University of Nottingham
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LECTURE 2B

Alternating Current

Electromechanical Devices
MMME2051

Module Convenor – Surojit Sen

Learning Outcomes

- Fundamentals of **Alternating Current – or AC**
  - DC v AC circuit study – waveforms a **function of time**!
  - **Sinusoidal** waveform – voltage & current
  - **Complex Numbers**
- AC circuits
  - **Phasor** study – simple way to solve time-varying circuits
  - Resistor, Inductor, Capacitor in phasor form - **CIVIL**
  - **Reactance** – Purely reactive circuits (just inductor/capacitor)
  - **Impedance** – Resistance & Reactance
- Power in AC circuits
  - **Active** v **Reactive** v **Apparent** Power
  - **Power Factor**
  - **Resonance**

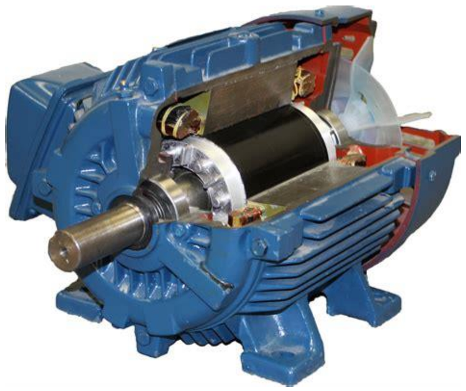
DC v AC

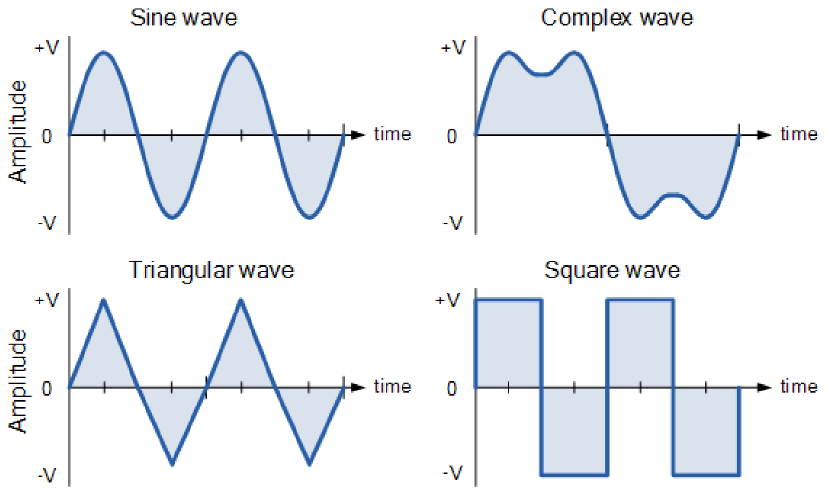
Direct Current
Current flowing in only one direction

e.g., battery

Alternating Current
Direction of current flow changes periodically

e.g., generator





Abbreviations AC and DC are often used to mean simply alternating and direct, i.e., reference to just current dropped

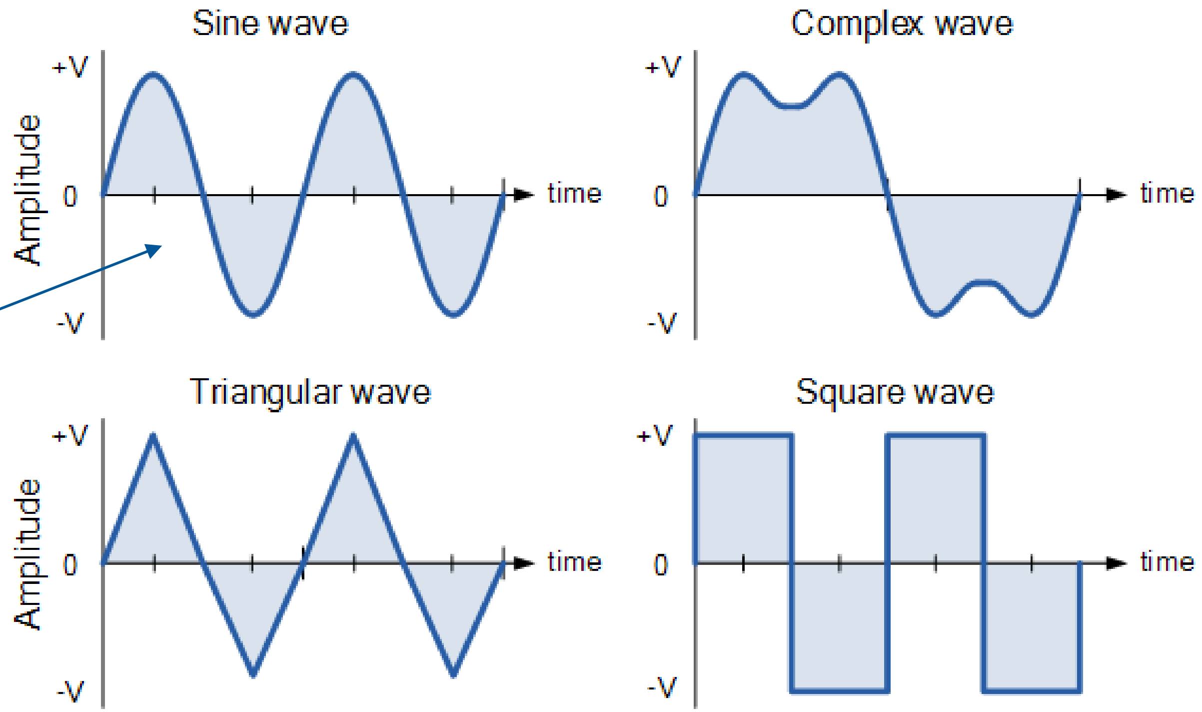
e.g., AC voltage, DC current etc.

Waveform

Representation of any **physical variable** as a **function of time** on a **graph**
(We would discuss only electrical variables like voltage and current)

Magnitude (y-axis) and **time** (x-axis)

Sine Wave (or sinusoid) is the most interesting – we would be studying this

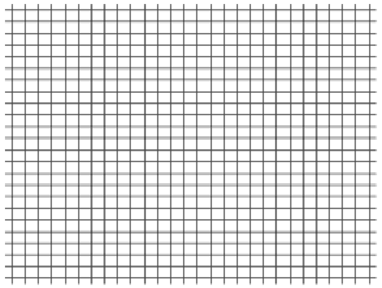


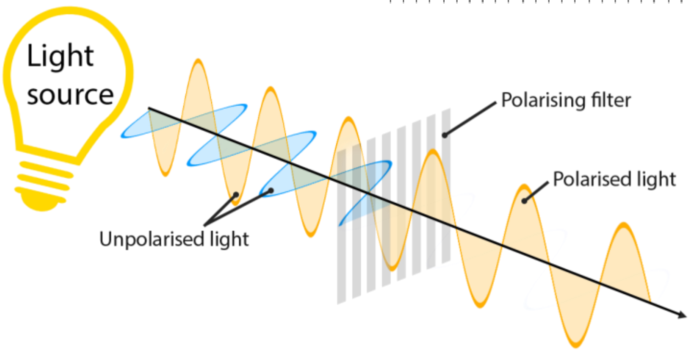
Other waveforms like **triangular**, **sawtooth**, and **square** are abundantly used in electrical engineering – they can all be represented as a sum of infinite number of sinusoids (check out **Fourier Series**!)

Waveform

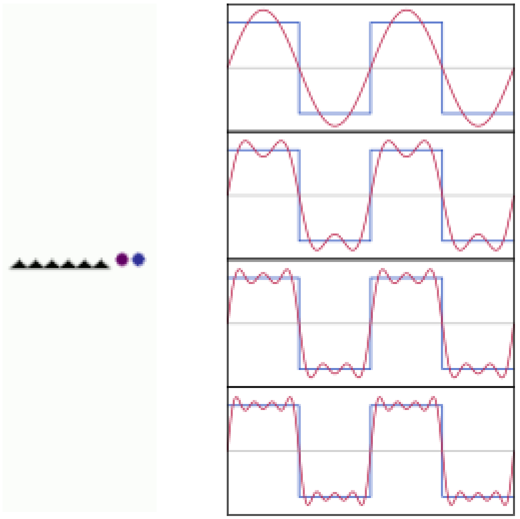
Why is Sine wave interesting?

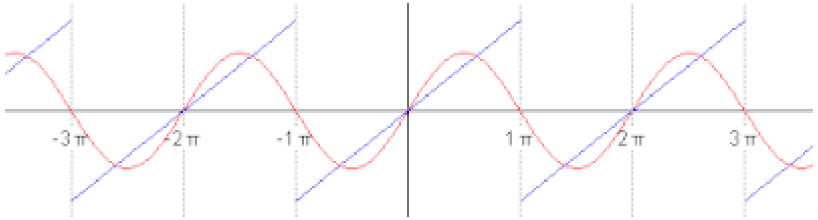
- Occurs in **nature**
- **Wind**, **sound** and **light** waves are sinusoidal



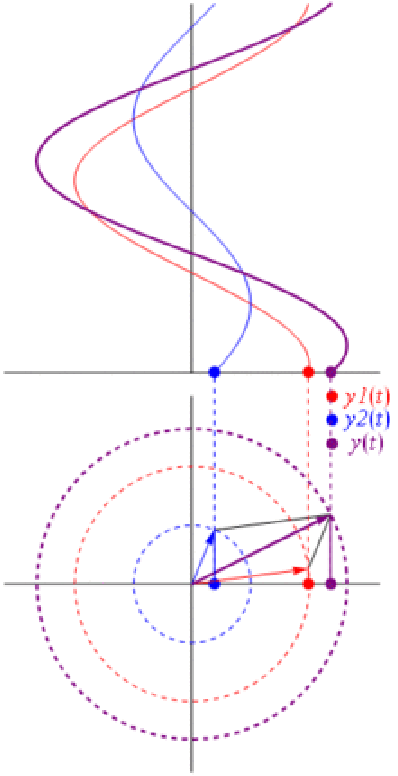


Fourier Series – Every waveform is made up of sinusoids





Motors & Generators translate rotation and voltage – **projection of a rotating object is a sinusoid**!



Sinusoid

Sinusoid is a mathematical curve defined in terms of the **sine trigonometric function**

Sine and **Cosine** are both examples of sinusoid

Cosine function is simply the Sine function, but **90° advanced**

We will use the **Cosine function to represent variables**

Frequency

Phase Angle

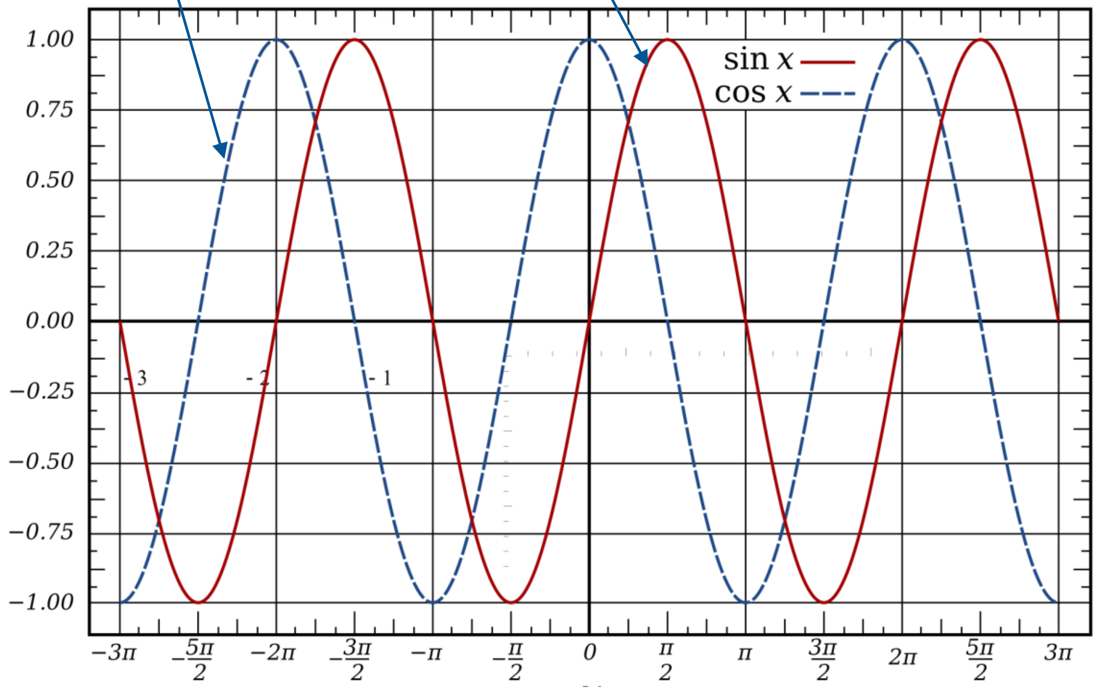
$$y(t) = A\cos(\omega t + \phi)$$

Variable as function of time

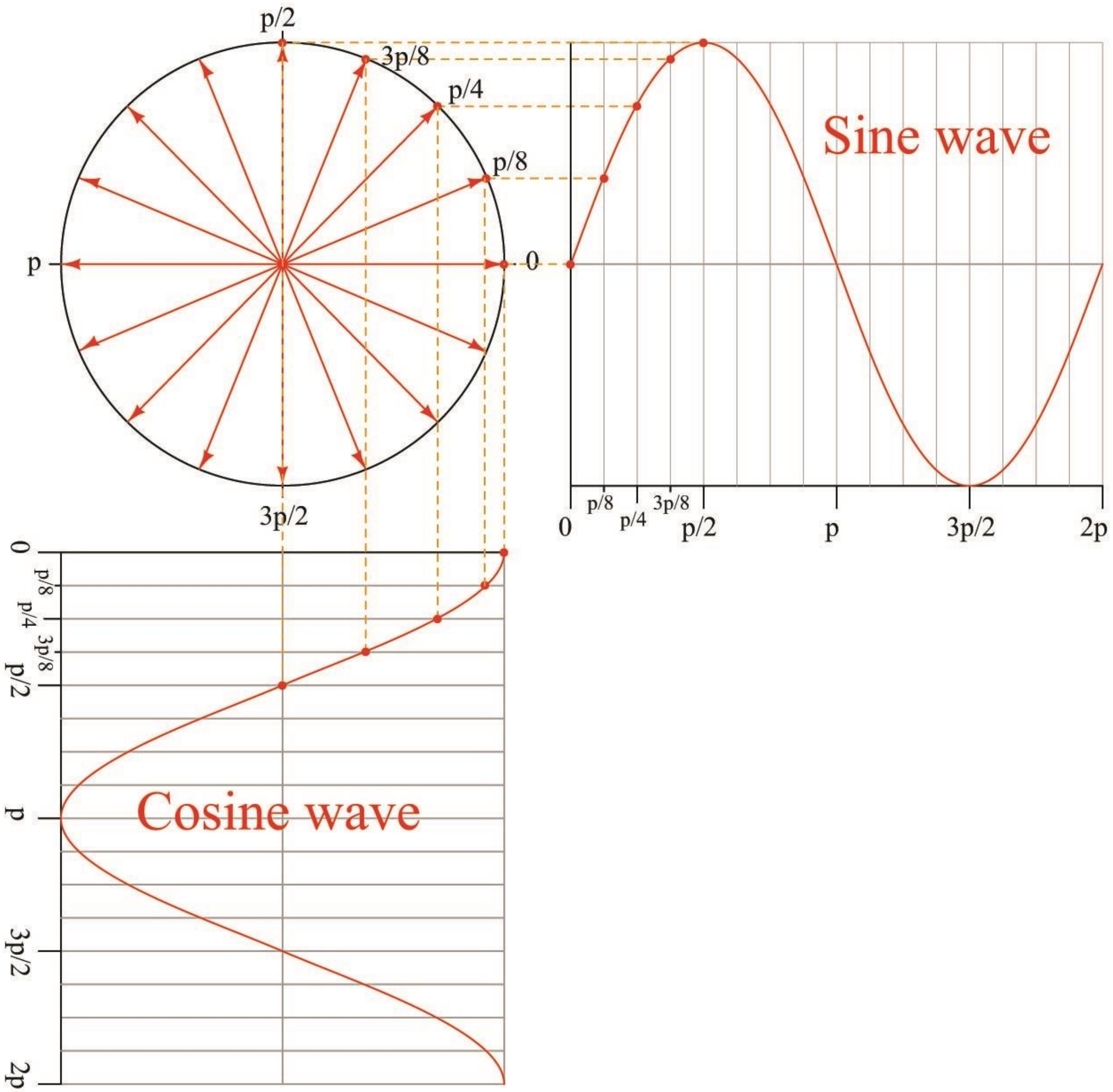
Amplitude

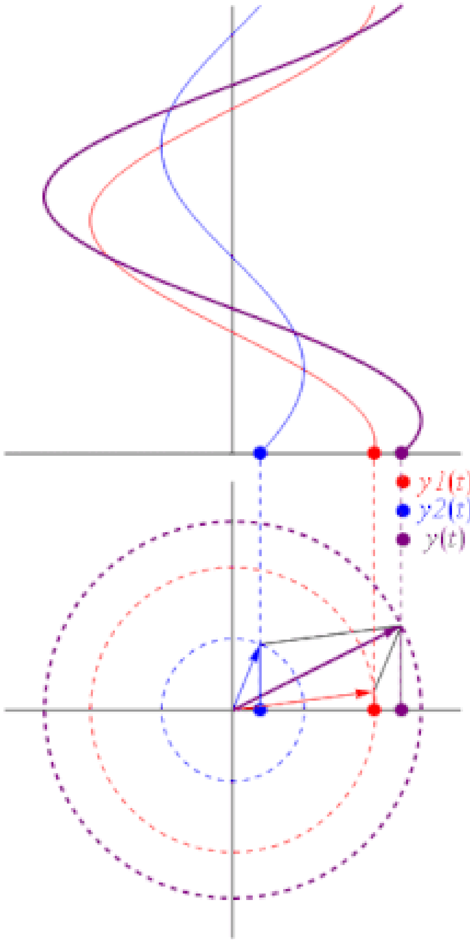
Phase offset

$x_1(t) = \cos t$ $x_2(t) = \sin t$

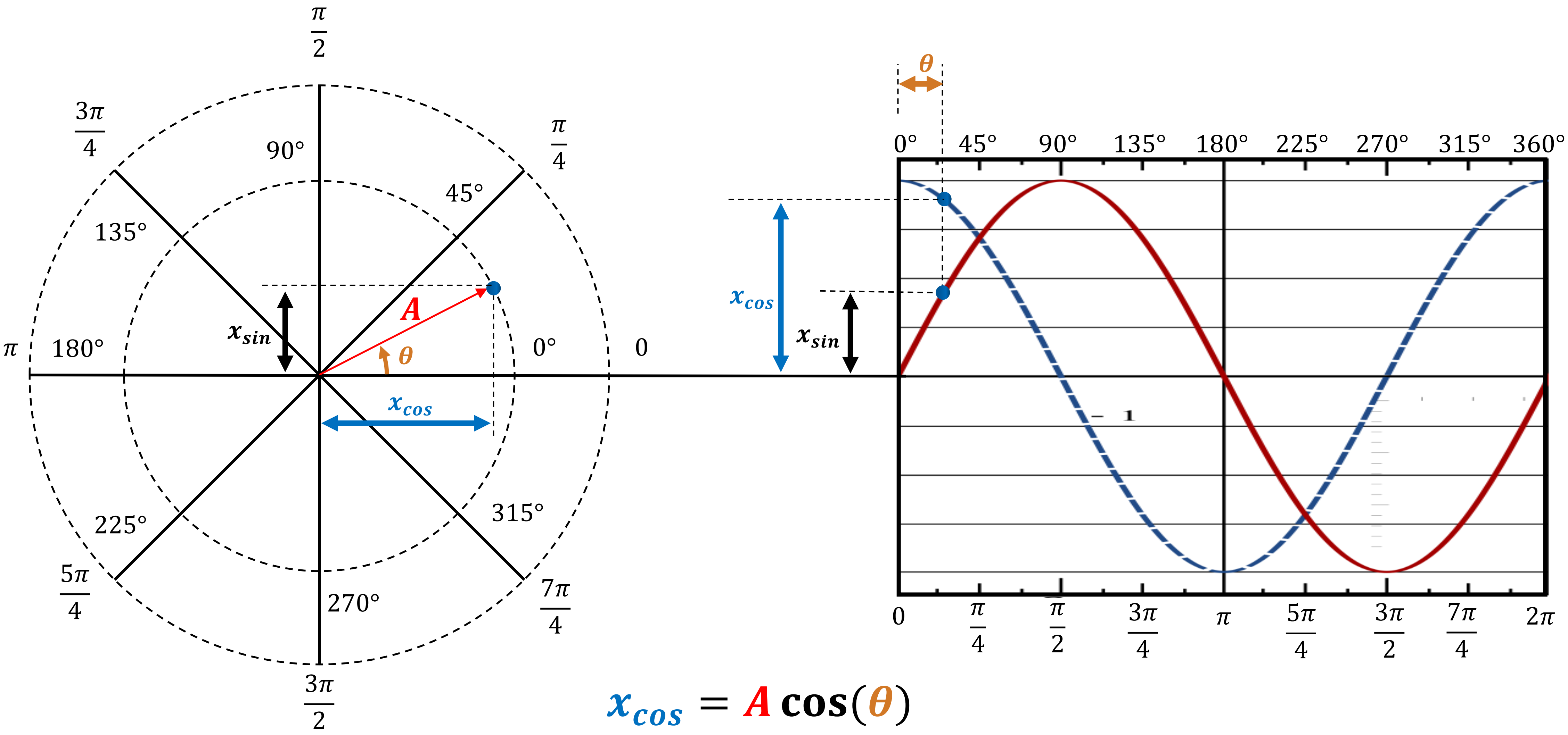


Sinusoid – Phase Angle





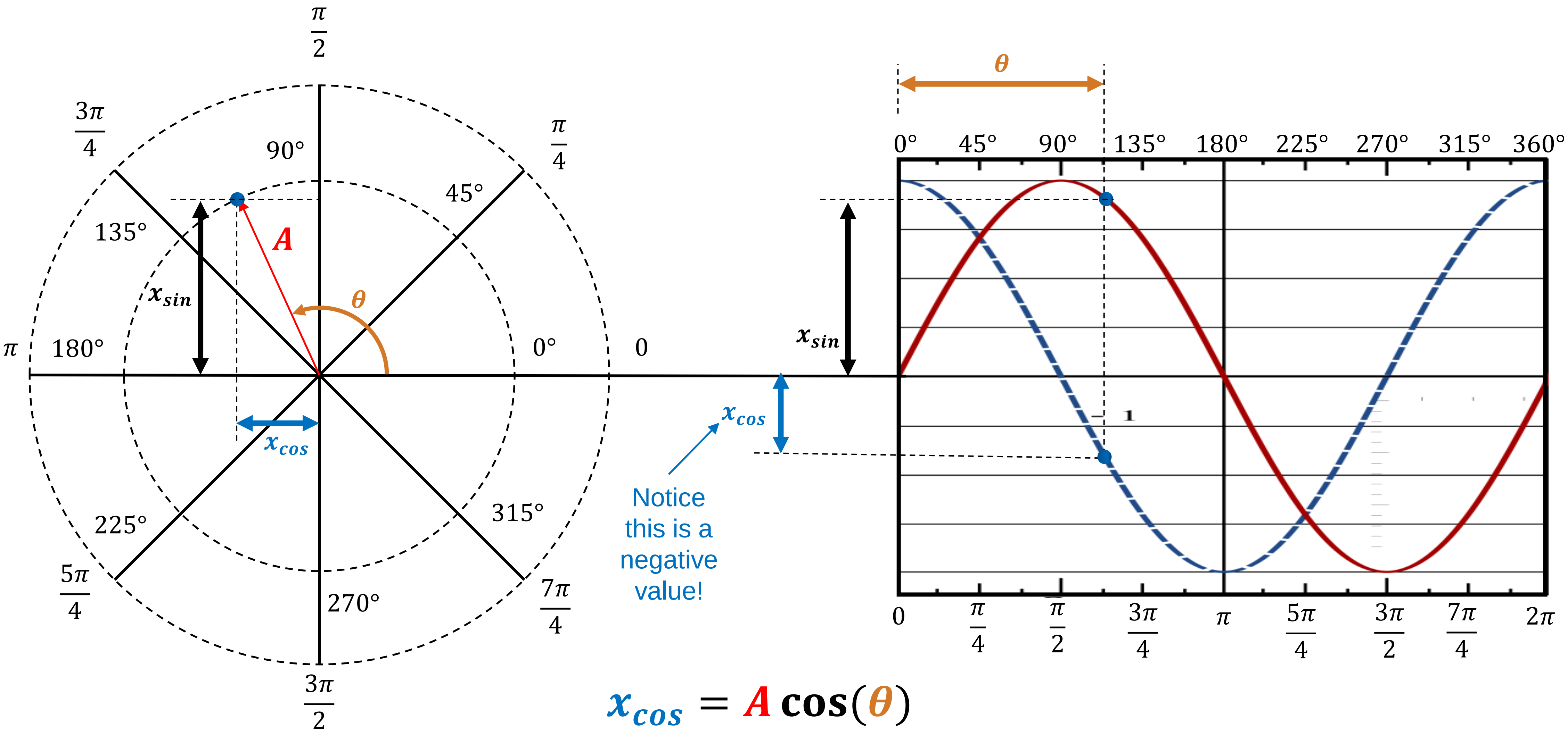
Sinusoid – Phase Angle



$$x_{cos} = A\cos(\theta)$$

$$x_{sin} = A\sin(\theta)$$

Sinusoid – Phase Angle



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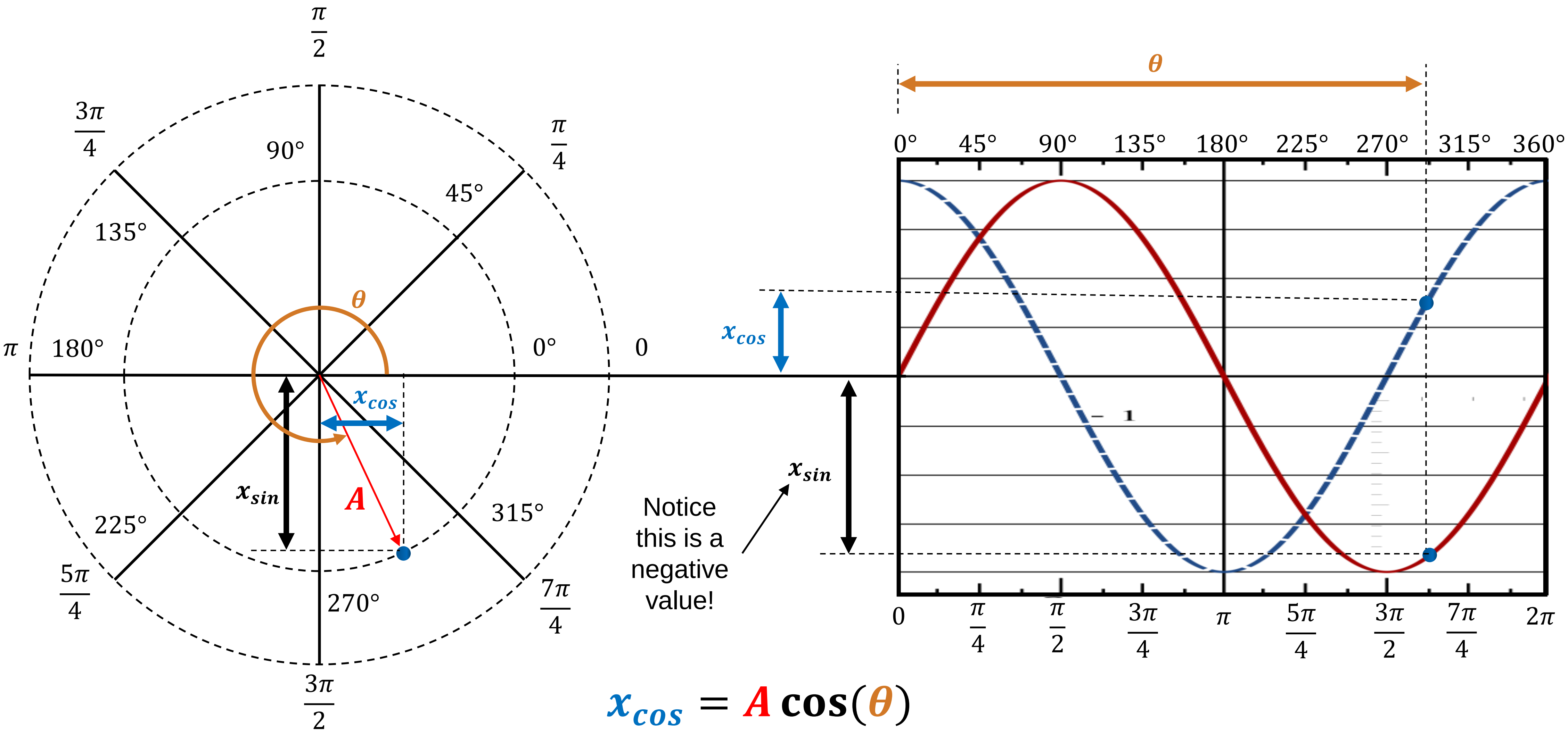
Sinusoid – Phase Angle

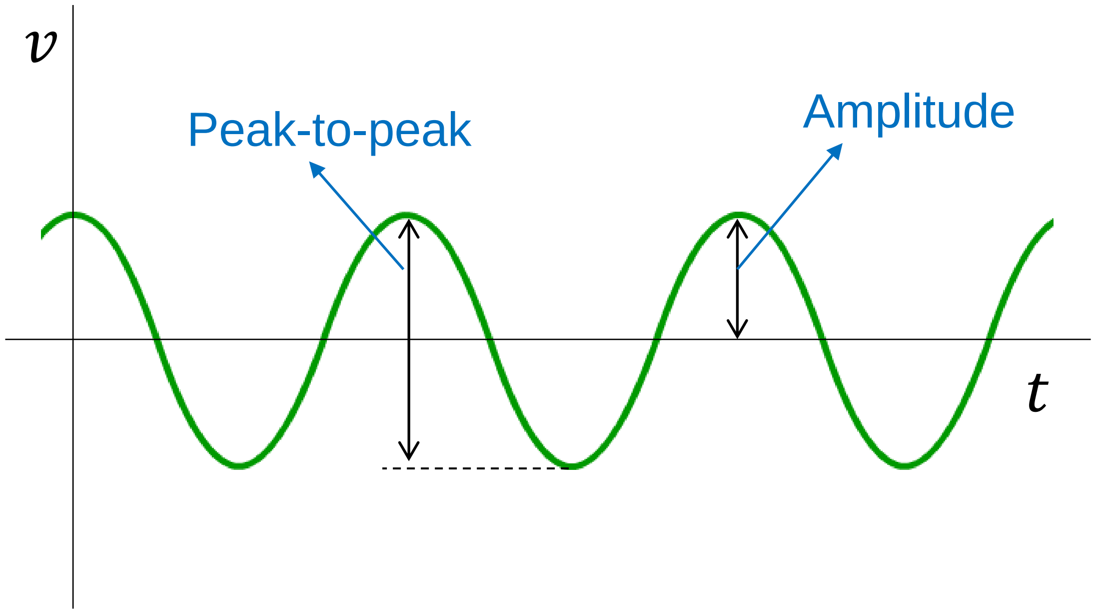


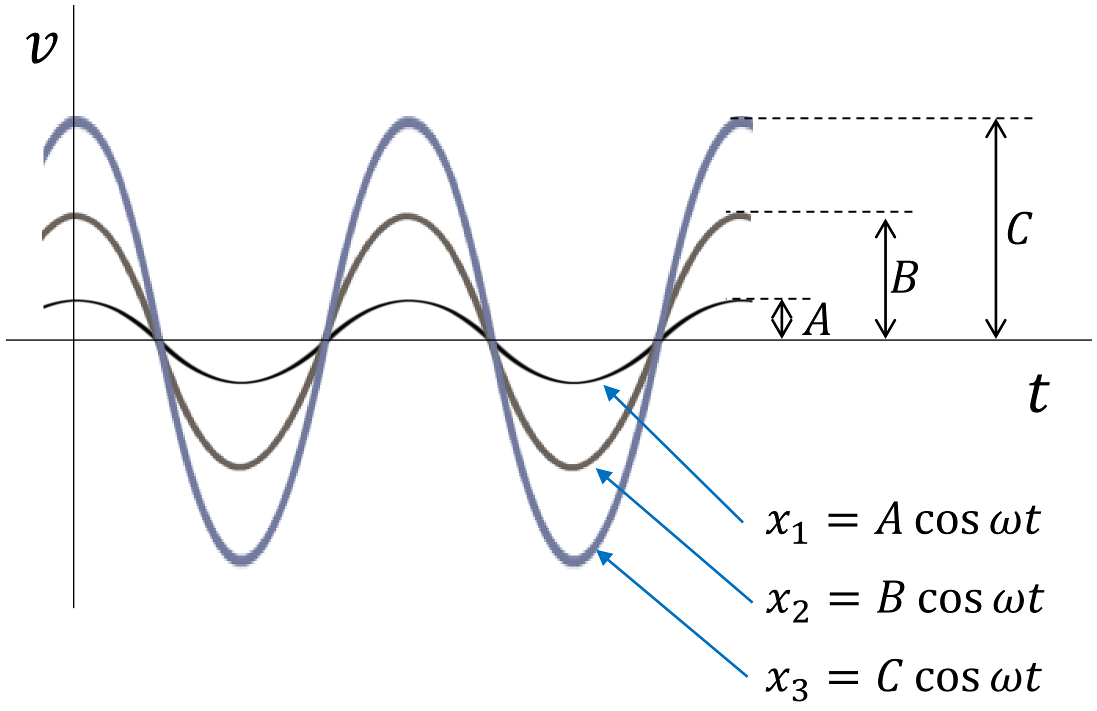
$$x_{cos} = A\cos(\theta)$$

$$x_{sin} = A\sin(\theta)$$

Sinusoid - Amplitude

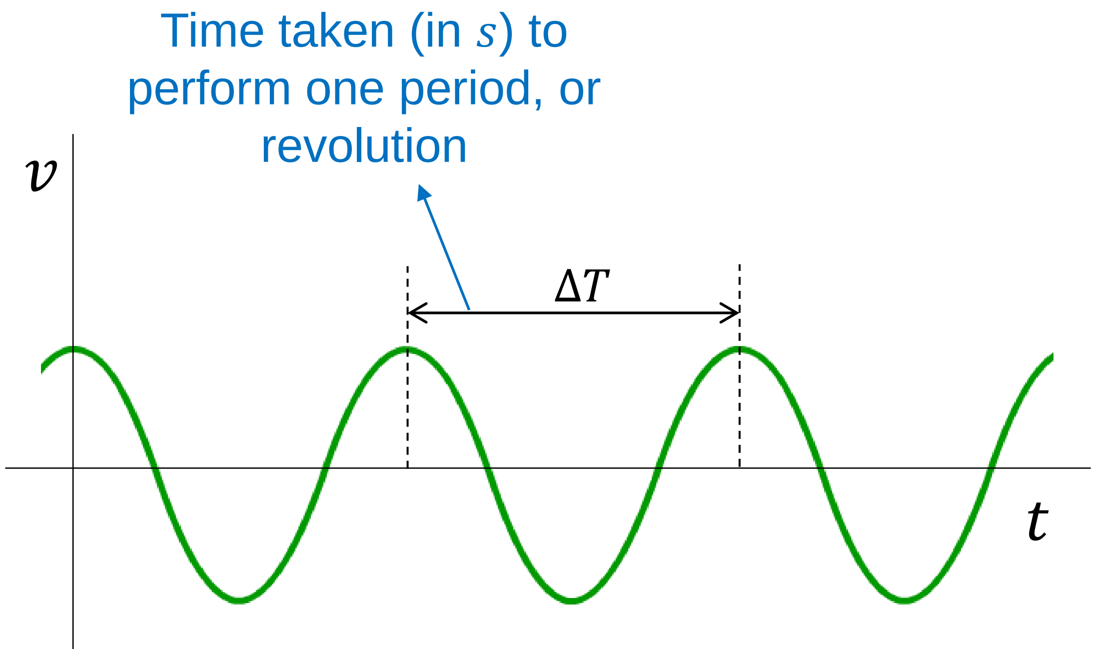
Maximum magnitude of the variable





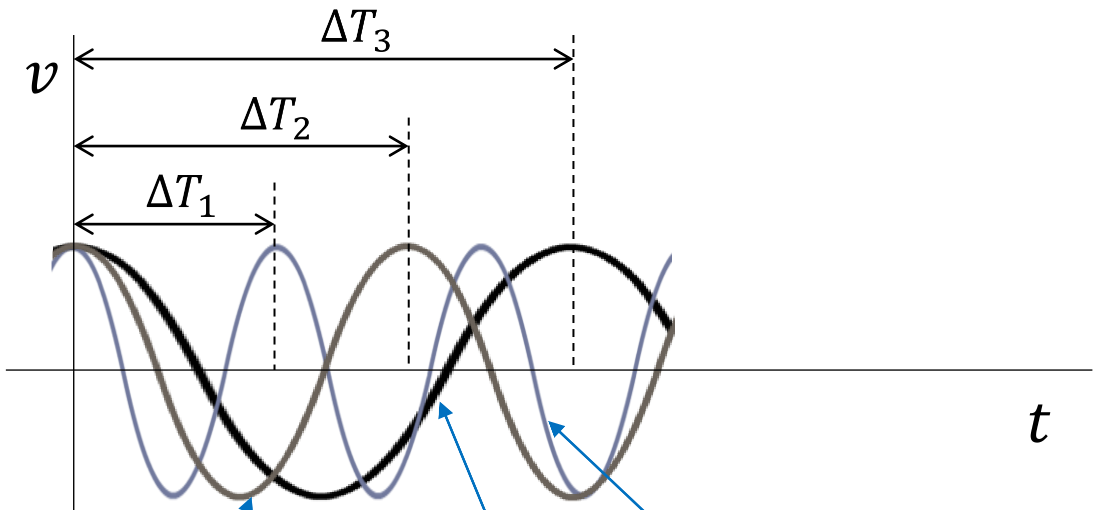
Sinusoid - Frequency or Angular Speed

Indicates how fast is the variable changing



$$Frequency(Hz) = f = \frac{1}{\Delta T}$$

$$Angular\ Speed\left(\frac{rad}{s}\right) = \omega = 2\pi f = \frac{2\pi}{\Delta T}$$



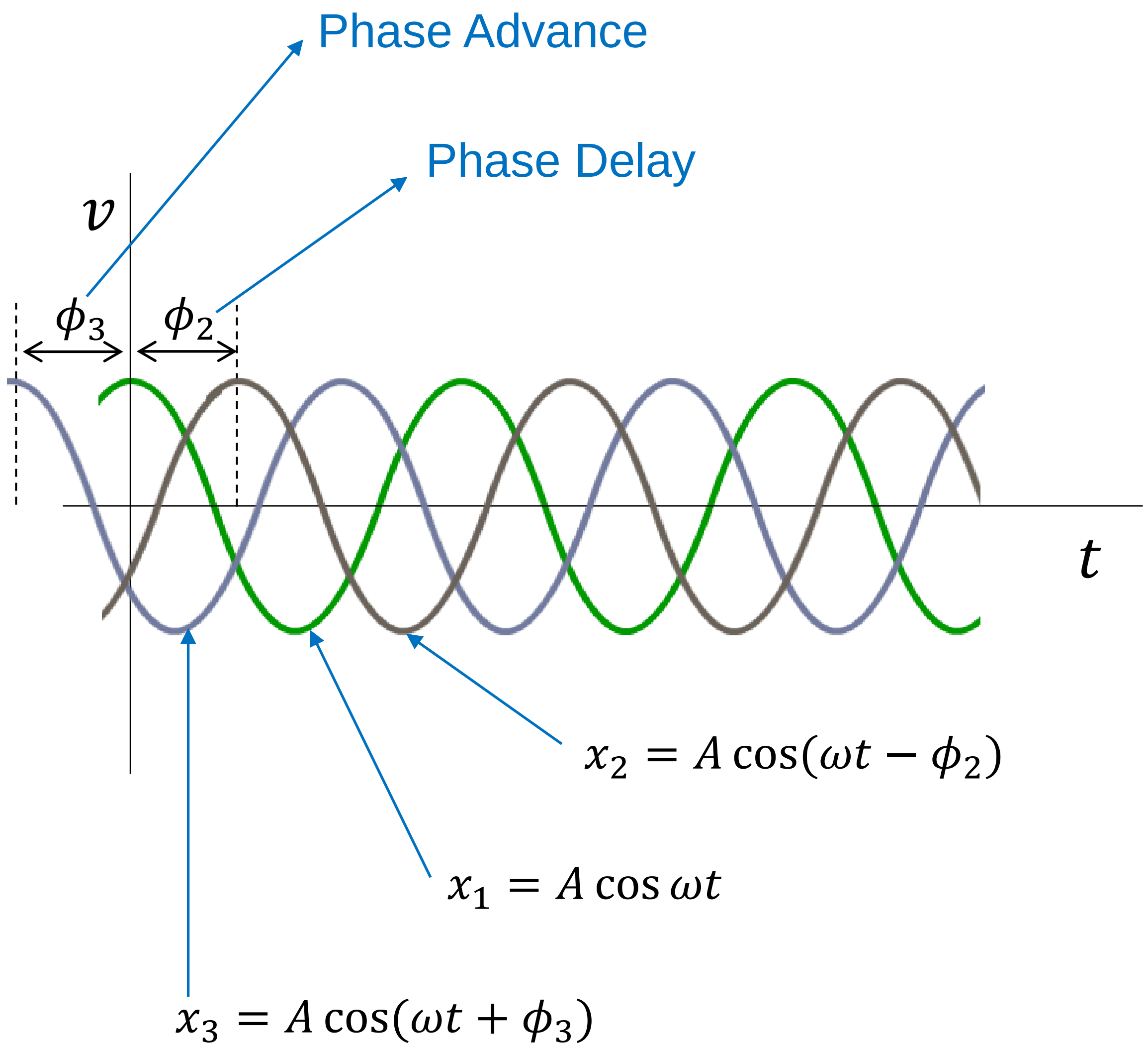
$$x_1 = A\cos\frac{2\pi}{\Delta T_1}t$$

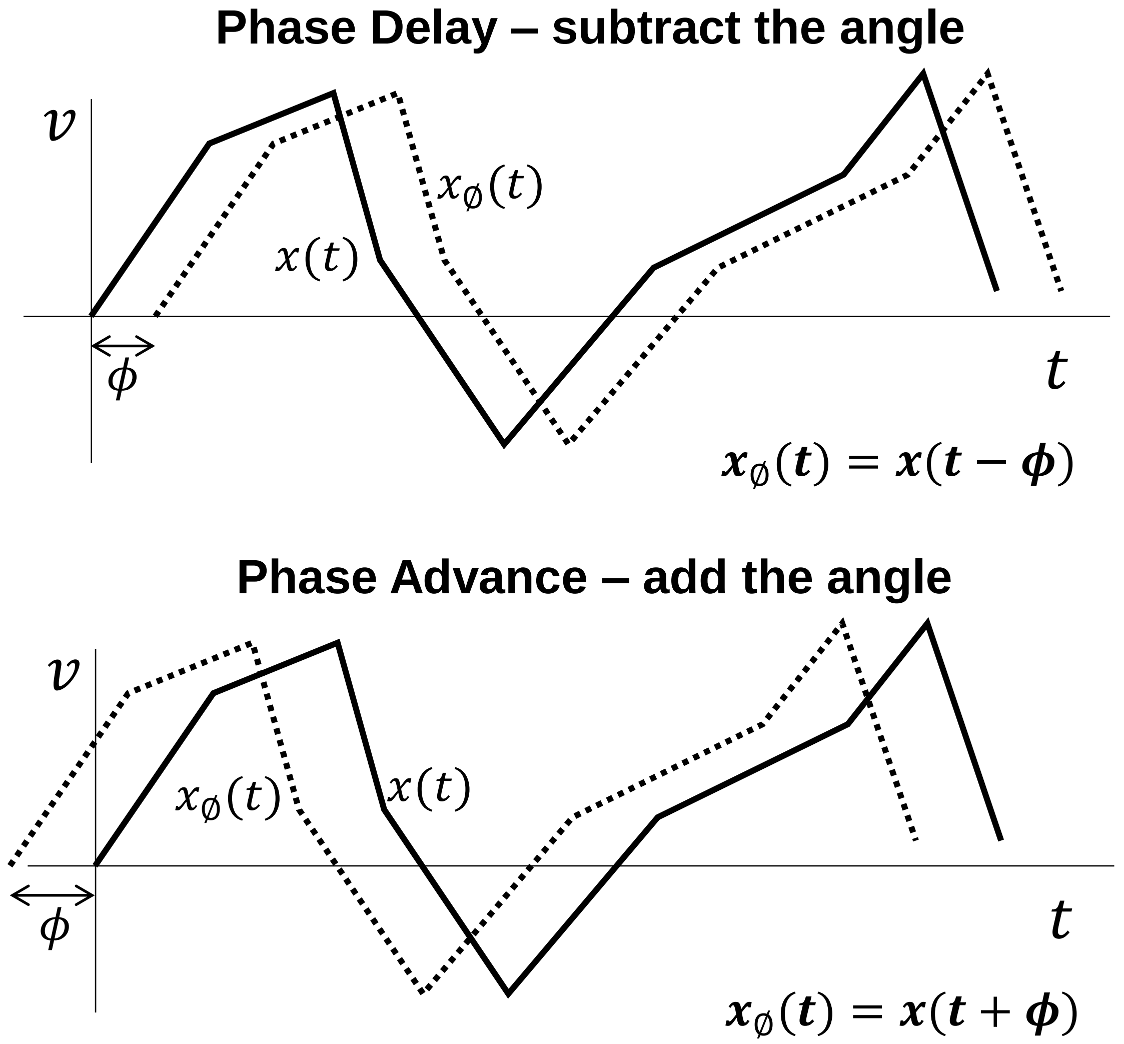
$$x_2 = A\cos\frac{2\pi}{\Delta T_2}t$$

$$x_3 = A\cos\frac{2\pi}{\Delta T_3}t$$

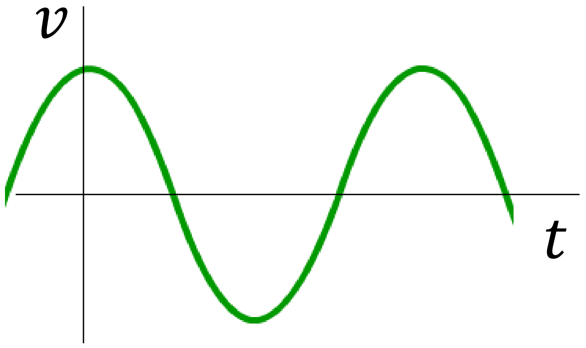
Sinusoid – Phase Offset

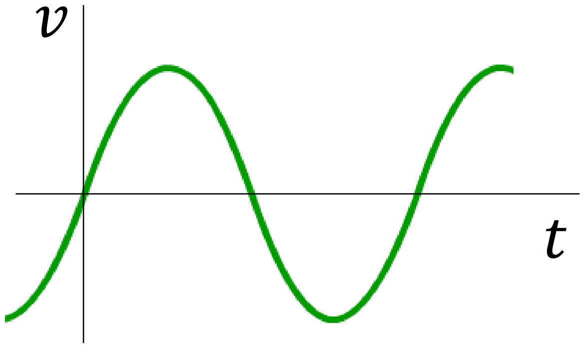
Phase angle at $t = 0$

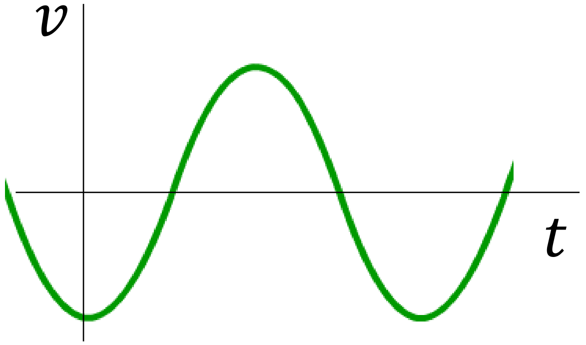




Sinusoid – Phase Offset







$$x_1 = \boldsymbol{A\cos\omega t}$$

Add a Phase Delay of $\frac{\pi}{2}$

$$x_2 = A\cos(\omega t - \frac{\pi}{2}) = \mathbf{Asin}\boldsymbol{\omega t}$$

Add a Phase Delay of $\frac{\pi}{2}$

$$x_3 = A\sin(\omega t - \frac{\pi}{2}) = \mathbf{-Acos}\boldsymbol{\omega t}$$

$$x_4 = \mathbf{Acos}\boldsymbol{\omega t}$$

Add a Phase Advance of $\frac{\pi}{2}$

$$x_5 = \mathrm{Acos}\left(\omega t + \frac{\pi}{2}\right) = \boldsymbol{-A\sin\omega t}$$

Add a Phase Advance of $\frac{\pi}{2}$

$$x_6 = -\mathrm{Asin}\left(\omega t + \frac{\pi}{2}\right) = \boldsymbol{-A\cos\omega t}$$

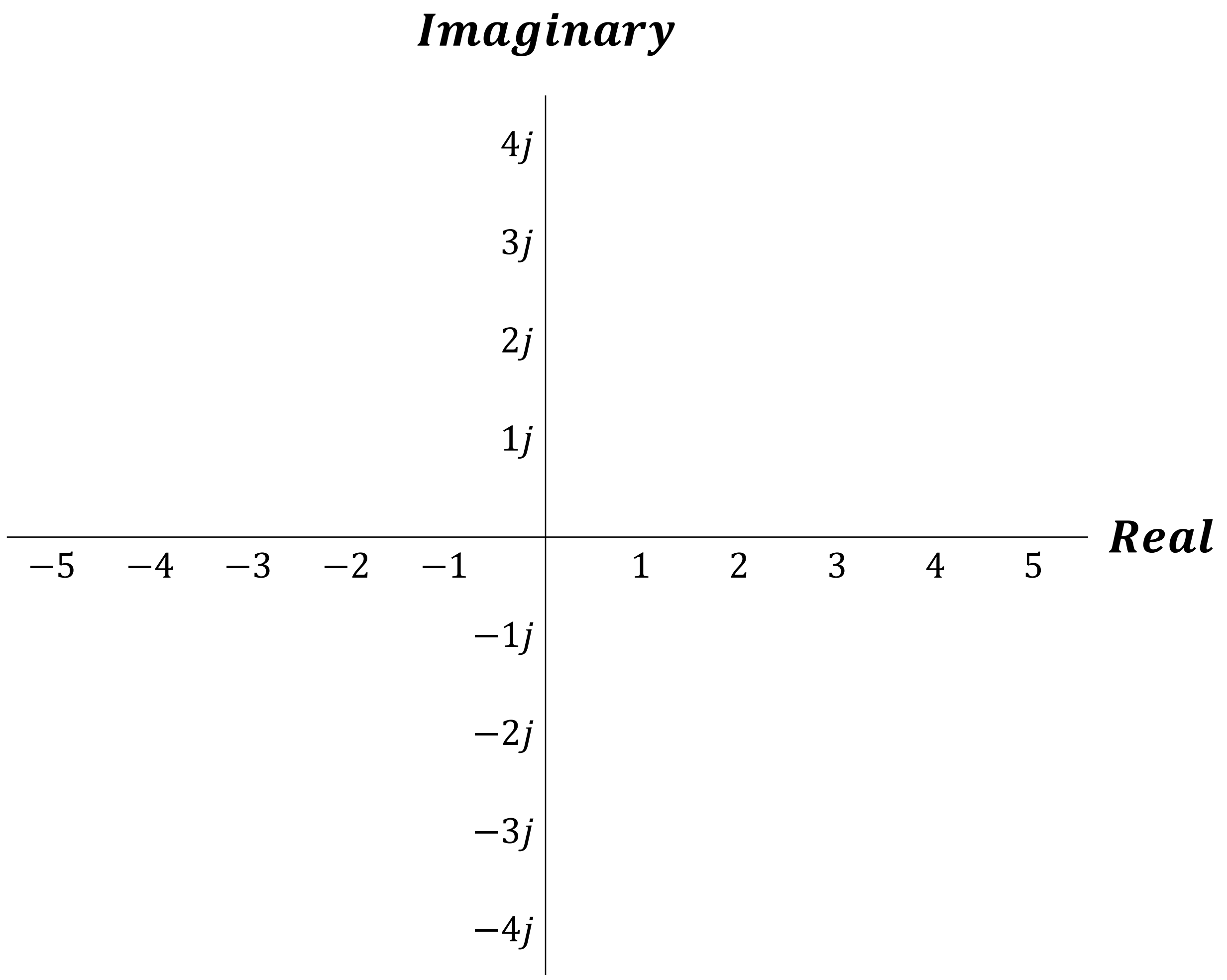
Complex Number

Solution of $x^2 = -1$

$$x = \sqrt{-1}$$

$$x = j$$

Argand Plane, or complex plane is used to represent complex numbers in the cartesian coordinate system



Complex Number

Cartesian Form – Use the x & y coordinates to represent the complex number

$$\mathbf{4 + j3}$$

$\boldsymbol{x + jy}$ (general form)

Polar Form – Use the magnitude & angle to represent the complex number

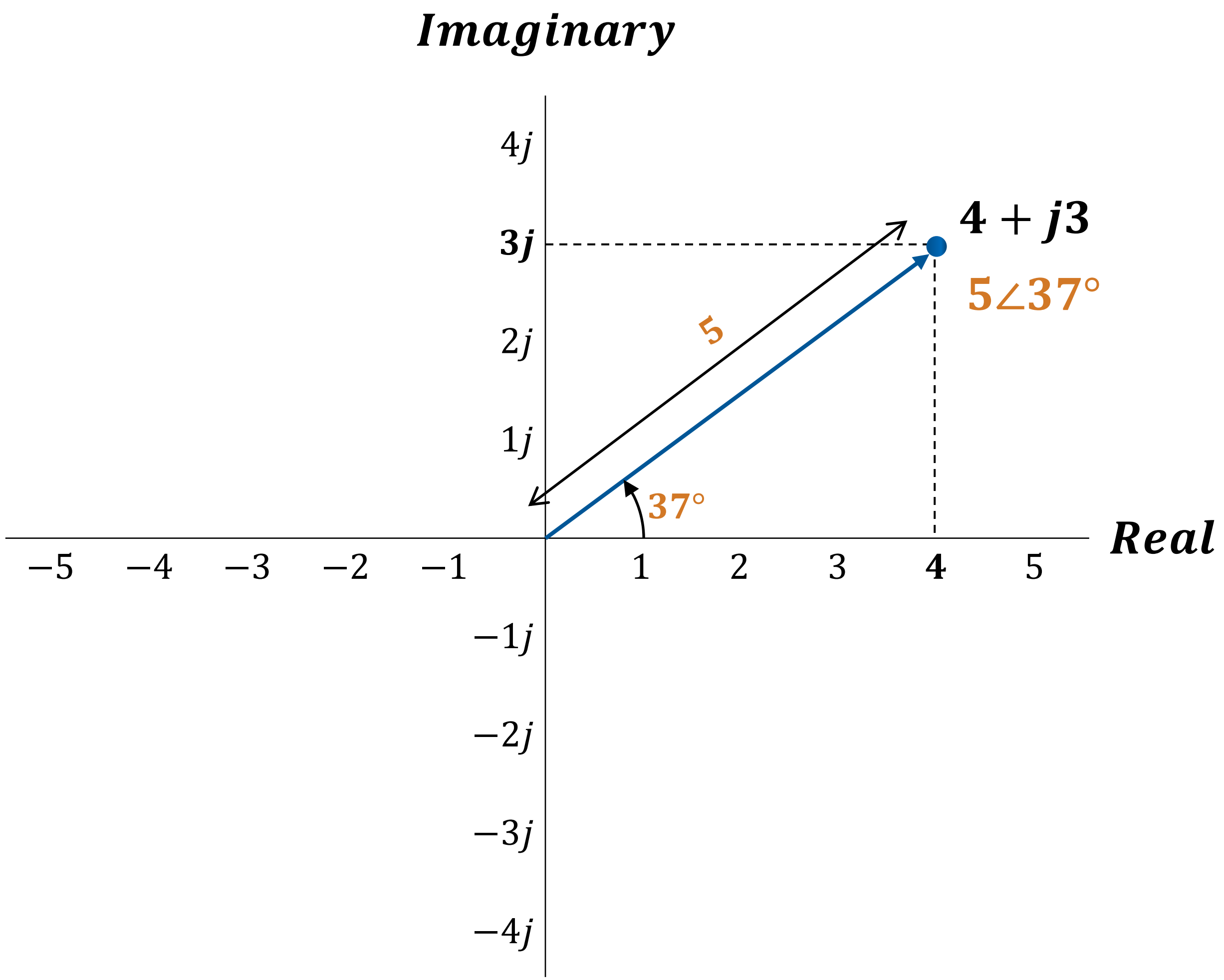
$$\mathbf{5\angle 37^\circ}$$

$\boldsymbol{|V|\angle\theta}$ (general form)

Exponential Form – Variation of Polar Form

$$\mathbf{5e^{j37^\circ}}$$

$\boldsymbol{|V|e^{j\theta}}$(general form)



Complex Number

Cartesian to Polar Conversion

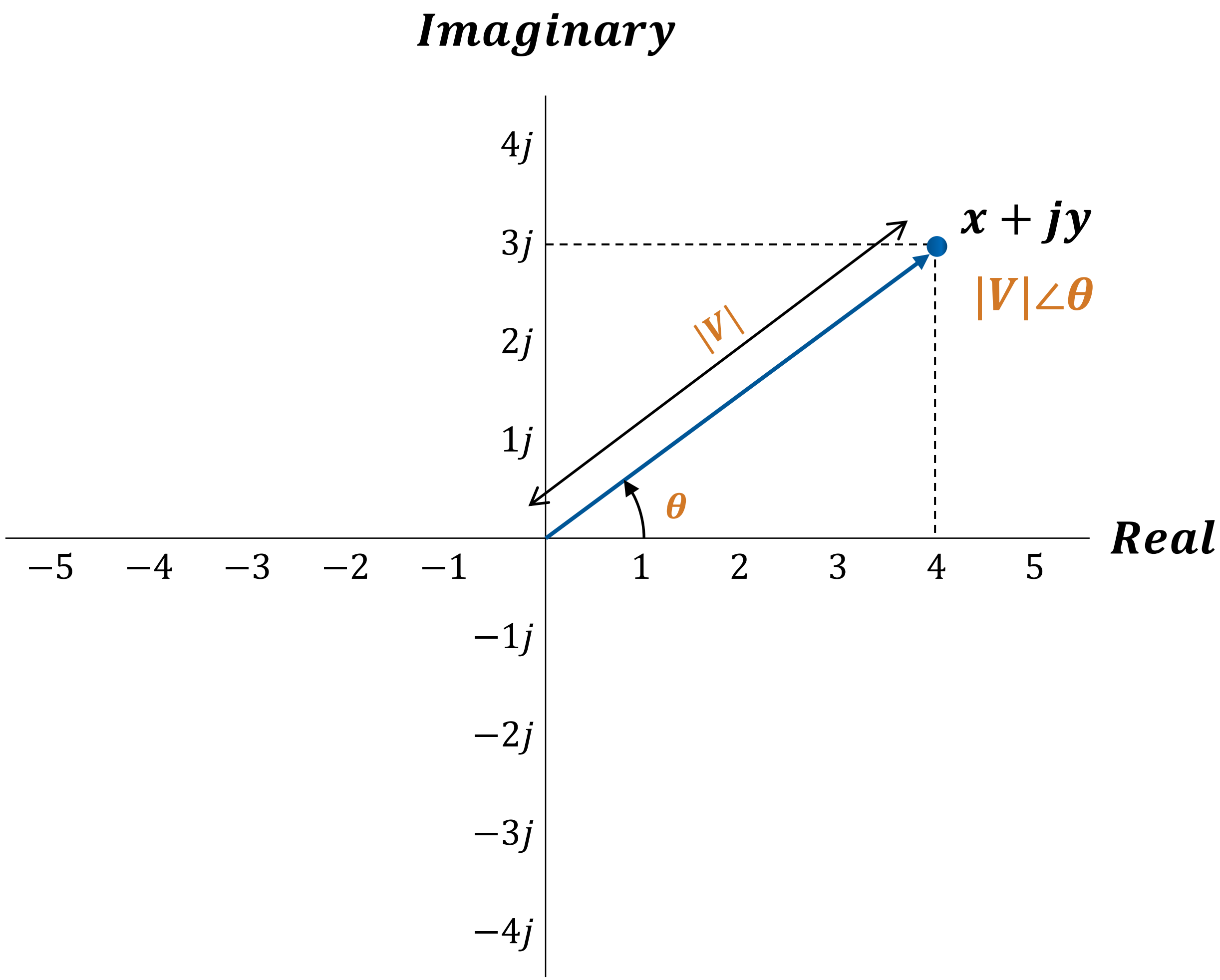
$$|V| = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\frac{y}{x}$$

Polar to Cartesian Conversion

$$x = |V|cos\theta$$

$$y = |V|sin\theta$$

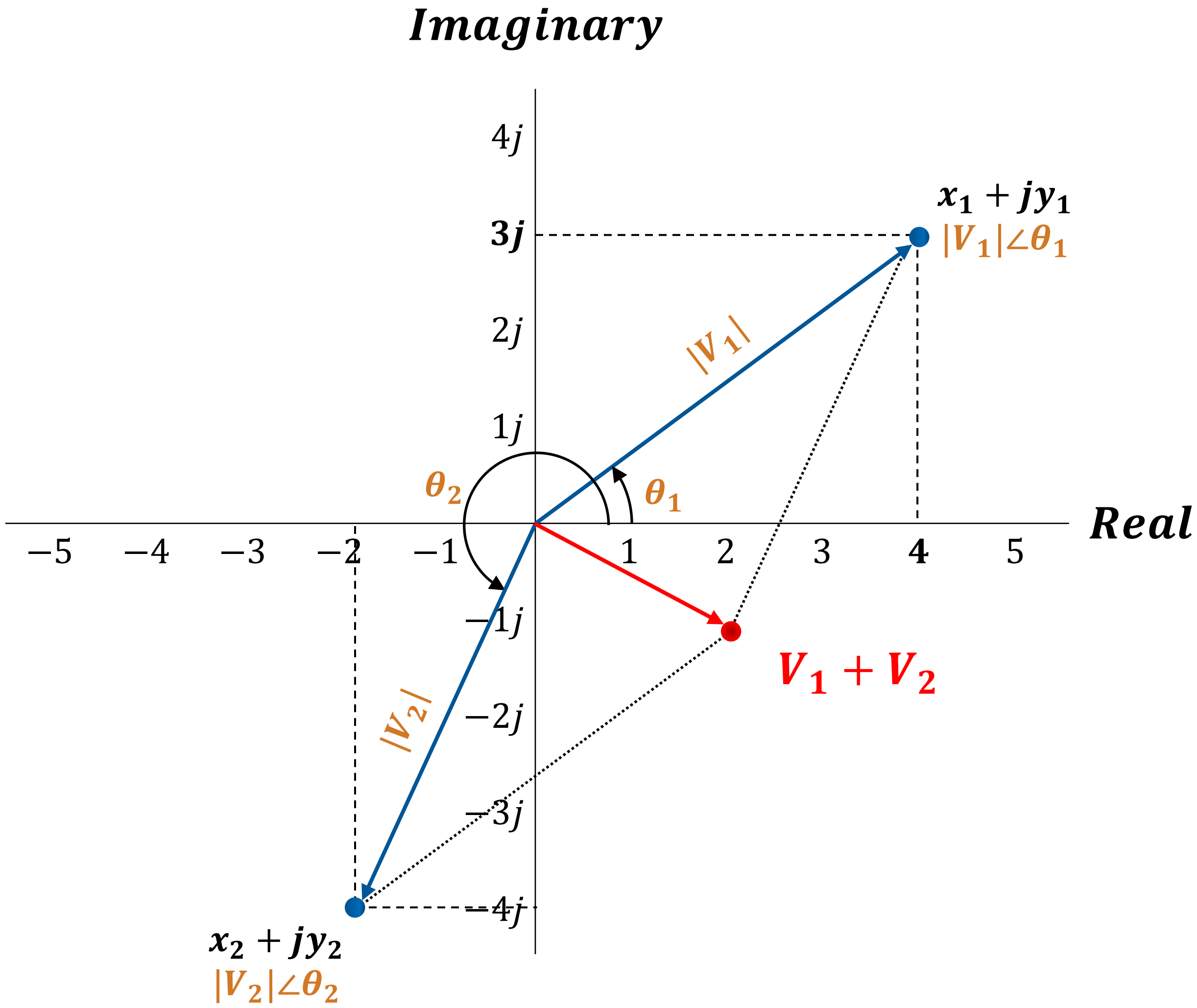


Complex Number

Addition

$$V_1 + V_2 = (x_1 + jy_1) + (x_2 + jy_2)$$
$$V_1 + V_2 = (x_1 + x_2) + j(y_1 + y_2)$$

Simply add the real terms and imaginary terms separately

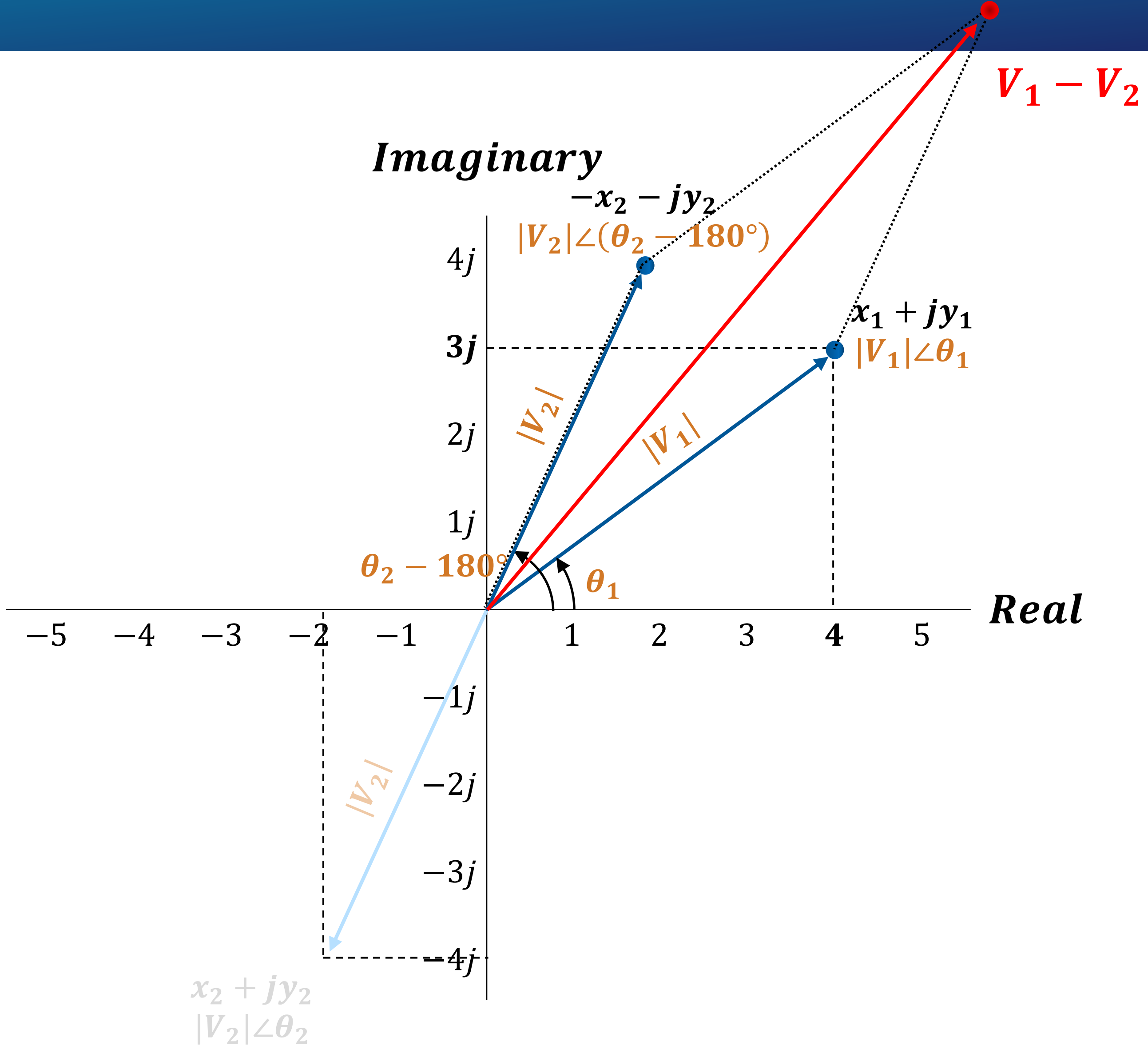


Complex Number

Subtraction

$$\boldsymbol{V_1 - V_2 = (x_1 + jy_1) - (x_2 + jy_2)}$$
$$\boldsymbol{V_1 - V_2 = (x_1 - x_2) + j(y_1 - y_2)}$$

Simply subtract the real terms and imaginary terms separately



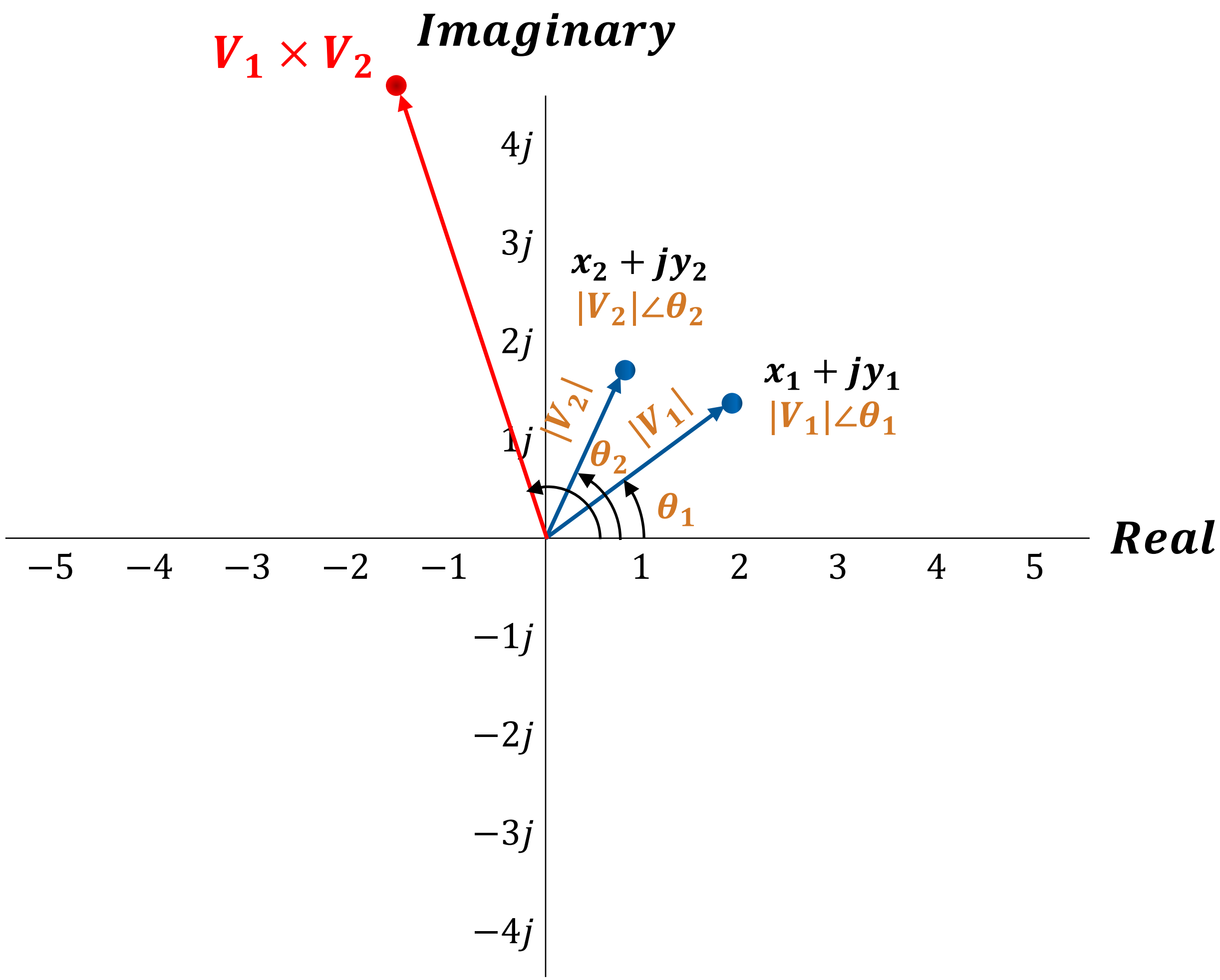
Complex Number

Multiplication

$$V_1 \times V_2 = (x_1 + y_1 j) \times (x_2 + y_2 j)$$
$$V_1 \times V_2 = x_1x_2 + x_1y_2 j + y_1x_2 + y_1y_2 j^2$$
$$V_1 \times V_2 = x_1x_2 + x_1y_2 j + y_1x_2 j + y_1y_2(-1)$$
$$V_1 \times V_2 = (x_1x_2 - y_1y_2) + (x_1y_2 + y_1x_2)j$$

Simpler method using Polar Form

$$V_1 \times V_2 = |V_1||V_2|\angle(\theta_1 + \theta_2) V_1 \times V_2 = |V_1|\angle\theta_1 \times |V_2|\angle\theta_2$$



Complex Number

Division

$$V_1 \div V_2 = \frac{(x_1 + jy_1)}{(x_2 + jy_2)}$$

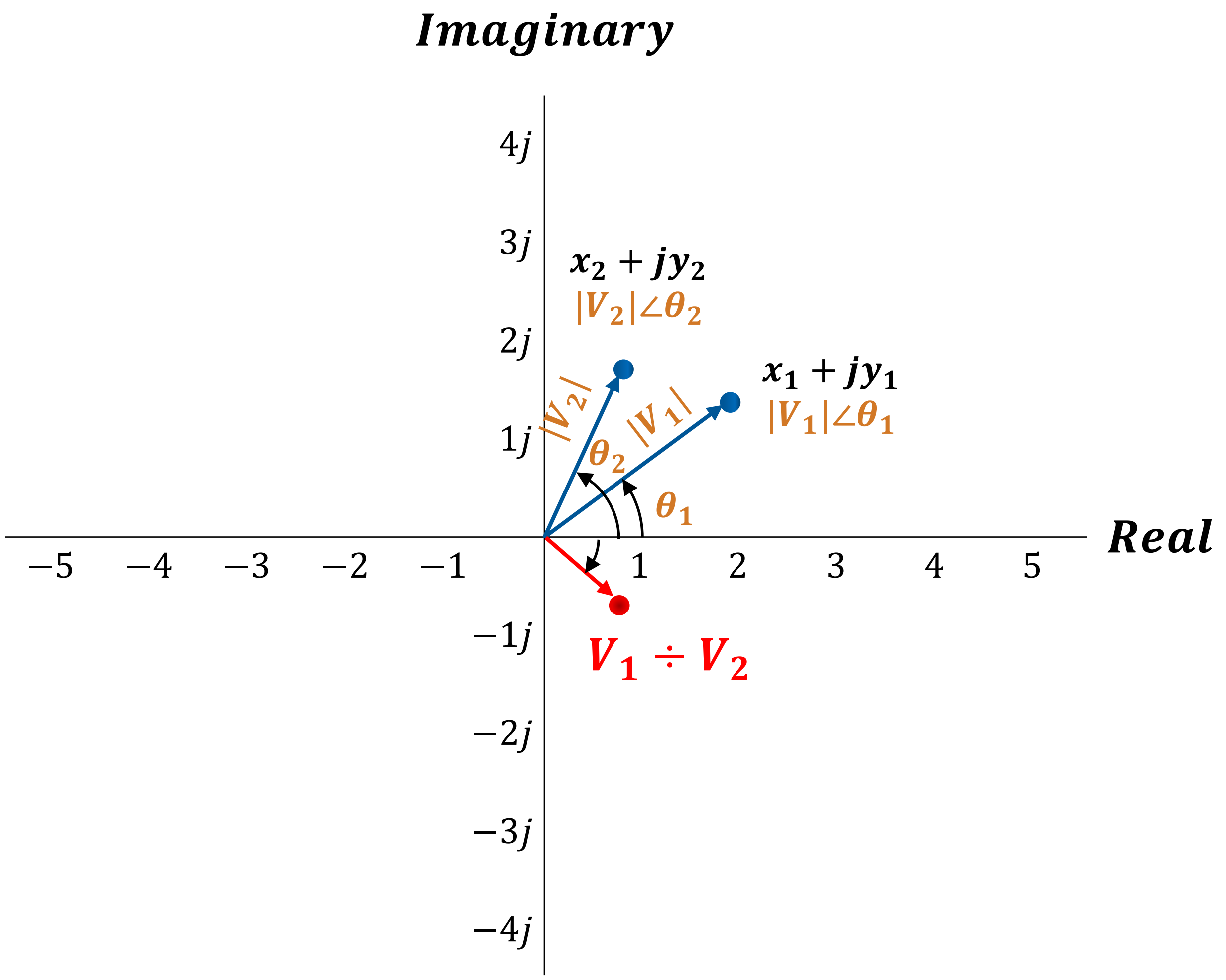
$$V_1 \div V_2 = \frac{(x_1 + jy_1) \times (x_2 - jy_2)}{(x_2 + jy_2) \times (x_2 - jy_2)}$$

$$V_1 \div V_2 = \frac{(x_1x_2 + y_1y_2) - j(x_1y_2 - y_1x_2)}{(x_2^2 - y_2^2)}$$

Simpler method using Polar Form

$$V_1 \div V_2 = |V_1|\angle\theta_1 \div |V_2|\angle\theta_2$$

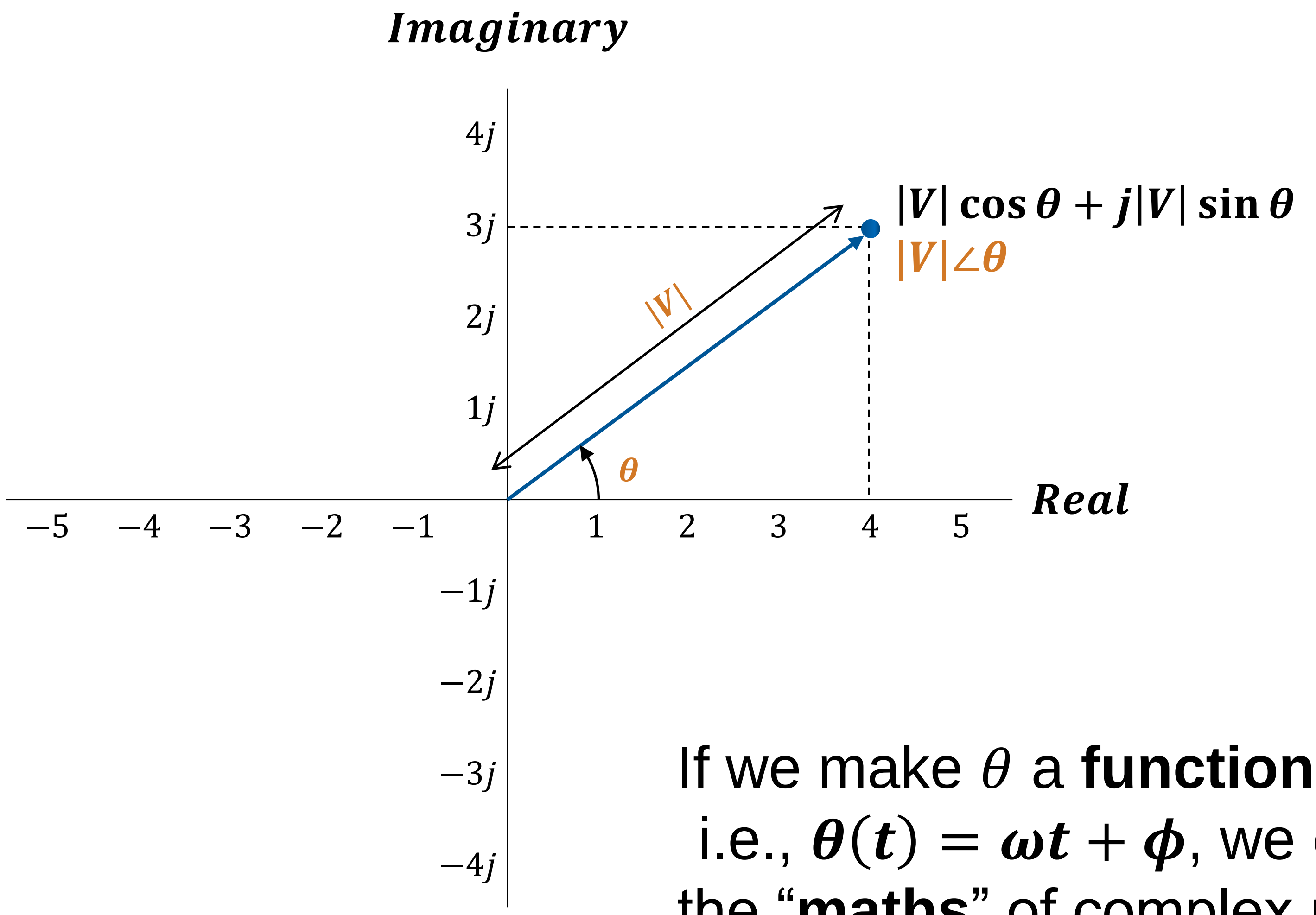
$$V_1 \div V_2 = \frac{|V_1|}{|V_2|}\angle(\theta_1 - \theta_2)$$

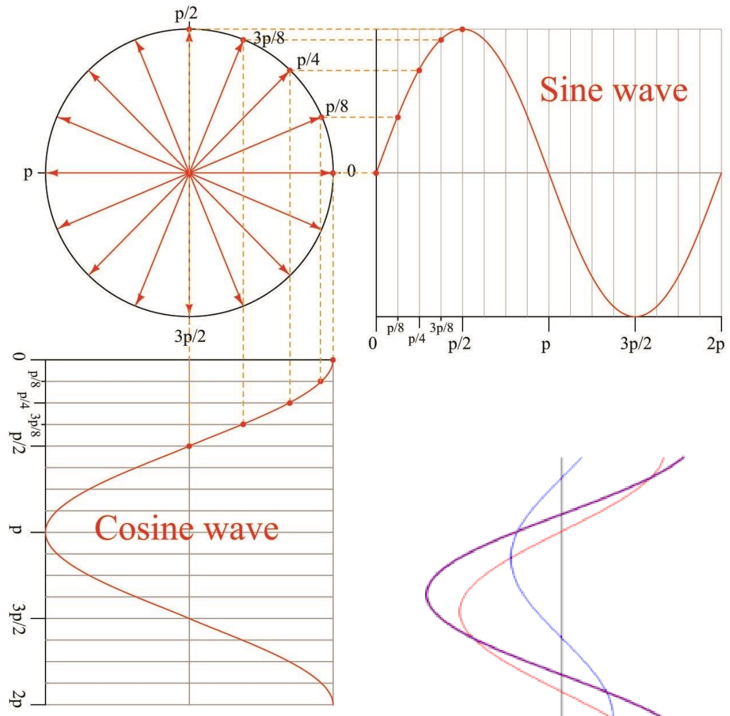


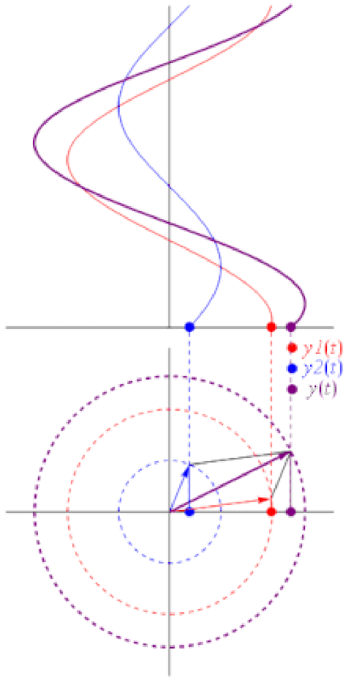
Learning Outcomes

- Fundamentals of **Alternating Current – or AC**
  - DC v AC circuit study – waveforms a **function of time**!
  - **Sinusoidal** waveform – voltage & current
  - **Complex Numbers**
- AC circuits
  - **Phasor** study – simple way to solve time-varying circuits
  - Resistor, Inductor, Capacitor in phasor form - **CIVIL**
  - **Reactance** – Purely reactive circuits (just inductor/capacitor)
  - **Impedance** – Resistance & Reactance
- Power in AC circuits
  - **Active** v **Reactive** v **Apparent** Power
  - **Power Factor**
  - **Resonance**

Phasor

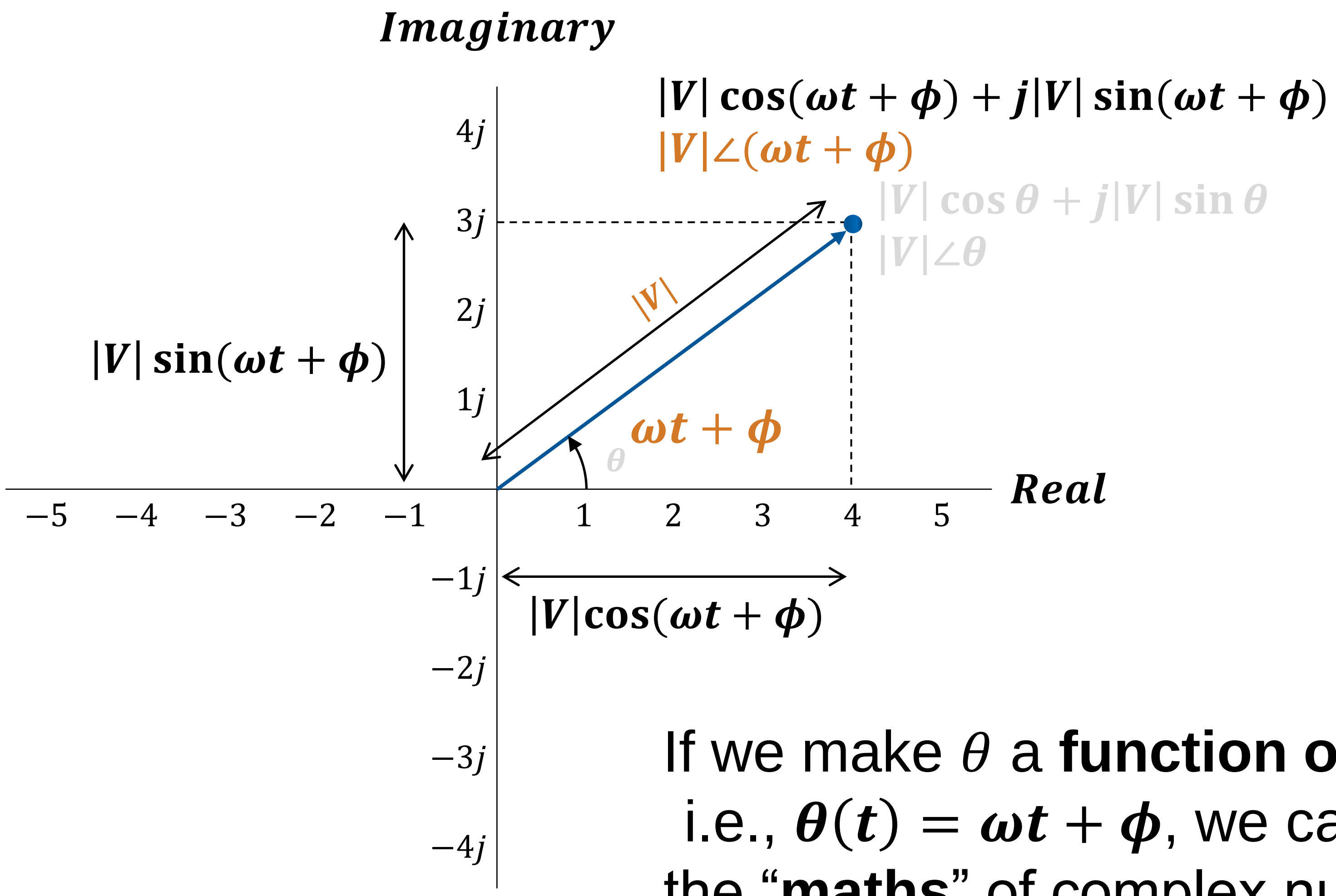


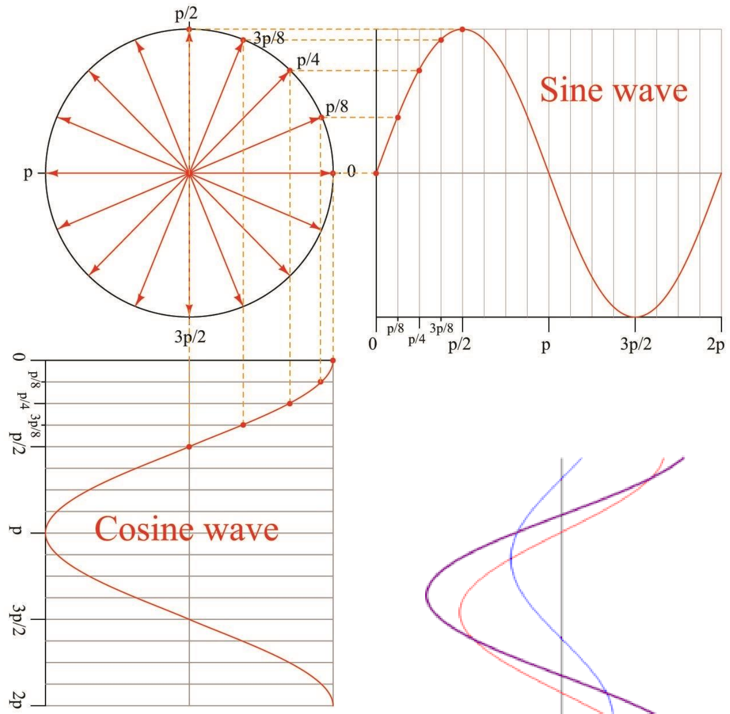


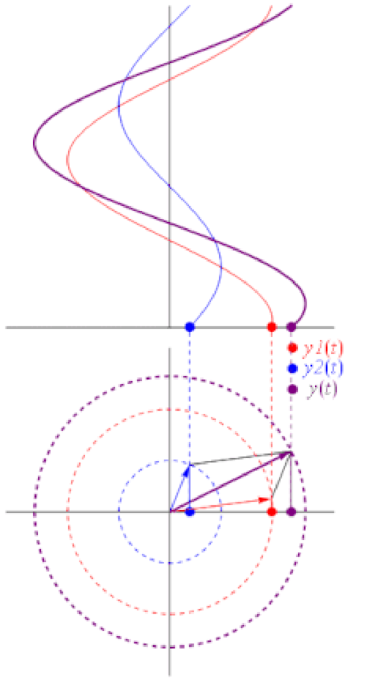


If we make θ a **function of time**, i.e., $\boldsymbol{\theta(t) = \omega t + \phi}$, we can use the "**maths**" of complex numbers to do the "**electrical**" of AC!

Phasor







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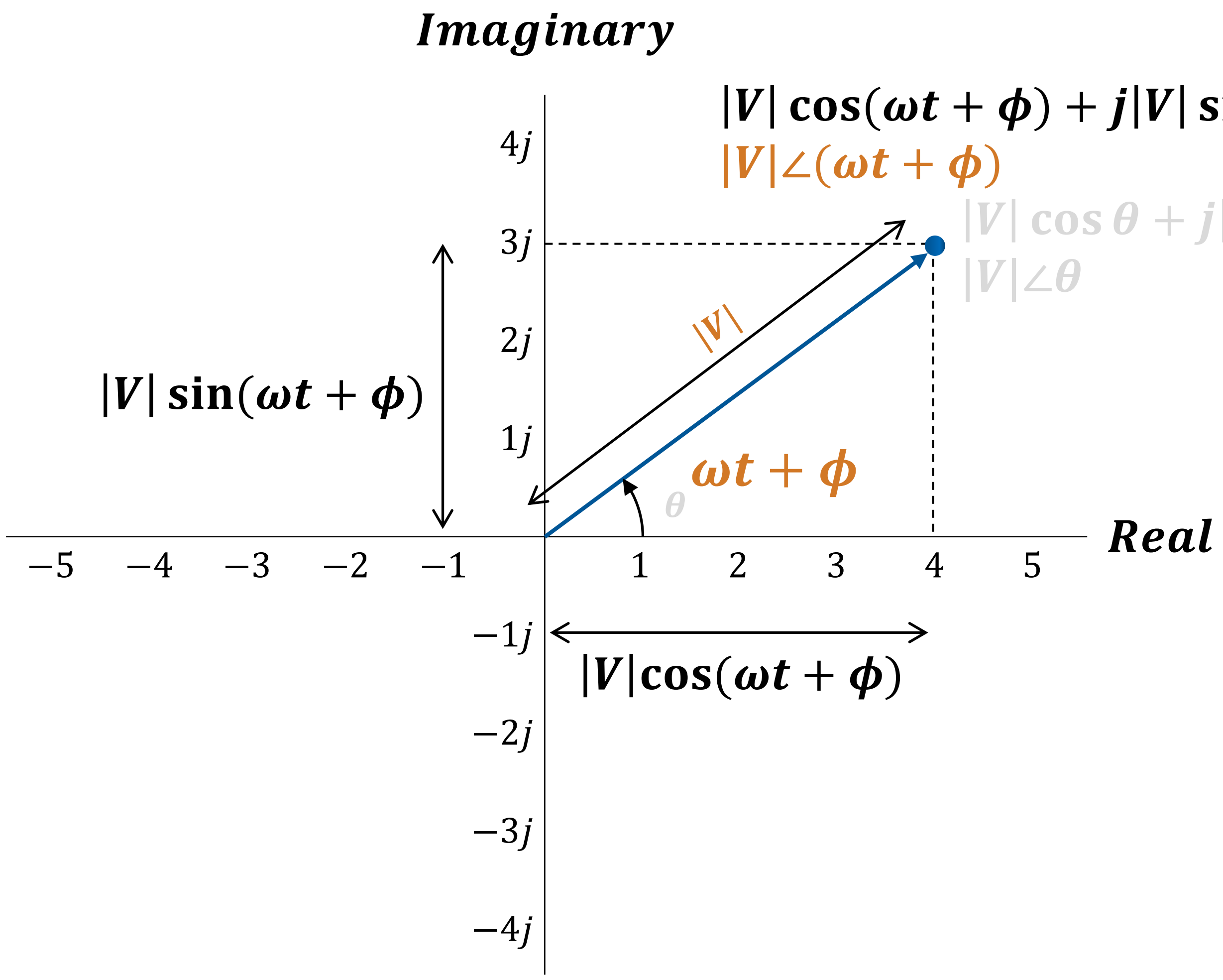
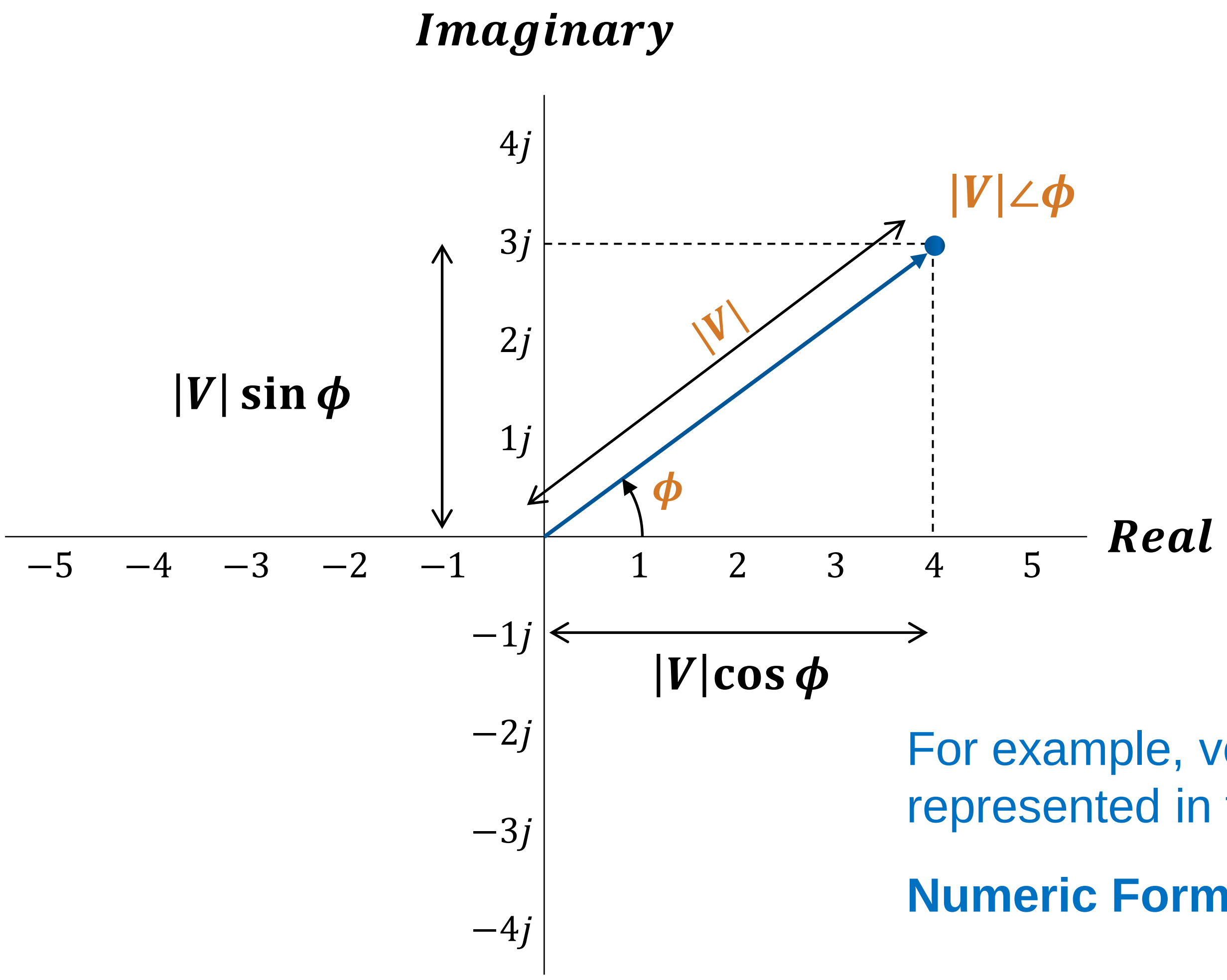
Phasor



Say we have a voltage variable $v=|V|cos(\omega t+\phi)$

We may represent it with a "phasor" which is nothing but a complex number that represents the initial position of the rotating vector (i.e., at $t=0$), and say the "projection on positive real axis" is the value of the physical variable

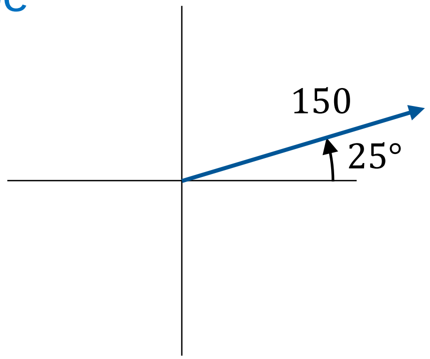
Phasor



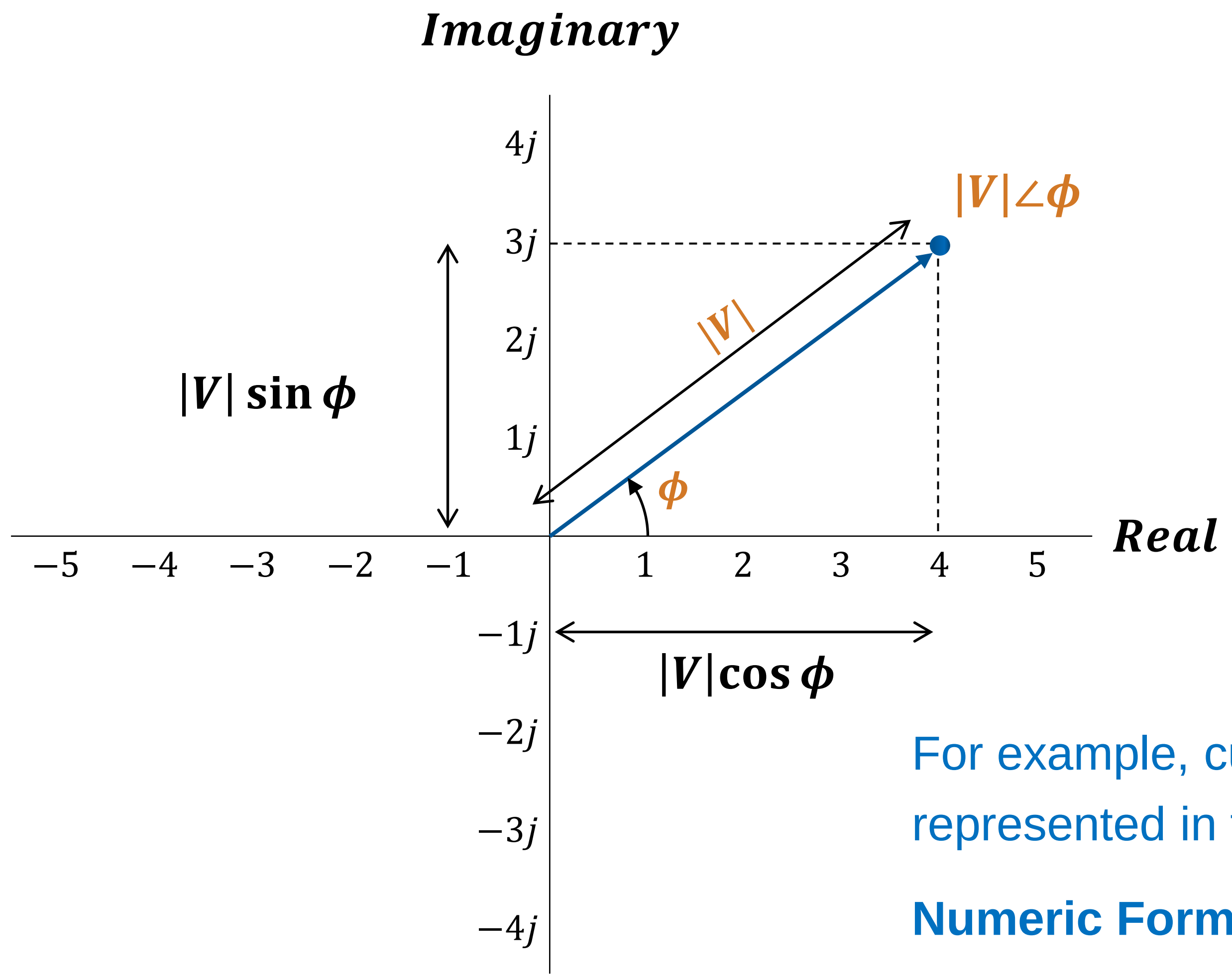
- A phasor is a complex number that represents the initial position of a rotating vector, i.e., at $t = 0$
- Use the amplitude ($|V|$) and phase offset (ϕ) of a cosine function
- For all AC steady-state analysis, $|V|$ and ϕ are all we need to get meaningful results
- AC steady-state analysis – this assumes frequency ω does not change

For example, voltage $\boldsymbol{v = 150\,cos(50t + 25°)}$ may be represented in the phasor form as follows:

Numeric Form – $\mathbf{150\angle 25°}$ **Visual Form –**



Phasor



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- Use the amplitude ($|V|$) and phase offset (ϕ) of a cosine function
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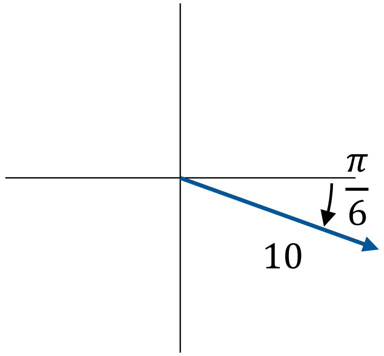
- AC steady-state analysis – this assumes frequency ω does not change

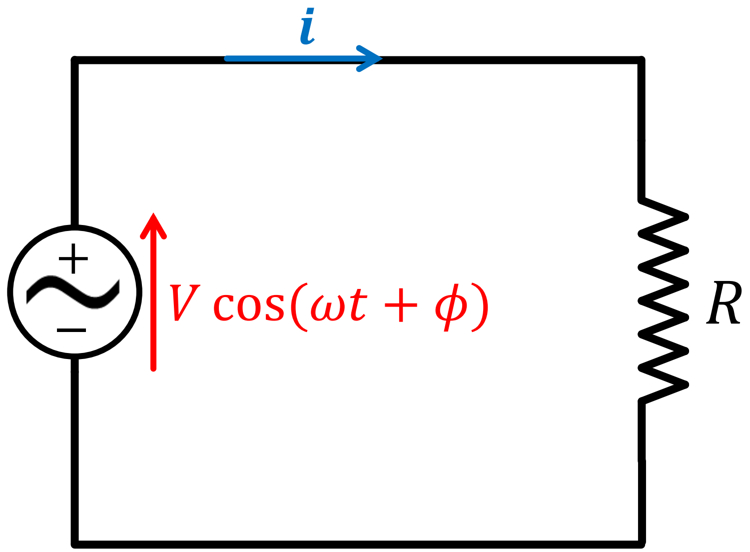
For example, current $\boldsymbol{i = 10\,cos(50t - \frac{\pi}{6})}$ may be represented in the phasor form as follows:

Numeric Form – $\boldsymbol{10\angle\frac{\pi}{6}}$

Visual Form –

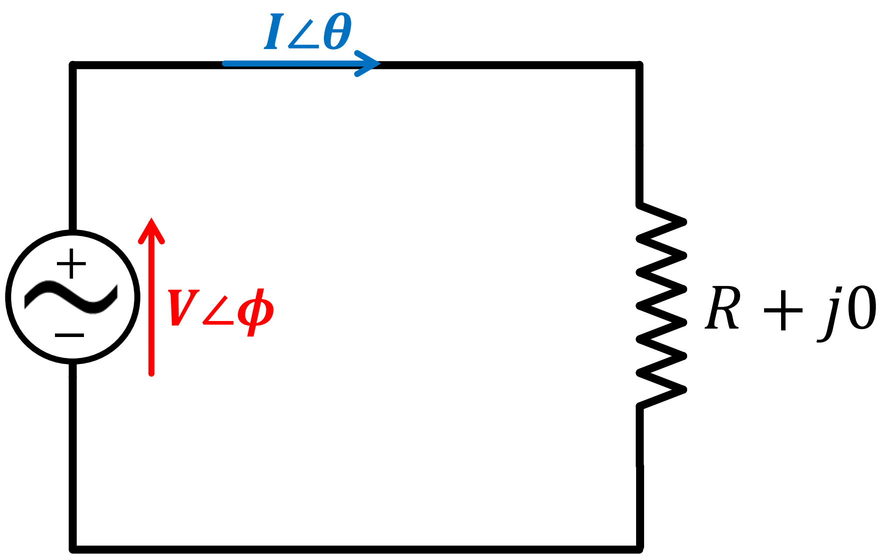


Phasors in Resistive Circuit



- Convert all variables to **phasors** or to **complex form**
- Apply the usual – **Kirchhoff's** & **Ohm's** Laws
- Solve the circuit like you did earlier – only difference being you are **now using complex numbers**!

Phasors in Resistive Circuit



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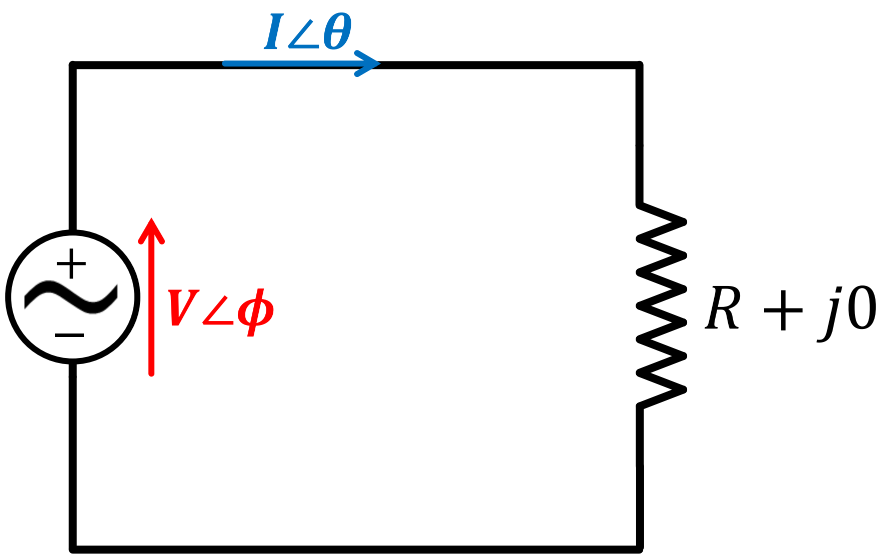
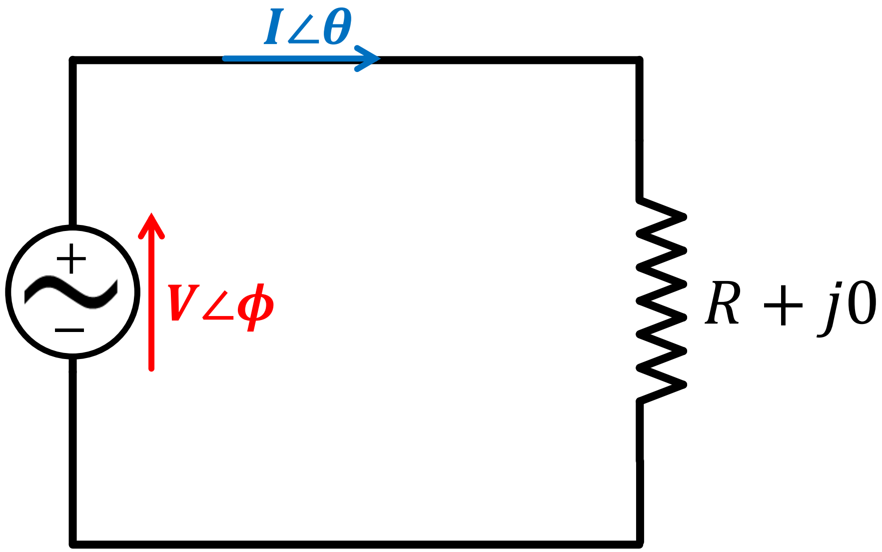
Phasors in Resistive Circuit



$$v = iR$$
$$V\angle\phi = IR\angle\theta$$
$$I\angle\theta = \frac{V}{R}\angle\phi$$

- ~~Convert all variables to **phasors** or to **complex form**~~
- ~~Apply the usual – **Kirchhoff's** & **Ohm's** Laws~~
- Solve the circuit like you did earlier – only difference being you are **now using complex numbers**!

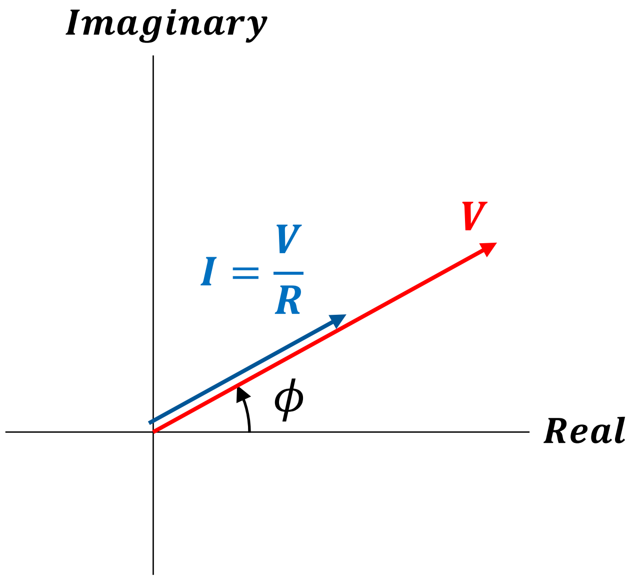
Phasors in Resistive Circuit

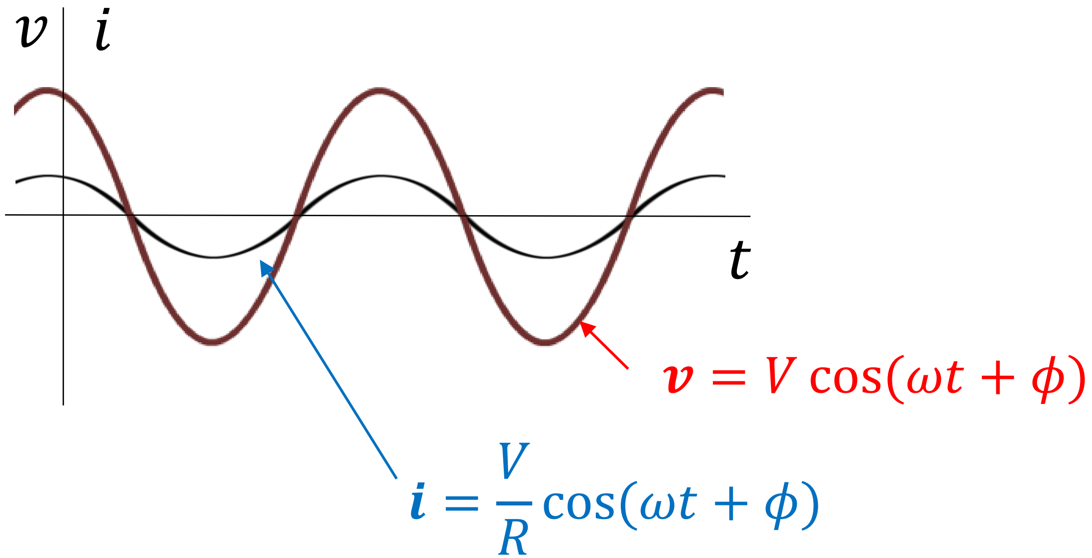


$$\boldsymbol{v = iR}$$

$$\boldsymbol{V\angle\phi = IR\angle\theta}$$

$$\boldsymbol{I\angle\theta = \frac{V}{R}\angle\phi}$$





Why Phasors?

Phasors are not very useful for purely resistive circuits!

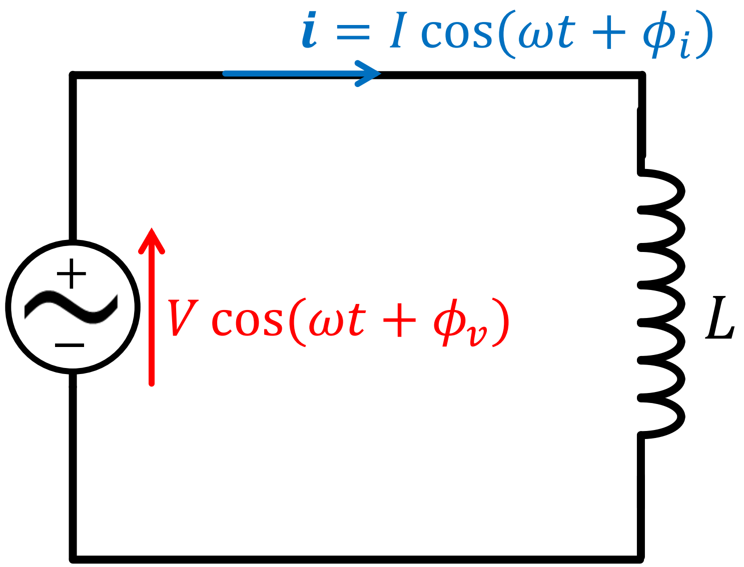
In resistive circuits, as there is no storage of energy in the resistive element, the current is always in phase with the voltage

But what about reactive elements?

Due to energy storage (and release) from inductors and capacitors, current is not in phase with voltage

This is where phasors come in handy – lets you avoid solving tedious differential equations

Phasors in Inductive Circuit



We know for an inductor: $\boldsymbol{v = L\frac{di}{dt}}$

$$\boldsymbol{v} = V\cos(\omega t + \phi_v) = \boldsymbol{Ve^{j(\omega t+\phi_v)}}$$

$$\boldsymbol{i} = I\cos(\omega t + \phi_i) = \boldsymbol{Ie^{j(\omega t+\phi_i)}}$$

Applying this:

$$Ve^{j(\omega t+\phi_v)} = L\frac{d(Ie^{j(\omega t+\phi_i)})}{dt}$$

$$Ve^{j(\omega t+\phi_v)}dt = Ld(Ie^{j(\omega t+\phi_i)})$$

$$V\int e^{j(\omega t+\phi_v)}dt = LI\int d(e^{j(\omega t+\phi_i)})$$

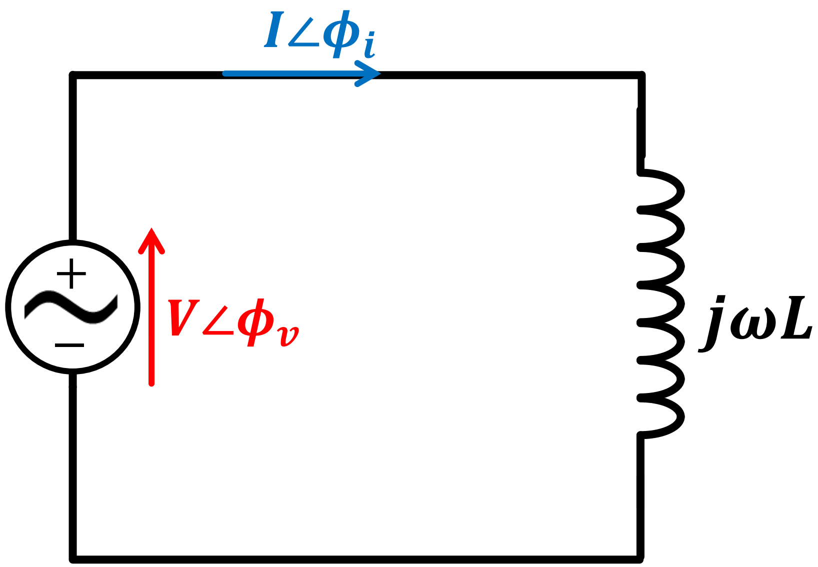
$$V\frac{e^{j(\omega t+\phi_v)}}{j\omega} = LIe^{j(\omega t+\phi_i)}$$

$$Ve^{j(\omega t+\phi_v)} = \mathrm{j}\omega LIe^{j(\omega t+\phi_i)}$$

$$\boldsymbol{v = j\omega Li}$$

You do not need to learn calculus here – there is an easy way!

Phasors in Inductive Circuit



Convert inductance to complex form

Solve using Ohm's & Kirchhoff's Laws

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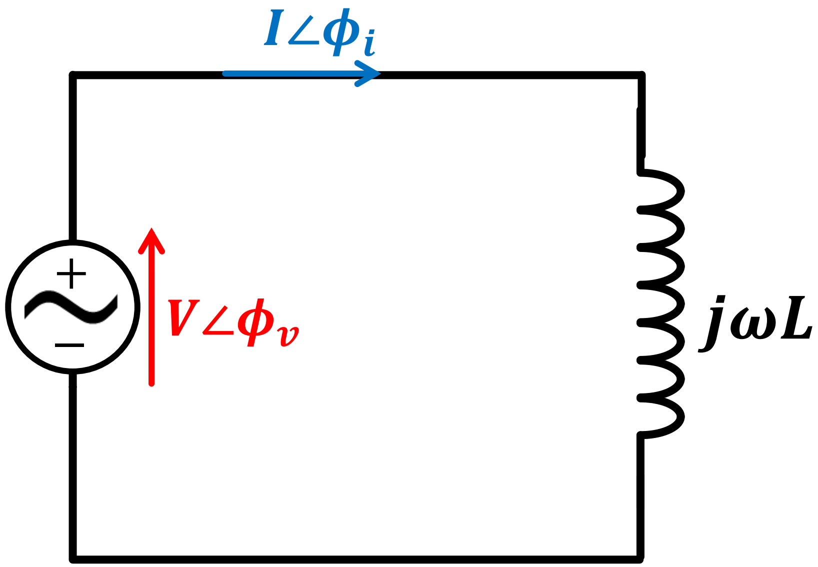
Phasors in Inductive Circuit



Convert inductance to complex form

Solve using Ohm's & Kirchhoff's Laws

Ohm's Law:

$$V = IR$$

But this needs to be generalised to incorporate complex "resistance" – **reactance** – symbol X

$$v = iX$$

$$V\angle\phi_v = I\angle\phi_i X$$

$$V\angle\phi_v = I\angle\phi_i j\omega L$$

$$\frac{V}{j\omega L}\angle\phi_v = I\angle\phi_i$$

Now remember complex number division:

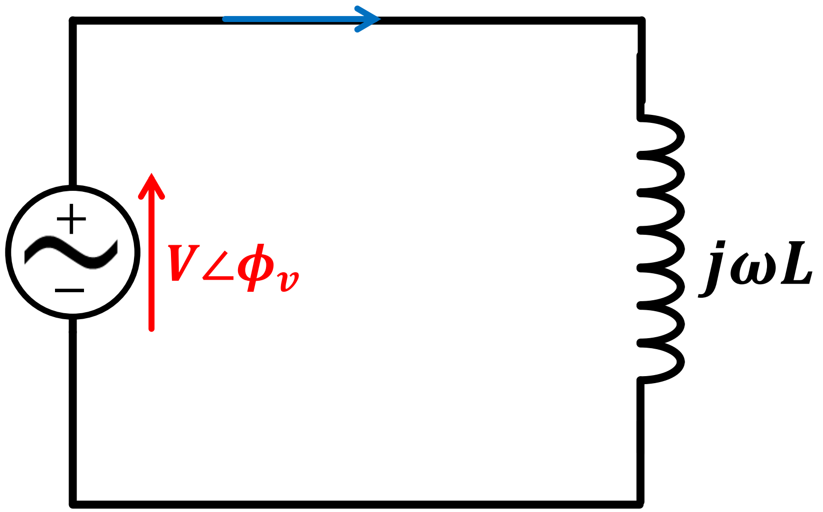
$$\left[\frac{V}{\omega L}\angle\phi_v\right] \div j1 = I\angle\phi_i$$

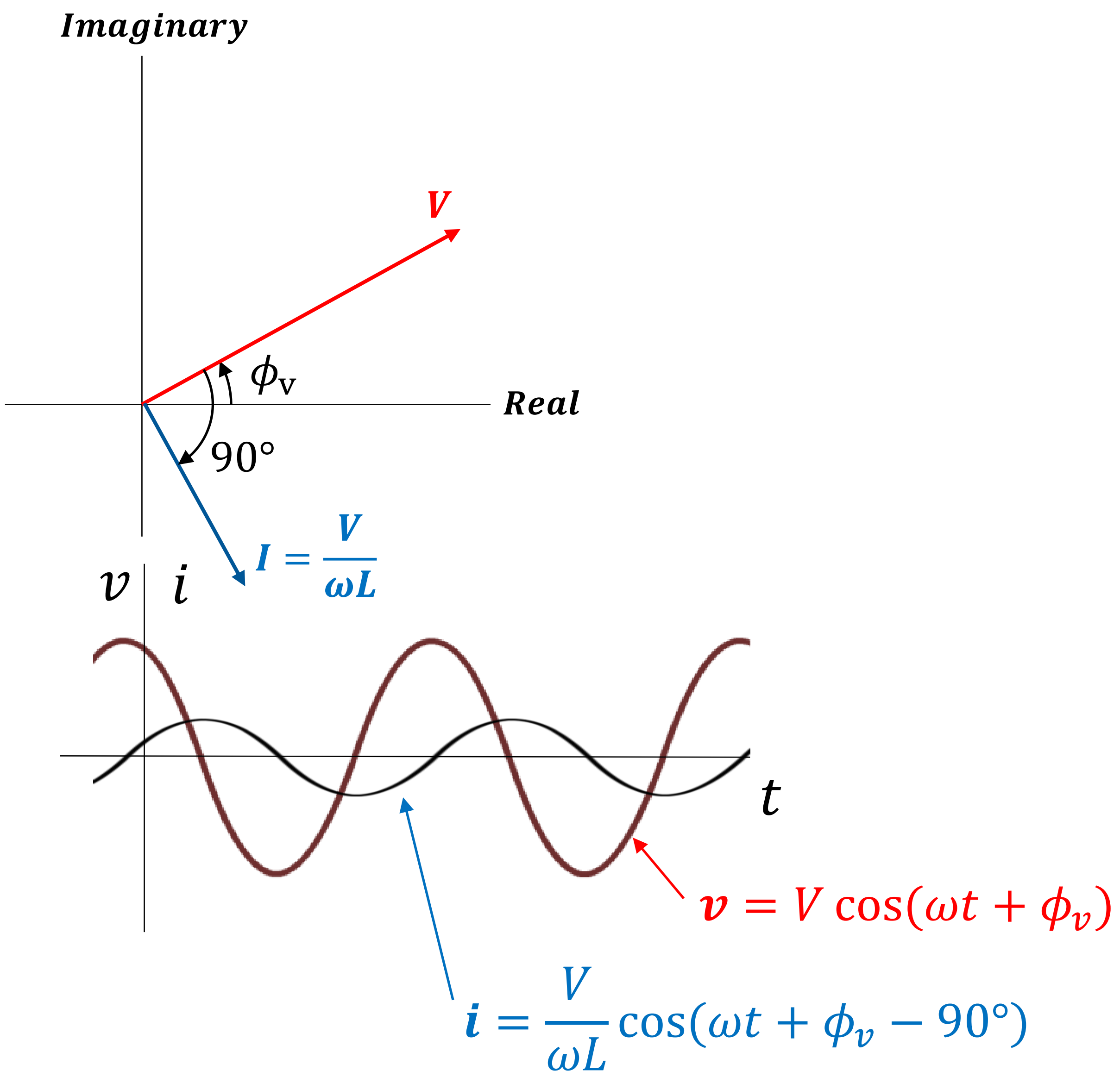
$$\left[\frac{V}{\omega L}\angle\phi_v\right] \div 1\angle 90° = I\angle\phi_i$$

$$\boldsymbol{\frac{V}{\omega L}\angle(\phi_v - 90°) = I\angle\phi_i}$$

Phasors in Inductive Circuit

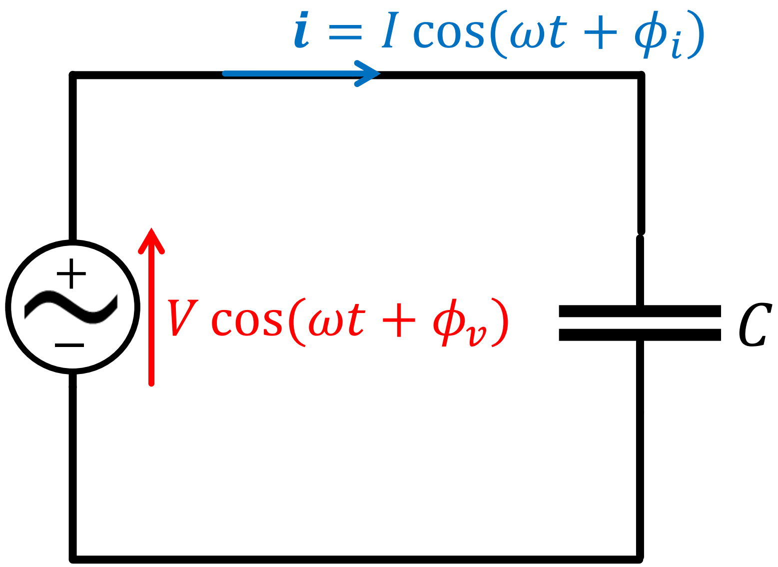
$$\boldsymbol{I\angle\phi_i = \frac{V}{\omega L}\angle(\phi_v - 90°)}$$





In **purely inductive** circuit, the **current** **LAGS** **voltage** by 90° or $\frac{\pi}{2}$ *radians*

Phasors in Capacitive Circuit



We know for a capacitor: $\boldsymbol{i = C\frac{dv}{dt}}$

$$\boldsymbol{v} = V\cos(\omega t + \phi_v) = \boldsymbol{Ve^{j(\omega t + \phi_v)}}$$

$$\boldsymbol{i} = I\cos(\omega t + \phi_i) = \boldsymbol{Ie^{j(\omega t + \phi_i)}}$$

Applying this:

$$i = C\frac{d(Ve^{j(\omega t + \phi_v)})}{dt}$$

$$i = CV\frac{d(e^{j(\omega t + \phi_v)})}{dt}$$

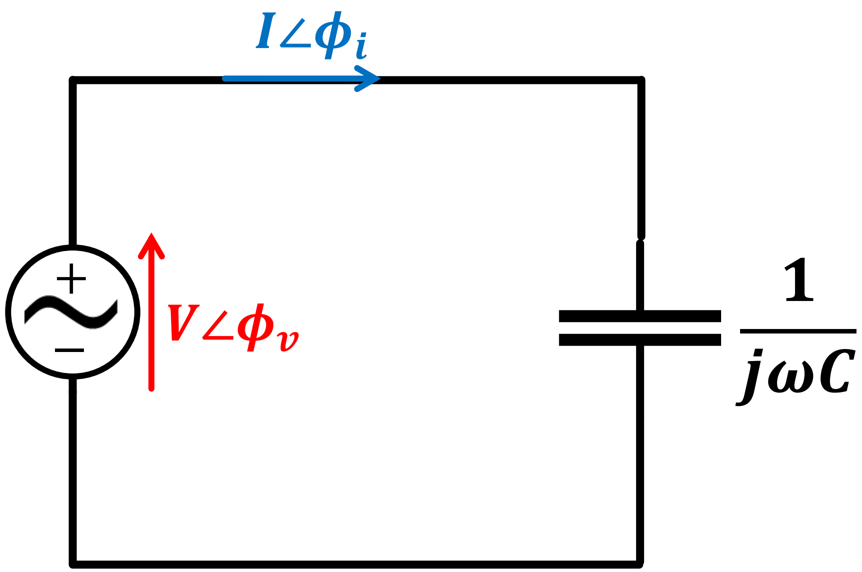
$$i = j\omega CVe^{j(\omega t + \phi_v)}$$

$$\frac{1}{j\omega C}i = Ve^{j(\omega t + \phi_v)}$$

$$\boldsymbol{v = \frac{1}{j\omega C}i}$$

You do not need to learn calculus here – there is an easy way!

Phasors in Capacitive Circuit



Convert capacitance to reactance

Solve using Ohm's & Kirchhoff's Laws

We know for a capacitor: $\boldsymbol{i = C\frac{dv}{dt}}$

$$\boldsymbol{v} = V\cos(\omega t + \phi_v) = \boldsymbol{Ve^{j(\omega t + \phi_v)}}$$

$$\boldsymbol{i} = I\cos(\omega t + \phi_i) = \boldsymbol{Ie^{j(\omega t + \phi_i)}}$$

Applying this:

$$i = C\frac{d(Ve^{j(\omega t + \phi_v)})}{dt}$$

$$i = CV\frac{d(e^{j(\omega t + \phi_v)})}{dt}$$

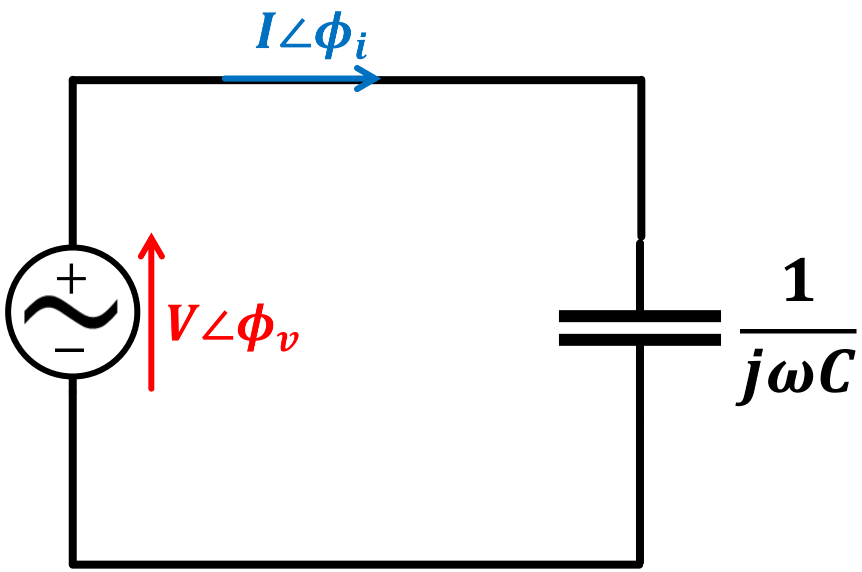
$$i = j\omega CVe^{j(\omega t + \phi_v)}$$

$$\frac{1}{j\omega C}i = Ve^{j(\omega t + \phi_v)}$$

$$\boldsymbol{v = \frac{1}{j\omega C}i}$$

You do not need to learn calculus here – there is an easy way!

Phasors in Capacitive Circuit



Ohm's Law:

$$V = IR$$

But this needs to be generalised to incorporate complex "resistance" – **reactance** – symbol X

$$v = iX$$

$$V\angle\phi_v = I\angle\phi_i X$$

$$V\angle\phi_v = I\angle\phi_i \frac{1}{j\omega C}$$

$$Vj\omega C\angle\phi_v = I\angle\phi_i$$

Now remember complex number multiplication:

$$[V\omega C\angle\phi_v] \times j1 = I\angle\phi_i$$

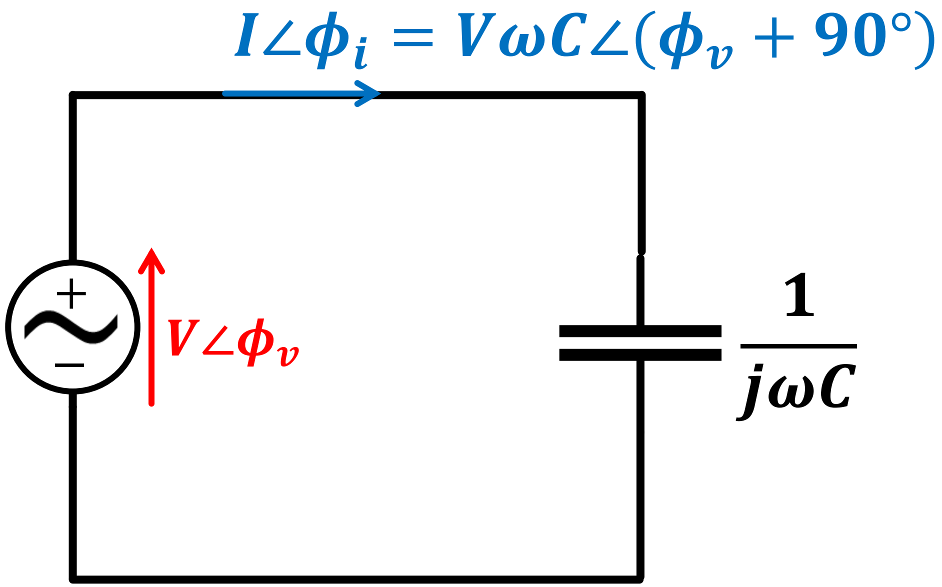
$$[V\omega C\angle\phi_v] \times 1\angle 90° = I\angle\phi_i$$

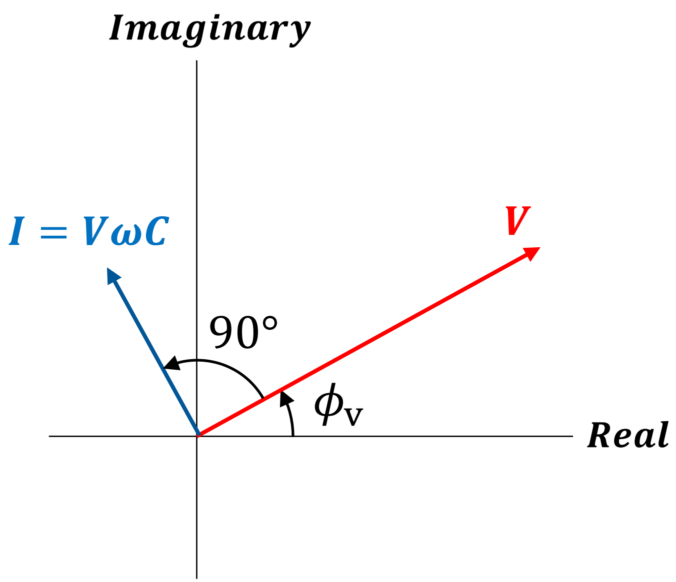
$$\boldsymbol{V\omega C\angle(\phi_v + 90°) = I\angle\phi_i}$$

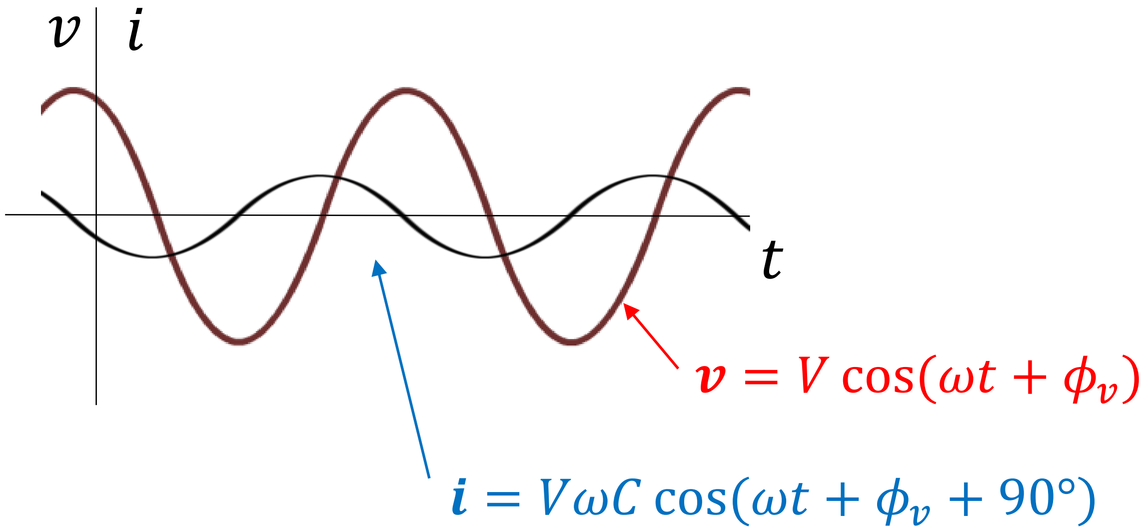
Convert capacitance to reactance

Solve using Ohm's & Kirchhoff's Laws

Phasors in Capacitive Circuit



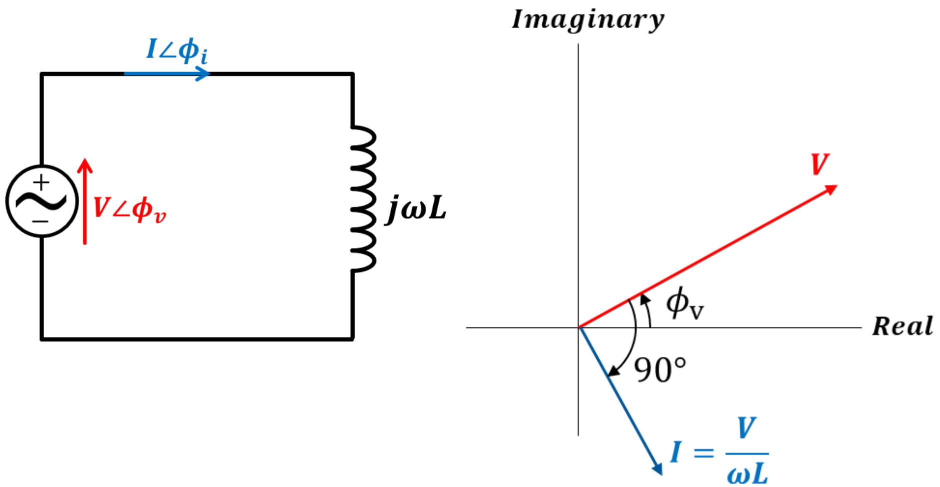




In **purely capacitive** circuit, the **current**

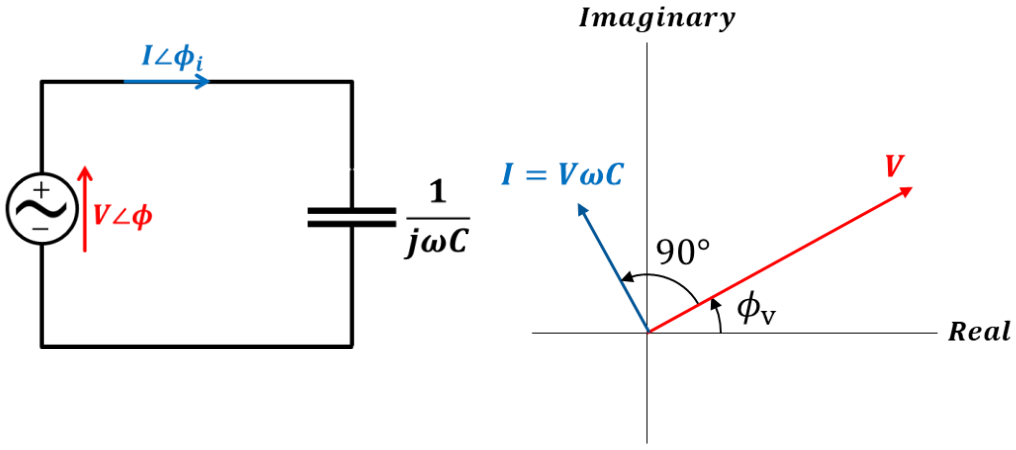
LEADS voltage by 90° or $\frac{\pi}{2}$ *radians*

Reactive Circuits – Summary



$$X_L = j\omega L$$

current **LAGS** **voltage** by 90° or $\frac{\pi}{2}\,radians$

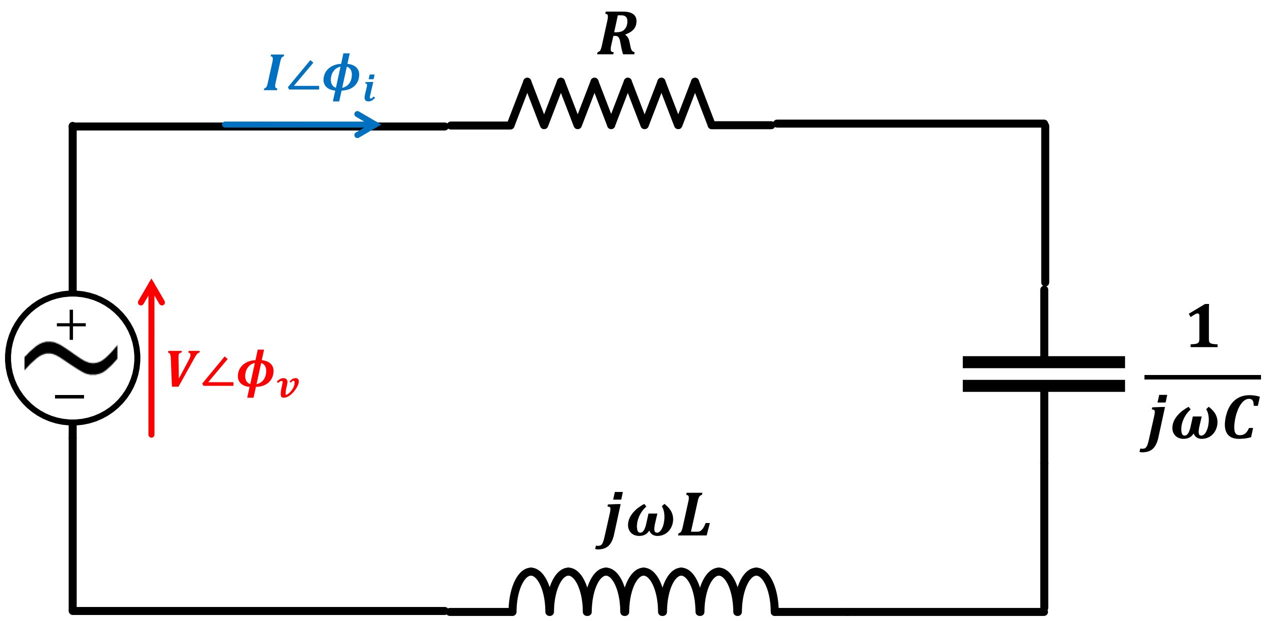


$$X_C = \frac{1}{j\omega C}$$

current **LEADS** **voltage** by 90° or $\frac{\pi}{2}\,radians$

CIVIL

The Real Circuit (Resistive + Reactive)



$$Impedance = Z = R + j\omega L + \frac{1}{j\omega C}$$

It is practically impossible to have a purely reactive circuit – any inductor or capacitor would have some **parasitic resistance** values

Remember we discussed Impedance in the previous lecture!

Impedance indicates how much a load "impedes" or **hinders** the **flow of current** through itself on application of a **set amount of voltage** across it

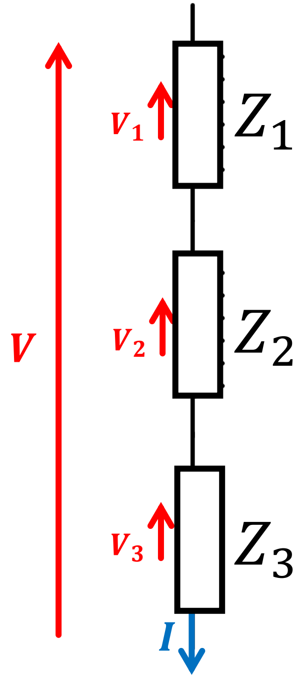
Generalisation of Resistance – now incorporates AC circuits as well

Ohm's Law still applies!

The Real Circuit (Resistive + Reactive)

Series

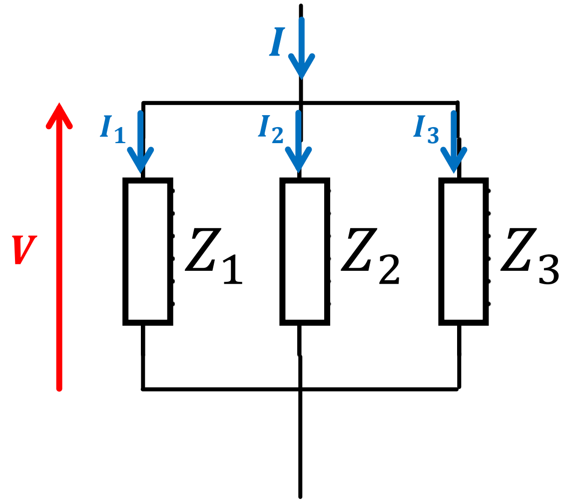
When two (or more) elements are connected together head-to-toe



$$Z = \sum Z_i$$

Parallel

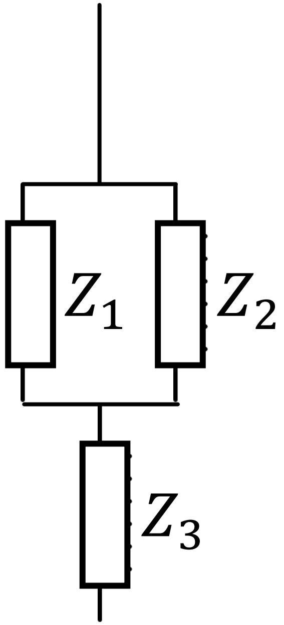
When two (or more) elements are connected head-to-head and toe-to-toe



$$\frac{1}{Z} = \sum \frac{1}{Z_i}$$

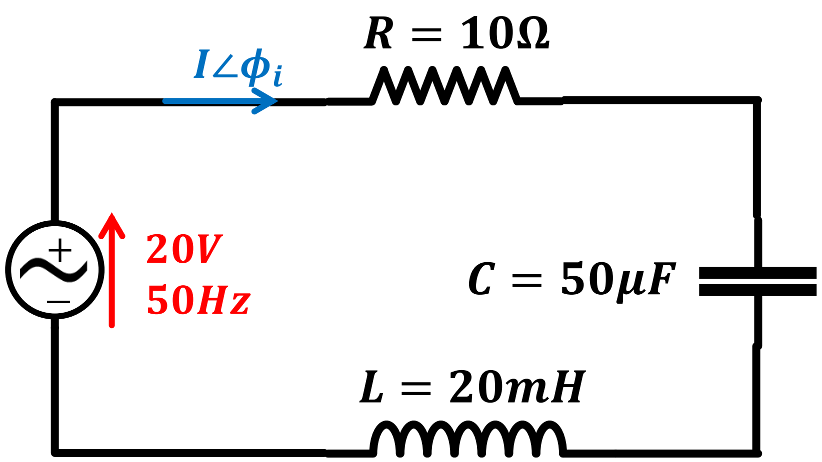
Series-Parallel

Combination of the both



Break the circuit up into series and parallel and solve individually

Example of Real Circuit



$$Z_R = 10\Omega$$

$$Z_L = j\omega L = j2\pi f L = j2 \times 3.14 \times 50 \times 20 \times 10^{-3} = j6.28\Omega$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j2\pi f C} = \frac{-j}{2 \times 3.14 \times 50 \times 50 \times 10^{-6}} = -j63.66\Omega$$

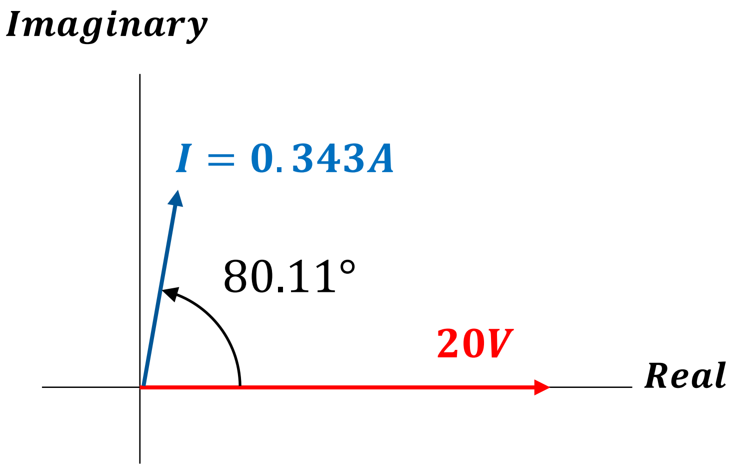
The **three elements** are clearly in **series**

$$Z = Z_R + Z_L + Z_C = 10 + j(6.28 - 63.69) = 10 - j57.38$$

Applying **Ohm's Law**, we need to divide V by Z, remember, for division, we need complex numbers in **polar form**

$$|Z| = \sqrt{10^2 + 57.41^2} = \sqrt{3395.91} = 58.24$$

$$\angle Z = \tan^{-1}\frac{-57.41}{10} = -80.11°$$



When no info on phase offset for voltage provided, no harm in setting it to 0°, makes calculations easier!

Applying **Ohm's Law**

$$I = \frac{V}{Z} = \frac{20\angle 0°}{58.24\angle -80.11°} = 0.343\angle 80.11°$$

Learning Outcomes

- Fundamentals of **Alternating Current – or AC**
  - DC v AC circuit study – waveforms a **function of time**!
  - **Sinusoidal** waveform – voltage & current
  - **Complex Numbers**
- AC circuits
  - **Phasor** study – simple way to solve time-varying circuits
  - Resistor, Inductor, Capacitor in phasor form - **CIVIL**
  - **Reactance** – Purely reactive circuits (just inductor/capacitor)
  - **Impedance** – Resistance & Reactance
- Power in AC circuits
  - **Active** v **Reactive** v **Apparent** Power
  - **Power Factor**
  - **Resonance**

Root Mean Square (RMS)

- In mathematics, the **root-mean-square** (or **RMS**) of a set of numbers x_i is defined as the square root of the arithmetic mean of the squares of the set

$$\underline{x} = \sqrt{\frac{x_1^2 + x_2^2 + x_3^2 + \cdots + x_n^2}{n}} = \sqrt{\frac{\sum x_i^2}{n}}$$

- When dealing with AC applications, the amplitude of voltage or current is seldom used (we will see shortly why – power)
- Hence, AC ammeters/voltmeters are invariably calibrated for RMS value – not peak/amplitude
- For all **sinusoidal waves**, the RMS value is $\frac{1}{\sqrt{2}} = 0.707$ times the amplitude
- It is much more convenient to make the **length of phasors** represent **RMS** instead of amplitude
- Going forward, we will deal with **only RMS values** when studying AC

$$\boldsymbol{RMS\ value\ of\ V = V_{rms} = \frac{|V|}{\sqrt{2}} = 0.707V}$$

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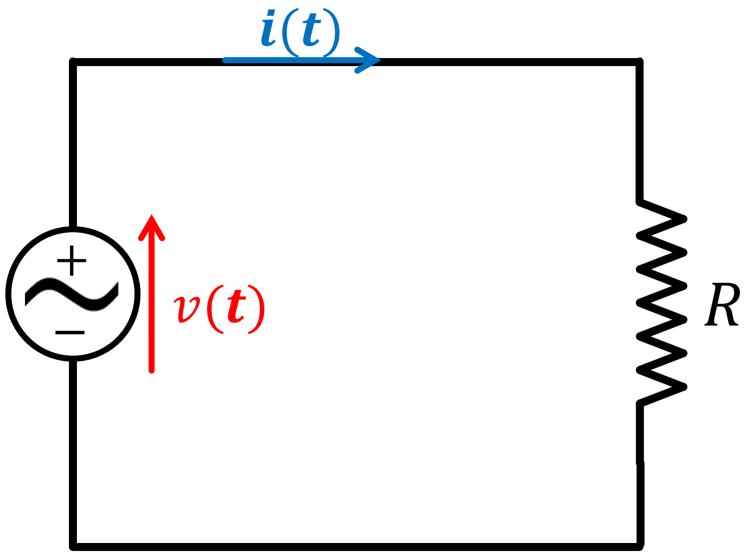
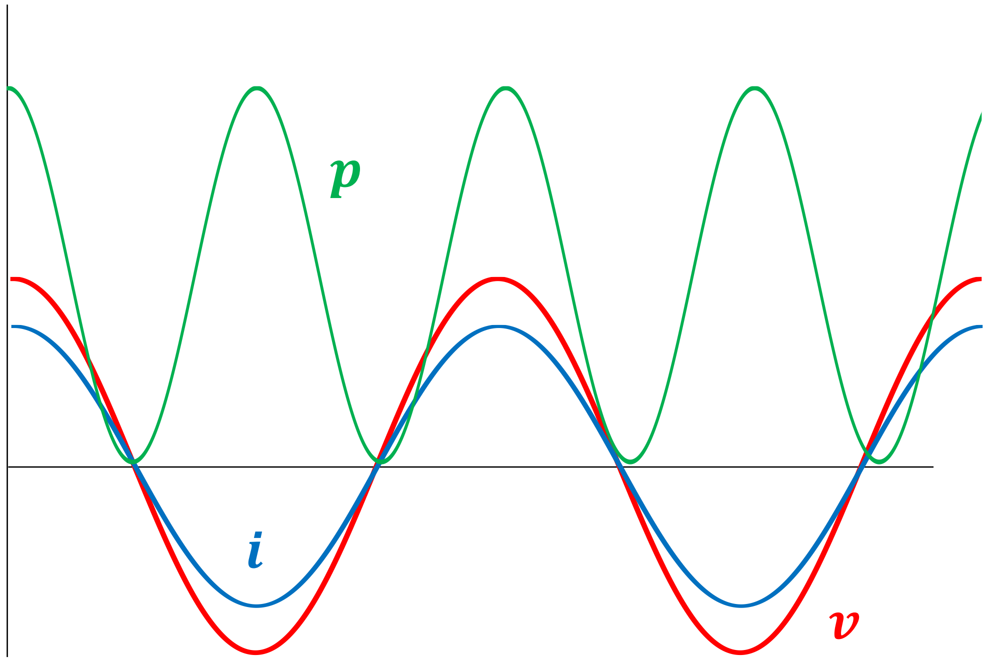
Power in Resistive Circuit





$$v(t) = V\cos\omega t$$

$$i(t) = I\cos\omega t$$

Instantaneous power

$$p(t) = v(t) \times i(t)$$

$$p(t) = V\cos\omega t \times I\cos\omega t$$

$$p(t) = \frac{VI}{2}(1+\cos 2\omega t) = \frac{I^2R}{2}(1+\cos 2\omega t) = \frac{V^2}{2R}(1+\cos 2\omega t)$$

Average power – integrate over full cycle

$$P_{avg} = \int \frac{VI}{2}(1+\cos 2\omega t)$$

$$P_{avg} = \frac{VI}{2} + 0$$

$$P_{avg} = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} = V_{rms}I_{rms}$$

Power in Resistive Circuit

Proof (don't learn)

$$\begin{aligned}
P_{avg} &= \frac{1}{T}\int_0^T p(t)\,dt \\
&= \frac{1}{T}\int_0^T v(t)i(t)\,dt \\
&= \frac{1}{T}\int_0^T V\cos(\omega t) I\cos(\omega t)\,dt \\
&= \frac{1}{T}\int_0^T \frac{VI}{2}\{1+\cos(2\omega t)\}\,dt \\
&= \frac{1}{T}\int_0^T \frac{VI}{2}\,dt + \frac{1}{T}\int_0^T \frac{V_m I_m}{2}\{\cos(2\omega t)\}\,dt \\
&= \frac{VI}{2} - \frac{1}{\omega T}\int_0^{2\pi} \frac{V_m I_m}{2}\{\cos(2\omega t)\}\,d\omega t \\
&= \frac{VI}{2} = V_{rms} I_{rms}
\end{aligned}$$

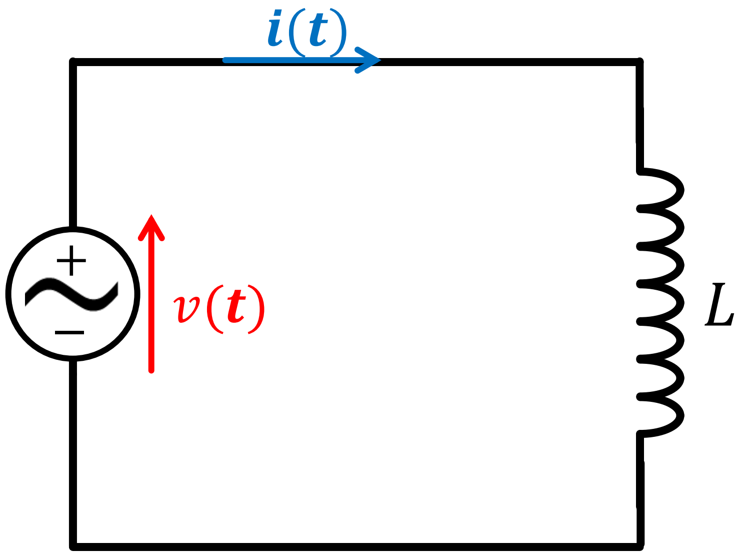
$$\boldsymbol{P_{avg} = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} = V_{rms} I_{rms}}$$

Remember that power in DC circuits $\boldsymbol{P = V_{dc} \times I_{dc}}$

Equivalently, the AC counterparts for $\boldsymbol{V_{dc}}$ **is** $\boldsymbol{V_{rms}}$ **and** $\boldsymbol{I_{dc}}$ **is** $\boldsymbol{I_{rms}}$

That is why we always use the RMS value of voltage and current

Power in Inductive Circuit



$$v(t) = V\cos\omega t$$

$$i(t) = I\sin\omega t$$

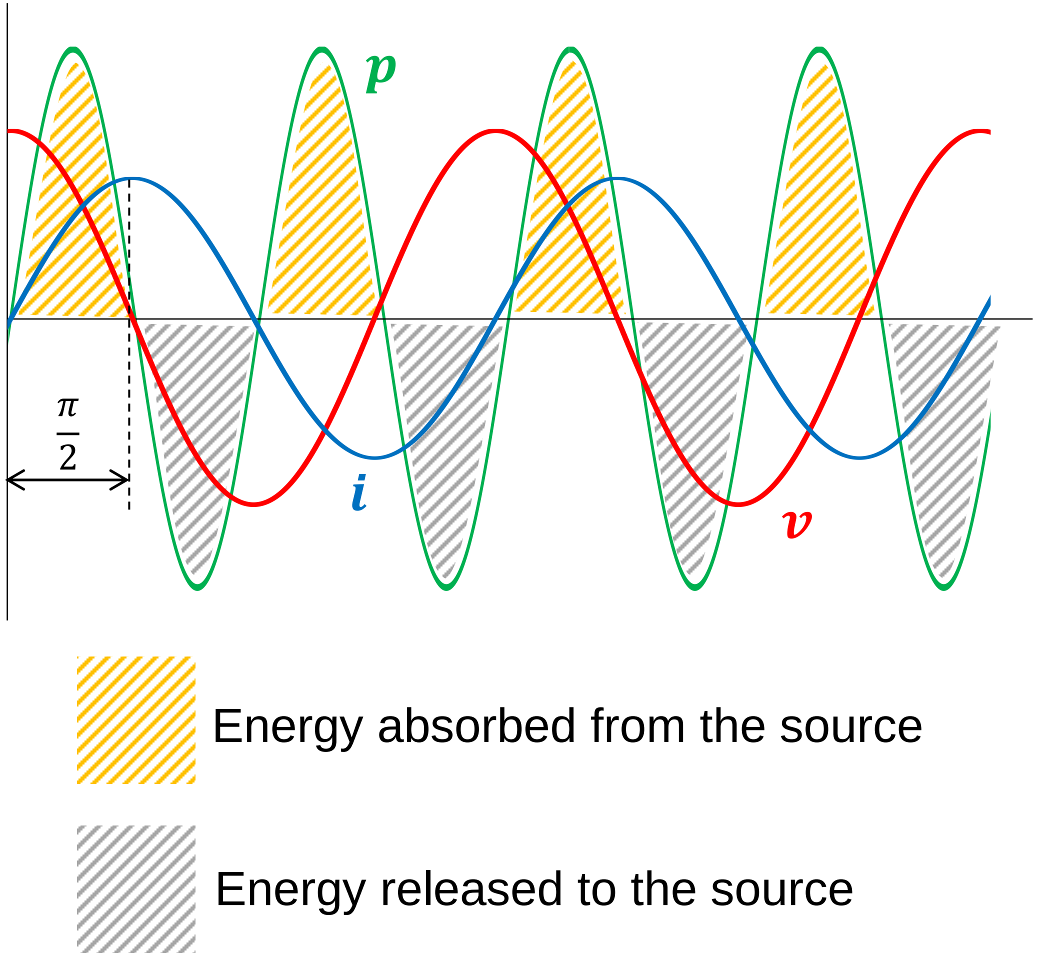
Do you know why?

Instantaneous power

$$p(t) = v(t) \times i(t)$$

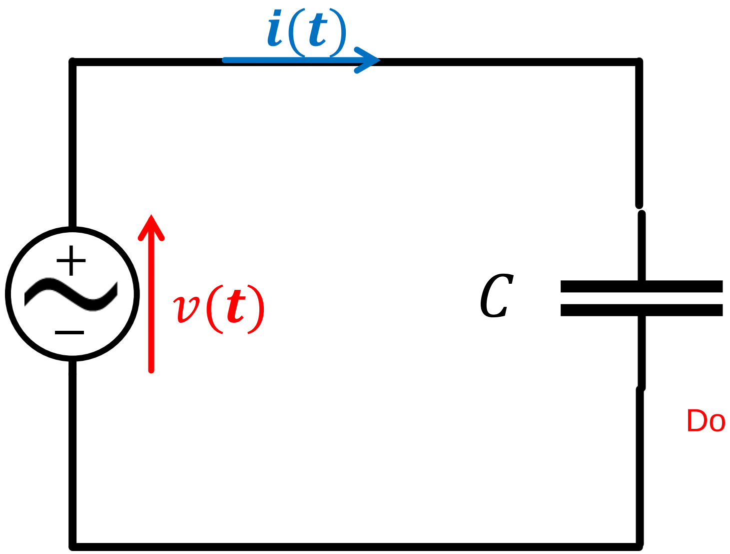
$$p(t) = V\cos\omega t \times I\sin\omega t$$

$$p(t) = \frac{VI}{2}\sin 2\omega t = \frac{\omega L I^2}{2}\sin 2\omega t = \frac{V^2}{2\omega L}\sin 2\omega t$$

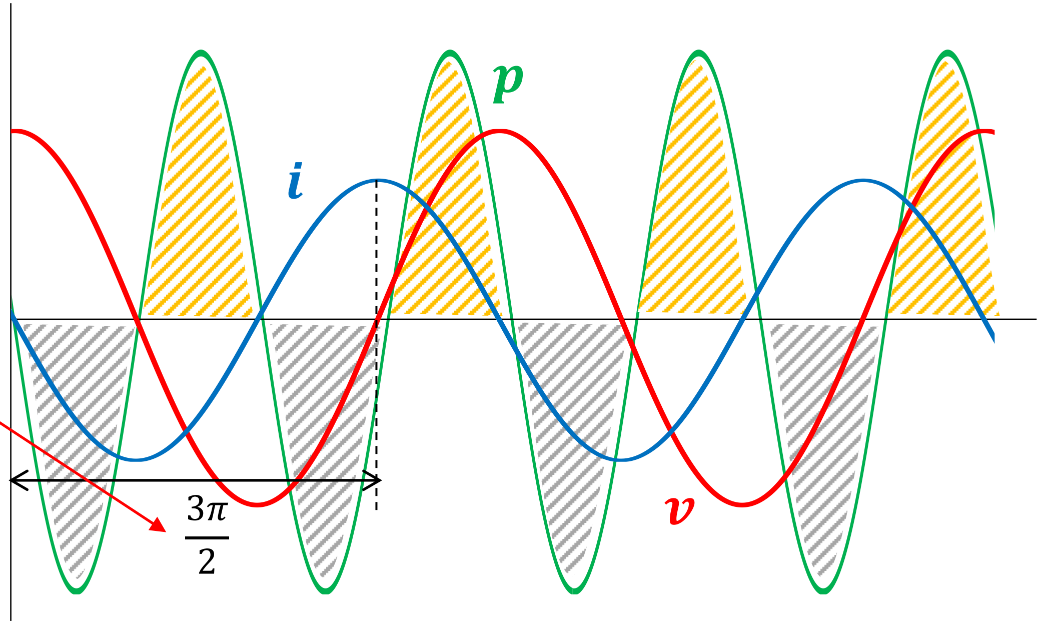


Average power is ZERO!

Power in Capacitive Circuit



Do you know why?



$v(t) = V\cos\omega t$

$i(t) = -I\sin\omega t$ Do you know why?

Energy absorbed from the source

Energy released to the source

Instantaneous power

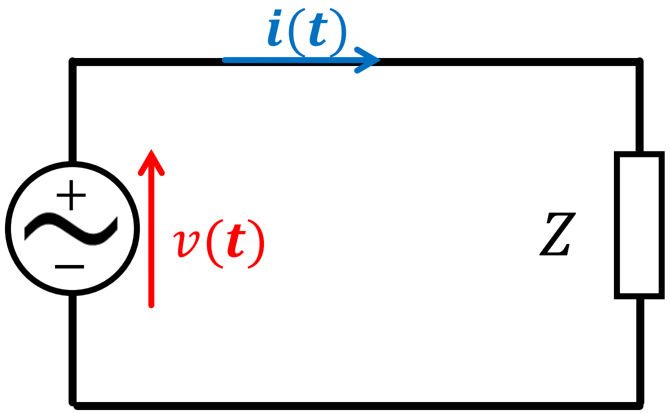
$$p(t) = v(t) \times i(t)$$

$$p(t) = -V\cos\omega t \times I\sin\omega t$$

$$p(t) = \frac{-VI}{2}\sin 2\omega t = \frac{I^2}{2\omega C}\sin 2\omega t = \frac{\omega C V^2}{2}\sin 2\omega t$$

Average power is ZERO!

Power in Real Circuit (Resistive + Reactive)



$$\boldsymbol{v(t) = V \cos \omega t}$$

$$\boldsymbol{i(t) = I \cos(\omega t + \gamma)}$$

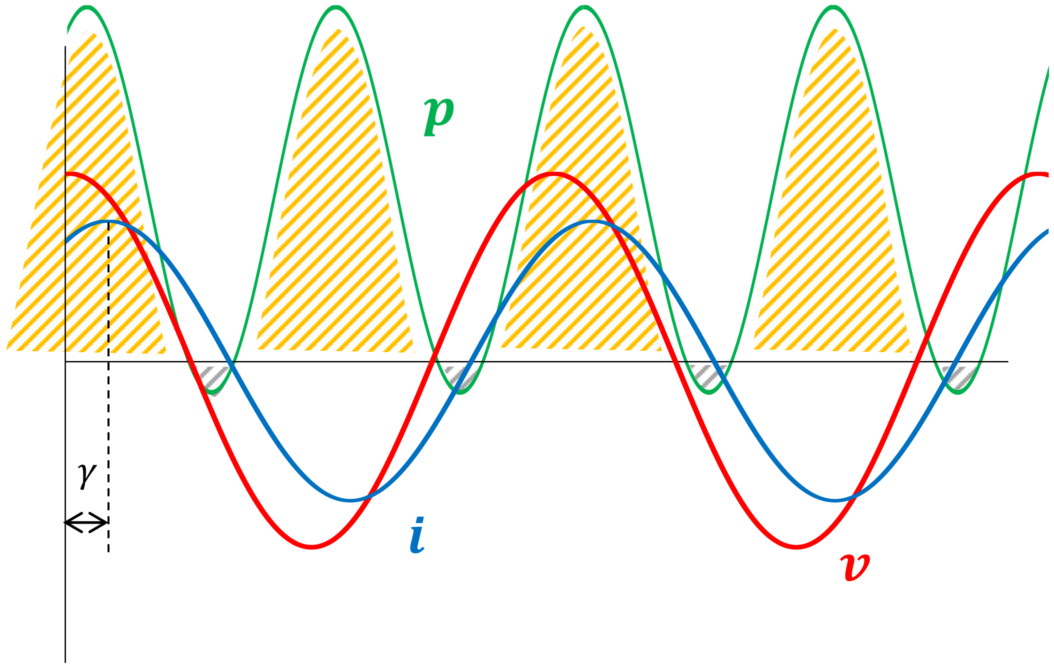
Instantaneous power

$$\boldsymbol{p(t) = v(t) \times i(t)}$$

$$\boldsymbol{p(t)} = V \cos \omega t \times I \cos(\omega t + \gamma)$$

$$p(t) = \frac{VI}{2}\{\cos(\omega t - \omega t - \gamma) + \cos(\omega t + \omega t + \gamma)\}$$

$$\boldsymbol{p(t) = \underbrace{\boxed{V_{rms} I_{rms} \cos \gamma}}_{\text{Average Power}} + \underbrace{\boxed{V_{rms} I_{rms} \cos(2\omega t + \gamma)}}_{\text{This term averages to zero over a cycle}}}$$

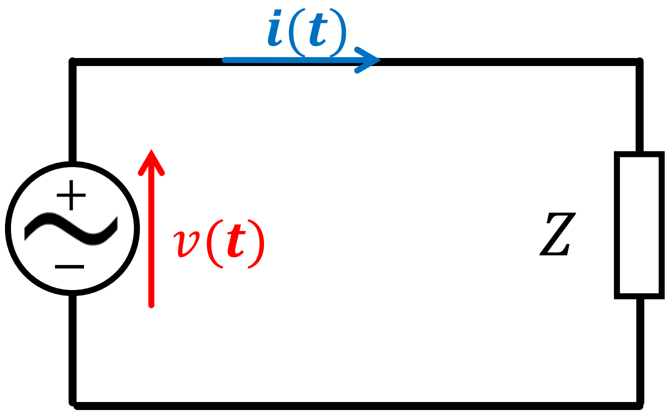


Energy absorbed from the source

Energy released to the source

Average power is $\boldsymbol{V_{rms} I_{rms} \boxed{\cos \gamma}}$ **Power Factor**

Power Factor



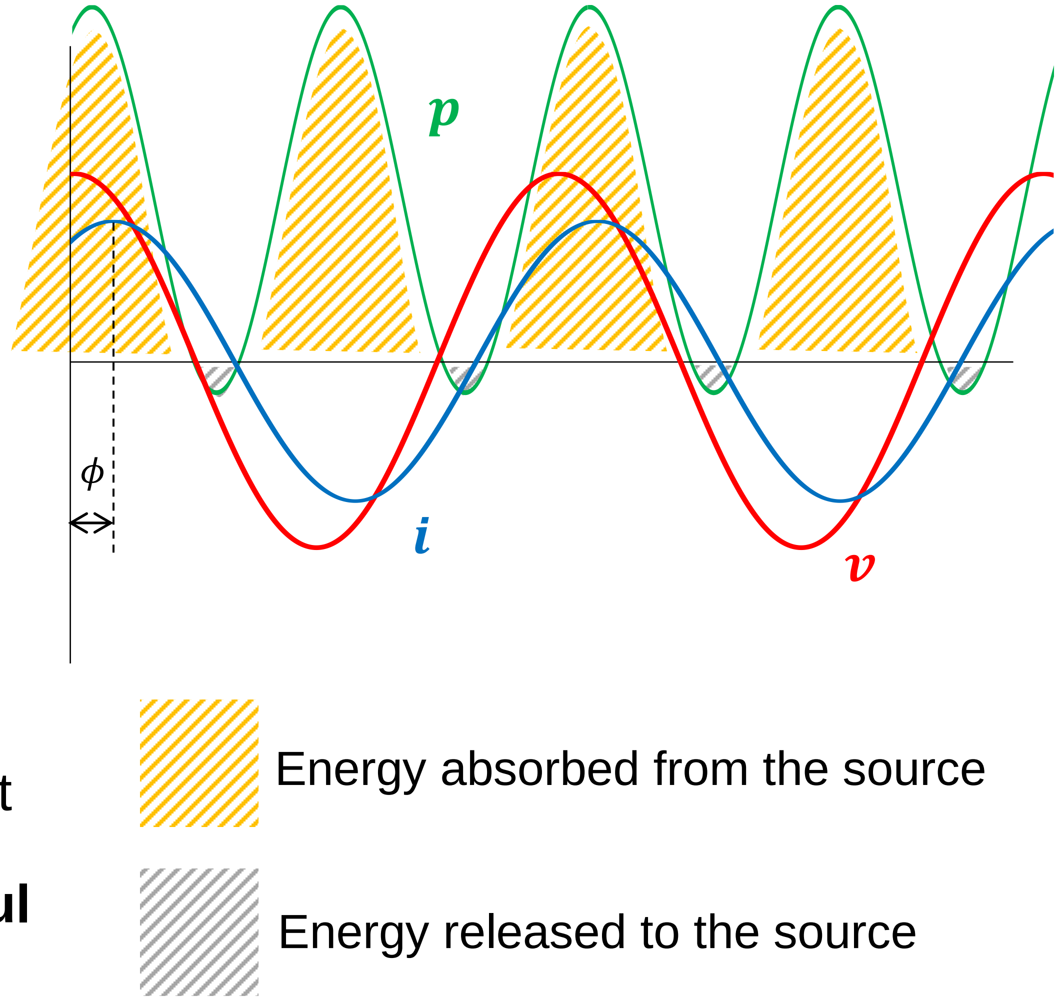
$$P_{avg} = V_{rms} I_{rms} \cos\gamma$$

$$\cos\gamma = Power\ Factor = PF$$

γ is the **phase deviation** between voltage & current

PF tells us **what fraction of the current does useful work**

Is it phase **advance/delay**? ***Does it matter?***



Power Factor

Purely Resistive Load R	$\gamma = 0°$ $\cos\gamma = 1$	All power consumed
Purely Reactive Load L or C	$\gamma = \pm 90°$ $\cos\gamma = 0$	No **real power** consumed
Real Inductive Load RL or RLC	$-90° < \gamma < 0°$ $0 < \cos\gamma < 1$	Part of **apparent power** consumed
Real Capacitive Load RC or RLC	$0° < \gamma < 90°$ $0 < \cos\gamma < 1$	

Active v Reactive v Apparent Power

Apparent Power (symbol S unit VA)

$$S = V_{rms} I_{rms}$$

- As the name suggests, this is the amount of power that appears to be flowing from source to load
- This is not the case as over a cycle, some (or all) of this power gets returned back to source
- As the power still flows (even if it is simply thrown back-forth between source and load), losses still occur
- A good circuit should have PF very close to unity
- However, AC equipment are rated for Apparent Power as it handles both used and unused power

Active Power (symbol P unit W)

$$P = V_{rms} I_{rms} \cos\gamma = V_{rms} I_{rms} PF = S \times PF$$

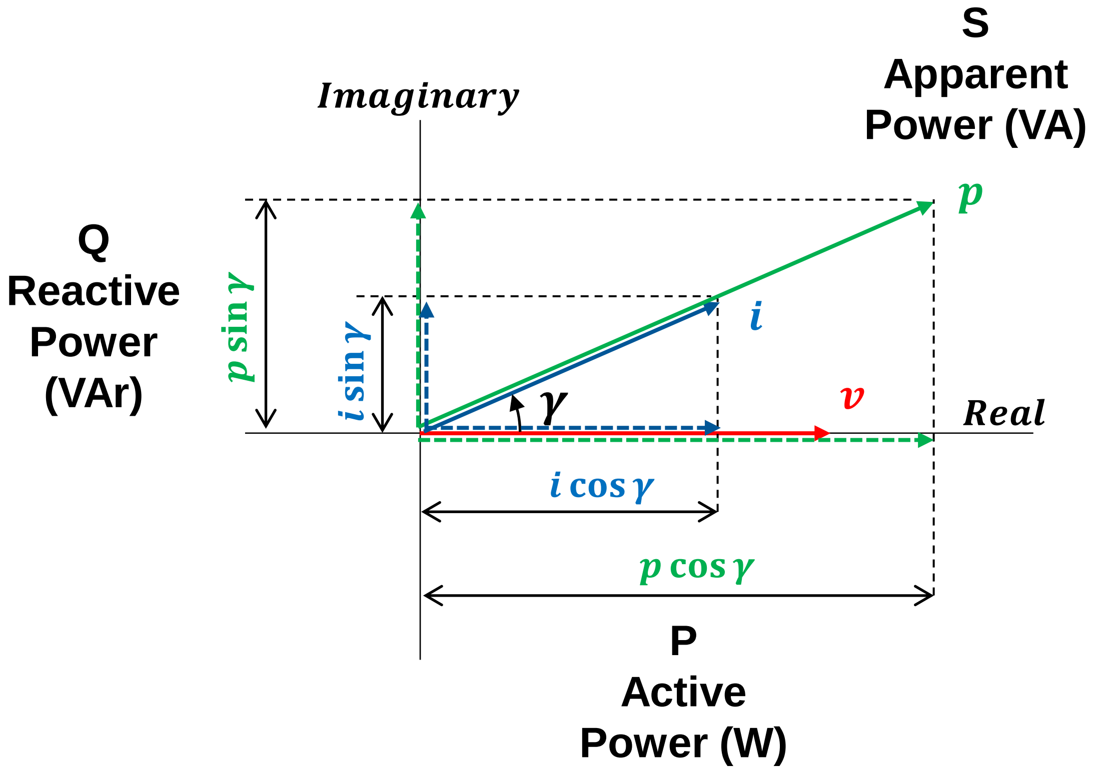
- This is the real power transferred to the load

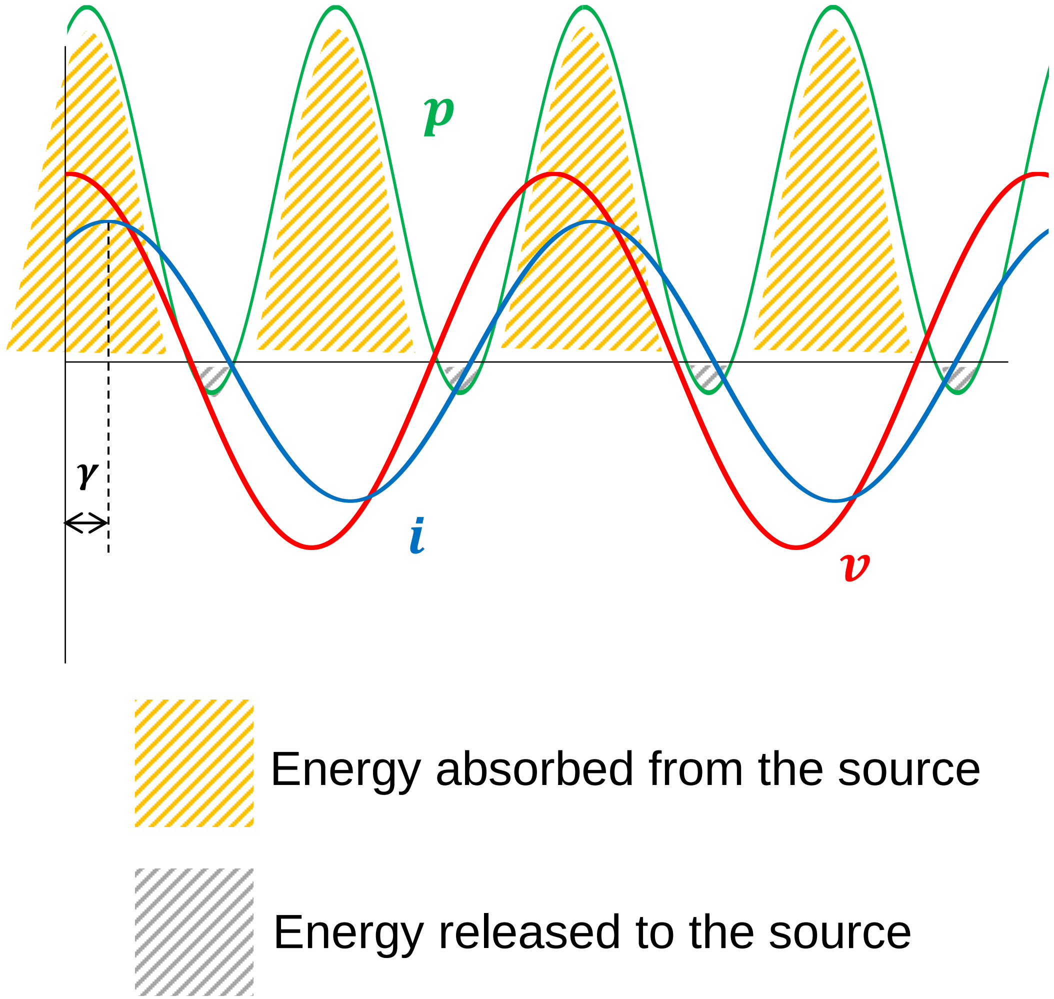
Reactive Power (symbol Q unit VAr)

$$P = V_{rms} I_{rms} \sin\gamma = V_{rms} I_{rms}\, sin\,\gamma = S\, sin\,\gamma$$

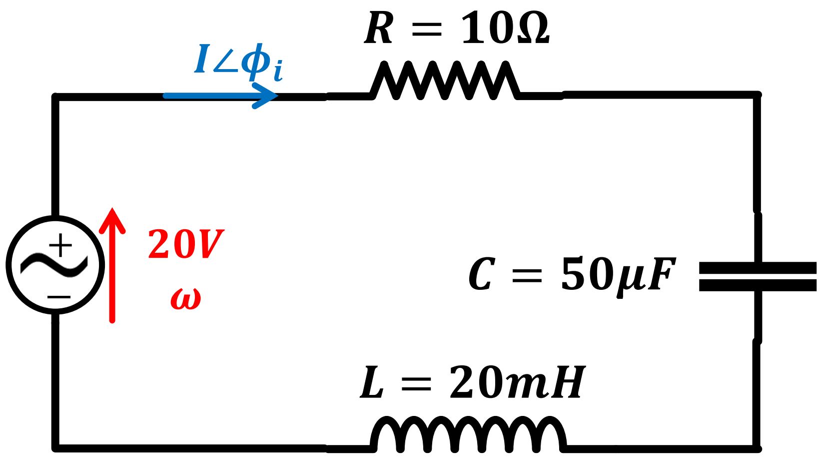
- This is the purely unused power exchanged between the source and load

Active v Reactive v Apparent Power





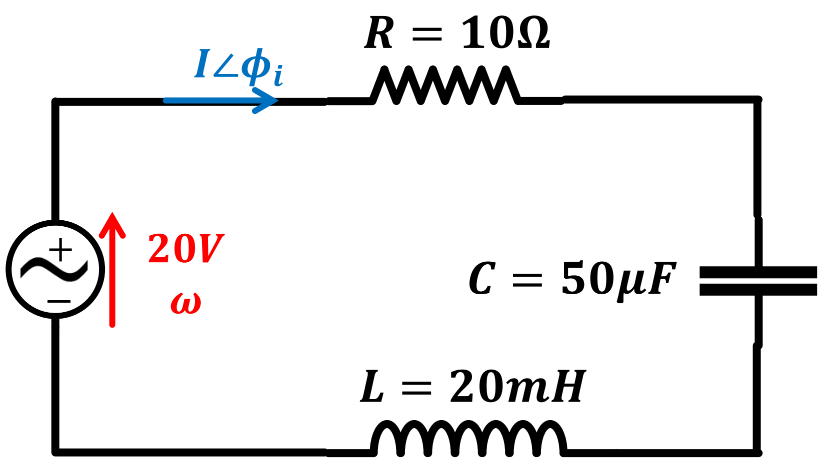
Resonance



- We have seen that inductor and capacitor **individually contribute** to delaying and advancing (respectively) the current waveform w/r/t the voltage
- When the inductance and capacitance value are equal (and opposite, inherently) they **nullify** each other – **Resonance**
- $Z_L = j\omega L$ **increases** with **increasing frequency**
- $Z_C = \frac{1}{j\omega C}$ **decreases** with **increasing frequency**

- We did this example earlier with frequency ($50\ Hz$), we saw that the **overall circuit was capacitive** (i.e., capacitance was overpowering inductance and resultant current was $80°$ leading)
- What happens if we **increase the frequency**?
- There will come a frequency when **inductance just matches capacitance** – this is **resonance**
- When this happens, you will be left with a **purely resistive circuit**, i.e., **overall impedance drops**!
- As you increase the frequency (from $50\ Hz$), you would see current rising gradually, then sharply at resonance, then again start falling

Resonance



$$Z_L = Z_C$$

$$j\omega L = \frac{1}{j\omega C}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$\omega = \frac{1}{\sqrt{20 \times 10^{-3} \times 50 \times 10^{-6}}} = \frac{1}{\sqrt{10^{-6}}}$$

$$\omega_{res} = 1000\frac{rad}{s}$$

Lets find out the current at resonant frequency and plot the phasor diagram

$$Z_L = j\omega_{res}L = j \times 1000 \times 20 \times 10^{-3} = j20\Omega$$

$$Z_C = \frac{1}{j\omega_{res}C} = \frac{-j}{1000 \times 50 \times 10^{-6}} = -j20\Omega$$

The **three elements** are clearly in **series**

$$Z = Z_R + Z_L + Z_C = 10 + j(20 - 20) = 10\Omega$$

Applying **Ohm's Law**

$$I = \frac{V}{Z} = \frac{20\angle 0°}{10\angle 0°} = 2\angle 0° A$$

$2\ A$ is significantly higher than $343\ mA$ that we calculated at $50\ Hz$ frequency

This is because the at resonance, inductive and capacitive impedances nullify each other

Example 1

A coil is connected to a 50 V AC supply at 400 Hz. If the current supplied to the coil is 200 mA and the coil has a resistance of 60 Ω , determine the value of inductance.

Like most practical forms of inductor, the coil in this example has both resistance *and* reactance. We can find the impedance of the coil from:

$$|Z| = \frac{V}{I} = \frac{50}{0.2} = 250\Omega$$

Since

$$|Z| = \sqrt{R^2 + X^2}$$

$$X = \sqrt{|Z|^2 - R^2}$$

$$X = \sqrt{250^2 - 60^2} = 243\Omega$$

Now since $XL = 2\pi f L$,

$$L = \frac{X}{2\pi f} = \frac{243}{100\pi} = 0.097H$$

Example 2

An AC load has a power factor of 0.8. Determine the active power dissipated in the load if it consumes a current of 2 A at 110 V.

Since active power

$$P = PF \times V_{rms} \times I_{rms}$$

$$P = 0.8 \times 110 \times 2$$

$$P = 176\ W$$

Example 3

A coil having an inductance of $150\ mH$ and resistance of $250\ \Omega$ is connected to a $115\ V$ $400\ Hz$ AC supply. Determine:

(a) the power factor of the coil
(b) the current I_{rms} taken from the supply
(c) the power dissipated as heat in the coil.

(a) First we must find the reactance of the inductor, X_L, and the impedance, Z, of the coil at 400 Hz.

$$X_L = 2\pi \times 400 \times 0.015 = 376\ \Omega$$

Thus

$$Z = R + jX_L = 250 + j376\ \Omega$$

The power factor is

$$\cos\gamma = \frac{R}{|Z|}$$

Since

$$|Z| = \sqrt{R^2 + X_L^2} = \sqrt{250^2 + 376^2} = 452\ \Omega$$

Thus $$\boldsymbol{\cos\gamma} = \frac{R}{|Z|} = \frac{250}{452} = \mathbf{0.553}$$

(b)

$$\boldsymbol{I_{rms}} = \frac{V_{rms}}{|Z|} = \frac{115}{452} = \mathbf{0.254\ A}$$

(c) The power dissipated as heat is the active power

$$P = V_{rms} I_{rms} \cos\gamma = 0.254 \times 115 \times 0.553$$

$$\boldsymbol{P = 16.15\ W}$$

Summary

- Fundamentals of **Alternating Current – or AC**
  - DC v AC circuit study – waveforms a **function of time**!
  - **Sinusoidal** waveform – voltage & current
  - **Complex Numbers**
- AC circuits
  - **Phasor** study – simple way to solve time-varying circuits
  - Resistor, Inductor, Capacitor in phasor form - **CIVIL**
  - **Reactance** – Purely reactive circuits (just inductor/capacitor)
  - **Impedance** – Resistance & Reactance
- Power in AC circuits
  - **Active** v **Reactive** v **Apparent** Power
  - **Power Factor**
  - **Resonance**

Attendance

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