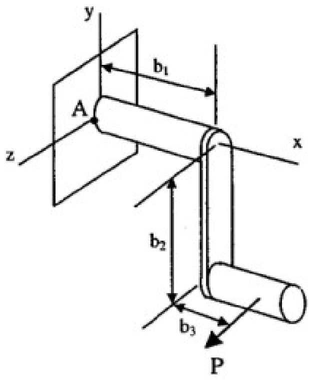


Figure Q5

The load P causes bending and torsion at point A.

The bending moment M can be calculated by:

$M = P(b_1 + b_3) = 1 \times (0.12)$ = 0.12 kNm = 120 Nm
The torque can be calculated by:
$T = Pb_2 = 1 \times 0.12$ = 0.12 kNm = 120 Nm

The bending moment causes a compressive stress at A which can be calculated using $\sigma = \frac{My}{I}$ where $y = \frac{d}{2}$ and I is given by $I = \frac{\pi D^4}{64}$ so:

$\sigma = \frac{64My}{\pi D^4} = \frac{64*120*0.01}{\pi \times 0.02^4}$ = 153 x 10^6 Pa = 153 MPa

The torque causes a shear stress at A which can be calculated using $\tau = \frac{Tr}{J}$ where $r = \frac{d}{2}$ and J is given by $J = \frac{\pi D^4}{32}$ so:

$\tau_{yx} = \frac{TR}{J} = \frac{32 \times 120 \times 0.01}{\pi \times 0.02^4}$ = 76.4 x 10^6 Pa = 76.4 MPa

Giving a stress state of:

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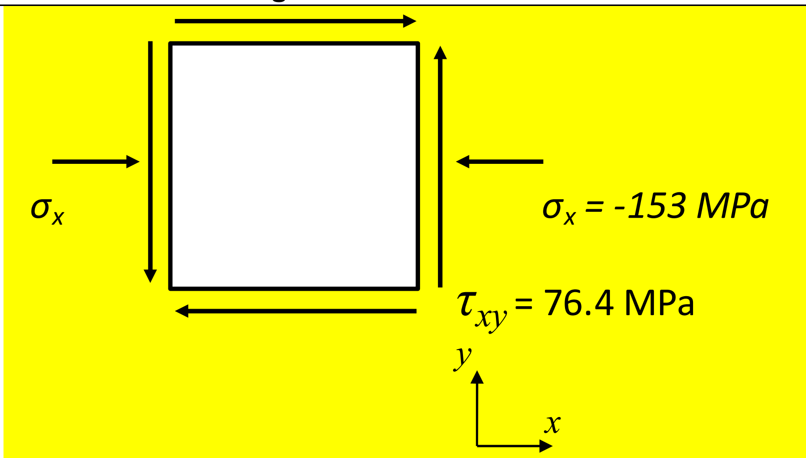
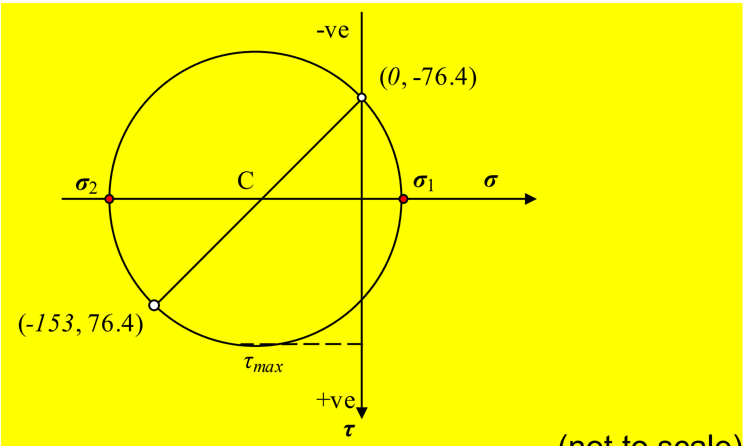
MM2MS2 - Mechanics of Solids 2
Exercise Sheet 1 - Combined Loading Solutions



Which leads to a Mohr's circle:



For this case, $C = \frac{-153}{2}$ = -76.5 MPa,

$R = \sqrt{\left(\frac{-153}{2}\right)^2 + 76.4^2} = \sqrt{-76.5^2 + 76.4^2}$ = 108.1 MPa

Giving the results for the in-plane values as:

$\sigma_1 = C + R =$ ***31.6 MPa***,

$\sigma_2 = C - R =$ ***-184.6 MPa***,

$\tau_{max} = R =$ ***108.1 MPa***