Asymmetrical Bending

Lecture 1 – Principal Axes & 2^{nd} Moments of Area

Department of Mechanical, Materials & Manufacturing Engineering

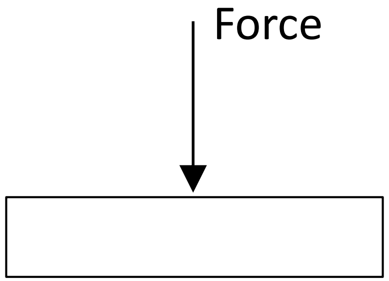
MMME2053 – Mechanics of Solids

University of Nottingham
UK | CHINA | MALAYSIA

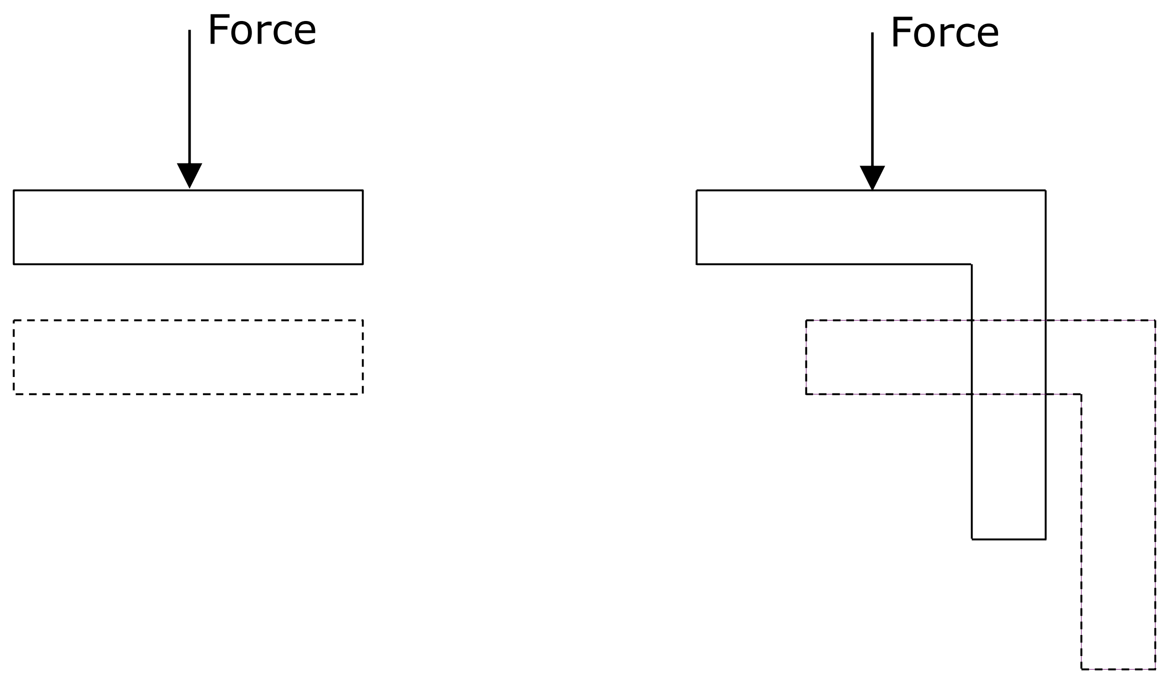
Asymmetrical Bending

Introduction

The beam bending equation, $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$, has been derived and is generally used to determine stresses in a beam with a **symmetrical** cross-section. The symmetry is usually about an axis perpendicular to the neutral axis of the section.



For a section where this symmetry does not apply, i.e. **asymmetric** sections, a complication arises, making bending analysis more difficult. In these cases, applying a bending moment will, in general, result not only in bending about the axis of the loading but also in simultaneous bending about the perpendicular axis i.e. there is an interaction effect.



Asymmetrical Bending

Learning Outcomes

1. Know that an asymmetric cross-section, in addition to its 2^{nd} moments of area about the x and y axes, I_x and I_y, possesses a geometric quantity called the Product Moment of Area, I_{xy}, with respect to these axes (knowledge);
2. Be able to calculate the 2^{nd} moments of area and the product moment of area about a convenient set of axes (application);
3. Know that an asymmetric section will have a set of axes, at some orientation, for which the product moment of area is zero, and that these axes are called the Principal Axes (knowledge);
4. Know that the 2^{nd} moments of area about the principal axes are called the principal 2^{nd} moments of area (knowledge);
5. Be able to determine the 2^{nd} moments of area and the product moment of area about any oriented set of axes, including the principal axes, using a Mohr's circle construction (application);
6. Understand that it is convenient to analyse the bending of a beam with an asymmetric section by resolving bending moments onto the principal axes of the section (knowledge);
7. Be able to follow a basic procedure for analysing the bending of a beam with an asymmetric cross-section (application).

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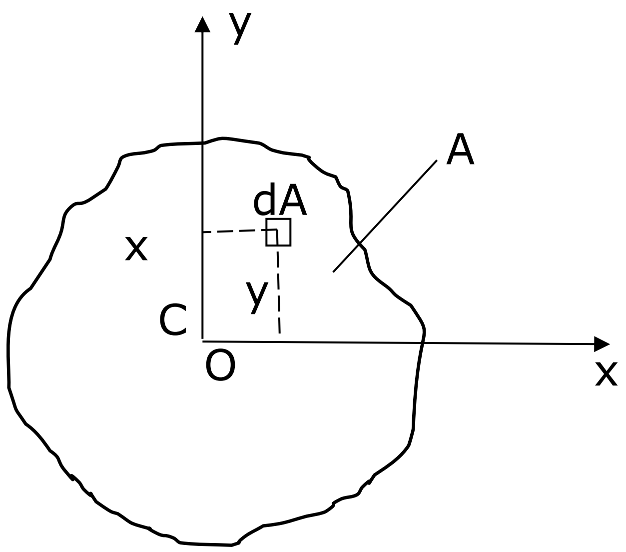
2^{nd} Moments of Area & Product Moment of Area

To analyse asymmetrically loaded sections, in addition to the 2^{nd} Moments of Area, I_x & I_y, we introduce a new geometric quantity called the Product Moment of Area, I_{xy}, as follows:



$$I_x = \int_A y^2 dA \qquad I_y = \int_A x^2 dA \qquad I_{xy} = \int_A xy dA$$

The summation of the elements of area, dA, multiplied by its perpendicular co-ordinate squared.

The summation of the elements of area, dA, multiplied by the product of their co-ordinates.

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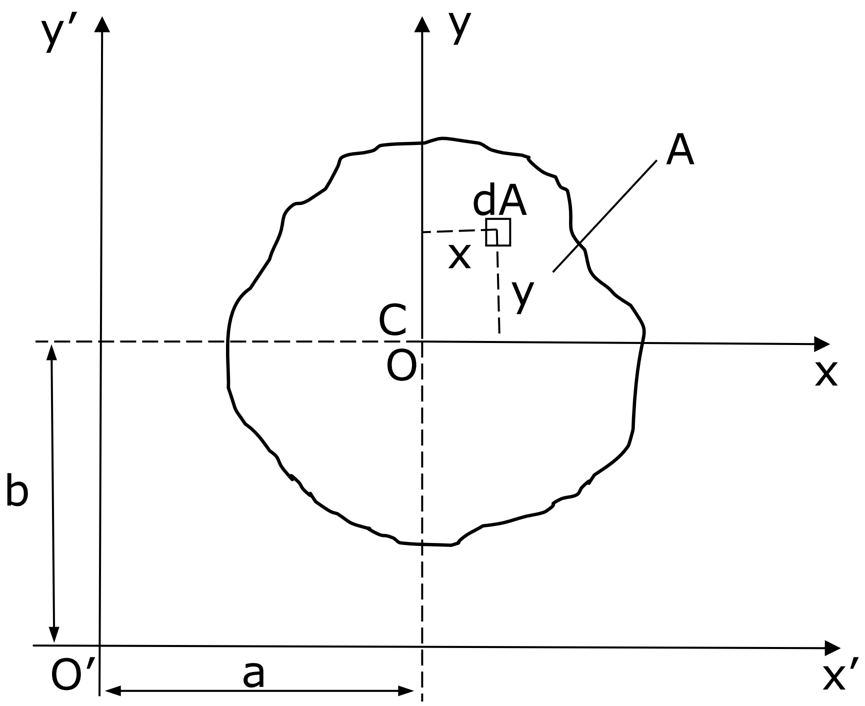
Parallel Axis Theorem

The Parallel Axis Theorem allows the calculation of the 2nd Moments of Area, $I_{x'}$ and $I_{y'}$ and the Product Moment of Area, $I_{x'y'}$ with respect to the x' and y' axes. For example, for $I_{x'}$:



$$I_{x'} = \int_A y'^2 dA$$

$$= \int_A (y+b)^2 dA$$

$$= \int_A y^2 dA + b^2 \int_A dA + \int_A 2by dA$$

$$\therefore I_{x'} = I_x + Ab^2$$

Similarly for $I_{y'}$ and $I_{x'y'}$:

$$I_{y'} = I_y + Aa^2$$

$$I_{x'y'} = I_{xy} + abA$$

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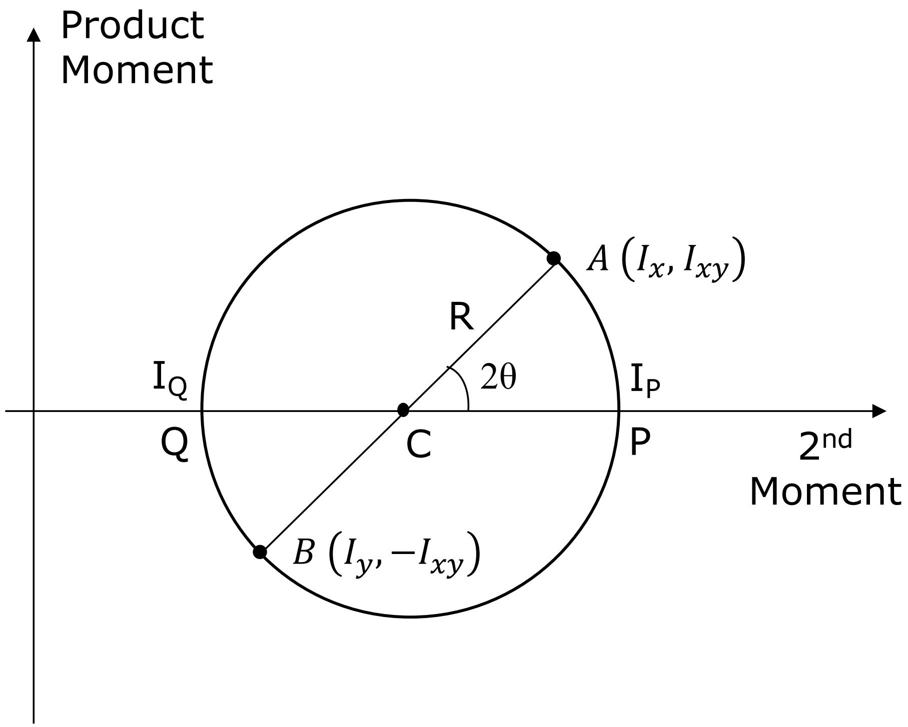
Principal Axes and Principal 2nd Moments of Area

Once values of I_x , I_y and I_{xy} (or $I_{x'}$, $I_{y'}$ and $I_{x'y'}$, if required) are known. These values can be used to plot a Mohr's circle.



Point A is plotted using the first 2nd Moment, I_x and the Product Moment, I_{xy}.

Point B is plotted using the second 2nd Moment, I_y and the negative Product Moment, $-I_{xy}$.

2nd moments are plotted on the x-axis and the product moments are plotted on the y-axis.

Points P and Q show the positions of the principal 2nd Moments of Area, I_p and I_q.

θ is the angular position e of the principal axes with respect to the x-y axes.

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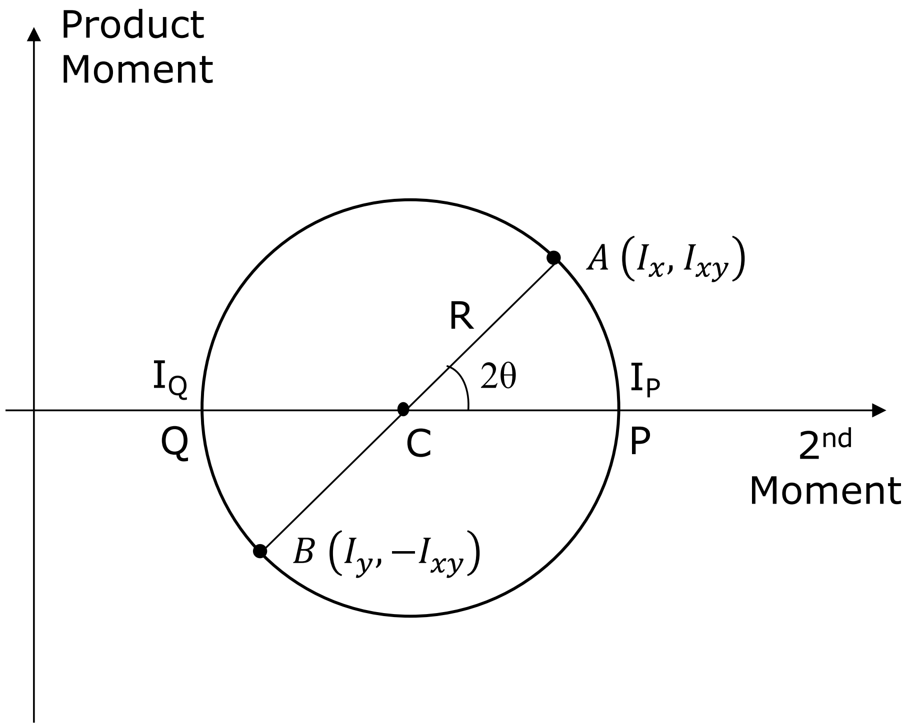

$$Centre, C = \frac{I_x + I_y}{2}$$

$$Radius, R = \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + {I_{xy}}^2}$$

The points P and Q on the circle correspond to the **Principal Planes** for which the Product Moment of Area is zero and the 2nd Moments of Area are the **Principal 2nd Moments of Area**, I_p and I_q.

$$I_p = C + R \qquad\qquad I_q = C - R$$

$$sin2\theta = \frac{I_{xy}}{R}$$

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