University of Nottingham

School of Mathematical Sciences

MTHS2007 Advanced Mathematics and Statistics for Mechanical Engineers

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Ordinary Differential Equations — Problem Sheet 1

1. Find the general solution of each of the differential equations given below.

   (a) $y'' - y = 0$ (b) $y'' + y = 0$

   (c) $y'' + 8y' + 15y = 0$ (d) $y'' + 6y' + 13y = 0$

   (e) $y'' - 6y' + 9y = 0$

2. Find the general solution of each of the differential equations given below.

   (a) $y'' + 4y' - 12y = 14\,e^x$ (b) $y'' + 4y' + 3y = 8\cos x - 6\sin x$

   (c) $y'' - 9y = 42\,e^{3x}$

3. Solve the following differential equations subject to the given boundary conditions.

   (a) $y'' + 4y' - 12y = 14\,e^x$, subject to $y(0) = 1, \quad y'(0) = -4$.

   (b) $y'' + 4y' - 12y = 3$, subject to $y(0) = 1, \quad y \to$ constant as $x \to \infty$.

4. Solve the system of equations given below for $y(t)$ and $z(t)$.

   $$y' + y - z = e^t, \quad -y + z' + z = e^t.$$

5. (a) Find the general solution of the ordinary differential equation

   $$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 5x + 3.$$

   (b) Find the solution of the ordinary differential equation

   $$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = 32e^t + 9,$$

   which satisfies the intial conditions $y(0) = 0$ $y'(0) = 1$.

6. The functions $x(t)$ and $y(t)$ satisfy the system of ordinary differential euqations

   $$\frac{dx}{dt} + x + y = 0, \quad \frac{dy}{dt} + 3x - y = \sin t.$$

   By eliminating $\dfrac{dy}{dt}$ and then y show that $x(t)$ satisfies the ordinary differential equation

   $$\frac{d^2x}{dt^2} - 4x = -\sin t.$$

   Hence find the general solutions for $x(t)$ and $y(t)$.

7. Find the general solution of each of the differential equations given below.

   (a) $y'' + y' - 2y = xe^{2x}$

   (b) $y'' + y' - 2y = x^2e^{2x}$