

MTHS2007 **Advanced Mathematics for Mechanical Engineers: exam feedback 2020-21**

1. This question was on the whole done well. Most students got full or nearly full marks for part (a) and did well also in part (b). A common problem here was mixing up the independent variable, for example writing functions of  $t$  instead of  $x$  in part (a). Sometimes this is just a slip of the pen (and something that lecturers are also known to do!), so wasn't penalised hard, but for some students it was done more seriously, leading in part (b) for example to differentiating with respect to the wrong variable when eliminating one of the dependent variables ( $y$  or  $z$ ). In the latter case it meant that subsequent calculations such as the auxiliary equation were completely different and lead to more significant losses of marks.
2. Part (a) was usually done well enough to get at least a couple of marks, but common errors included plotting the function as if it was  $1 + x$  (i.e. shifting it upwards) or constant in the interval  $-1 < x < 1$  or incorrectly extending it using the periodicity property.

Most students went wrong somewhere along the way in part (b) but did enough correctly to get most of the marks, once initial errors were accounted for (with "marks carried forward"). Common errors included incorrectly identifying or using the half period and integrating  $x$  between  $-2$  and  $2$  rather than  $-1$  and  $1$ . A lot of students failed to use symmetry properties correctly, for example deciding the function was even or neither odd nor even. Full marks were available even if the function's odd-ness was not spotted or used, but this led to longer calculations and in practice students taking this route more commonly got other things wrong. *Note that except for the half period and limits on the eventual integral, this was very similar to an example done in Minilecture 3E. An example of a function that is even more similar (except that it vanishes over a different part of the cycle) was also covered in detail in Examples Class 2.*

Part (c) was just a short test of being able to distinguish between the value obtained by a Fourier series at points where the function was discontinuous rather than continuous, but a lot of students didn't attempt it or tried to do complicated calculations involving the expressions found for Fourier coefficients. *Note that this topic was covered in detail in Minilecture 4B. Problems like this were also done in QUIZ 4B.*

Part (d) was often not attempted or had only an initial couple of lines of solution. Students who got as far as writing equations for Fourier coefficients of the solution generally got most of the marks available, with "marks carried forward". *Note that a very similar first-order ODE problem was solved in Minilecture 4D.*

3. In part (a), the sketch was usually done right, and most students correctly identified that the change in form of the function involved  $H(t - 1)$  in some way, but it was common to get the detailed expression wrong. *Note that similar examples were given in Minilecture 6A and in Examples Class 3. Problems like it were also done in QUIZ 6A.*

A lot of students did not attempt part (b) but those that did usually got at least a couple of marks. *Note that similar calculations were done as part of the worked examples of in Minilectures 6C and 6D, and in Examples Class 3.*

Most of the marks were available for part (c), even for students who did not do parts (a) and (b). Some students lost a small amount of credit for not stating clearly how the Laplace transform of the RHS entered in the solution.

A common mistake in part (d) was to omit the  $\frac{\text{const}}{s}$  term in the partial fraction expansion, and to mishandle the parts involving  $e^{-s}$  and the Second shifting theorem. *Note that similar worked examples involving discontinuous RHSs (and Heaviside step functions) were given in Minilectures 6C and 6D, and in Examples Class 3. They were also the topic of QUIZ 6C.*

4. Most students who attempted part (a) did a good job of justifying the given ODEs for the functions of separate variables, but a significant minority did not address the relationship between  $\lambda$  and  $\lambda'$ . *Problems with similar relabelling of constants came up in QUIZ 7D and Example Class 4 and it was otherwise similar to the calculations in minilecture 8B and the worked example in Problem Class 4.*

Most responses for part (b) followed the suggested pattern of checking positive, negative and zero values for  $\lambda$  and clearly understood the general approach to answering this question. But there were some minor errors that came up fairly frequently. Specifically, (i) lots of responses used boundary conditions different from those given in the exam paper, (ii) lots of responses did not correctly or completely solve  $\cos(mL) = 0$  to get  $kL = n\pi/2$  for  $n$  an odd integer. *Note that an identical ODE and boundary conditions were solved in Minilecture 8C, albeit the in the context of the Laplace rather than the heat equation.*

Errors in part (c) were even more common: lots of responses treated the first-order ODE  $T' = -\lambda'T$  as though it were second-order  $T'' = -\lambda'T$ , and also many responses did not sum the many independent solutions found in (b) to give the correct general solution. *Except for having a different label for the constant, this was the same as calculations in Minilecture 8C and in Problem Class 4. Even if incorrect expressions had previously been found for  $\lambda'$ , partial credit was available for solving the rest of the problem in a similar manner.*

5. (a) There were lots of errors that arose from using the conditional probability formula incorrectly, e.g.  $P(A|B) = P(A \cap B)/P(A)$  instead of  $P(B)$  being in the denominator. Also quite a few errors seemed to arise from not reading the question carefully, for example not stating which of  $A$  or  $C$  was more likely for (ii) and in (iii) lots of responses checked independence of  $A$  and  $B$ , not  $A$  and  $C$  as was asked.

(b) Most responses to (i) were good, but a significant minority did not correctly interpret the language of the question in terms of probabilities and conditional probabilities.

Like (a)(ii) and (iii) above, many of the errors in (ii) seemed to arise from not carefully reading the question. For example, many used the failure probability from part (i), not the probability for DMC as the question asked. Similarly, a significant minority of responses gave the probability that more than two capacitors fail, not no more than two as the question asks.

(c) Part (i) was generally done well, though a significant minority didn't use the formula for  $s^2$  correctly. A significant number of students also went on to use the value of  $s^2$  as if it were  $s$  in later parts - forgetting to take the square root.

Many responses to (ii) found the probability of a component meeting specification but did not then give the probability of it not meeting specification. Another important common error in this question was not giving any details of working (presumably as a result of using a calculator). If the answer given was not correct then I could not award many marks for working.

For (iii) the latter comment for (ii) above also applies - lots of responses gave very little working or justification for their answers. Also the comments made at the end included many incorrect interpretations of the CI and also many responses which did not link back to part (ii) as the question asked.