Deflection of Beams

Lecture 3 – Alternative Loading Types

Department of Mechanical, Materials & Manufacturing Engineering

MMME2053 – Mechanics of Solids

University of Nottingham
UK | CHINA | MALAYSIA

Deflection of Beams

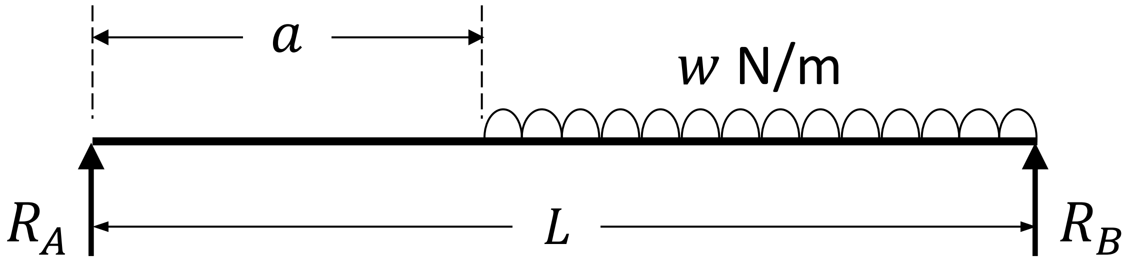
Learning Outcomes

1. Know how to derive the differential equation of the elastic line (i.e. deflection curve) of a beam (synthesis);
2. Employ Macaulay's method, also called the method of singularities, to determine bending moment expressions for beams where there are discontinuities in the bending moment distribution arising from discontinuous loading (application);
3. Be able to solve this equation by successive integration in order to yield the slope, $\frac{\mathrm{d}y}{\mathrm{d}x}$, and the deflection, y, of a beam at any position, x, along its span (application);
4. Recognise and use different singularity functions in the bending moment expression, relating to different loading conditions, including point loads, uniformly distributed loads and point bending moments (comprehension);
5. Employ Macaulay's method for statically indeterminate beam problems (application).

Alternative Loading Types

Uniformly Distributed Load

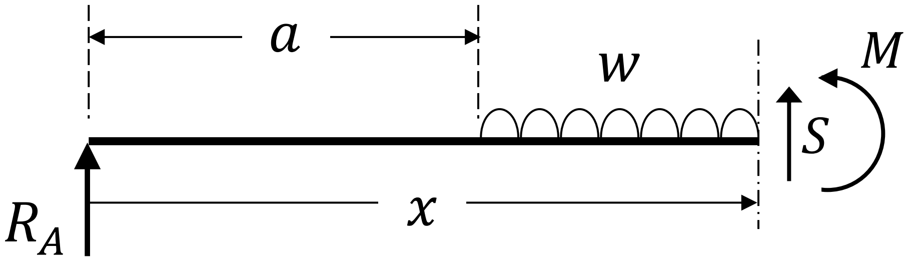
Consider a uniformly distributed load (UDL), w Nm^{-1}, acting over part of a beam's span.



The UDL runs from distance a from the origin (left-hand end) of the beam, all the way to the right-hand end of the beam.

A discontinuity occurs at the position where the UDL commences.

Drawing a free body diagram of the beam sectioned after the discontinuity:



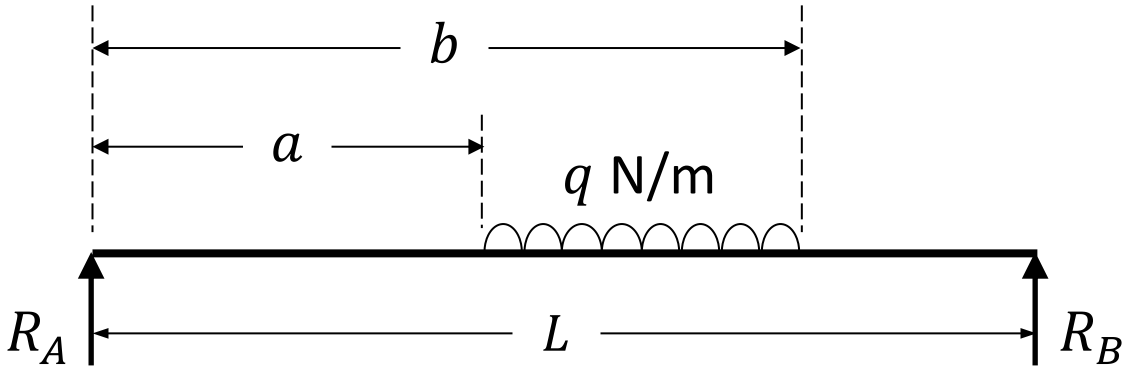
Taking moments about the section position in order to determine an expression for the bending moment, M:

$$M = R_A x - \frac{w\langle x - a\rangle^2}{2}$$

As can be seen, when taking moment equilibrium, the contribution of the UDL is calculated by first turning the UDL, w (unit Nm^{-1}), into a force (unit N) by multiplying it by the distance over which it acts, $x - a$ (unit m). This force is then turned into a moment by multiplying it by the distance to the centre position of the UDL, $\frac{\langle x-a\rangle}{2}$ (unit m).

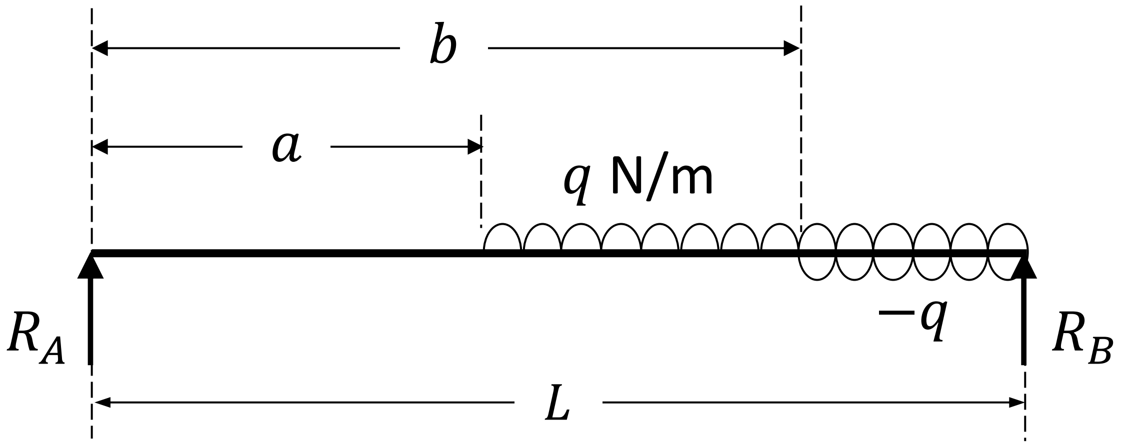
Discontinuous Uniformly Distributed Load

Consider a discontinuous uniformly distributed load (UDL), q Nm^{-1}, acting over part of a beam's span.



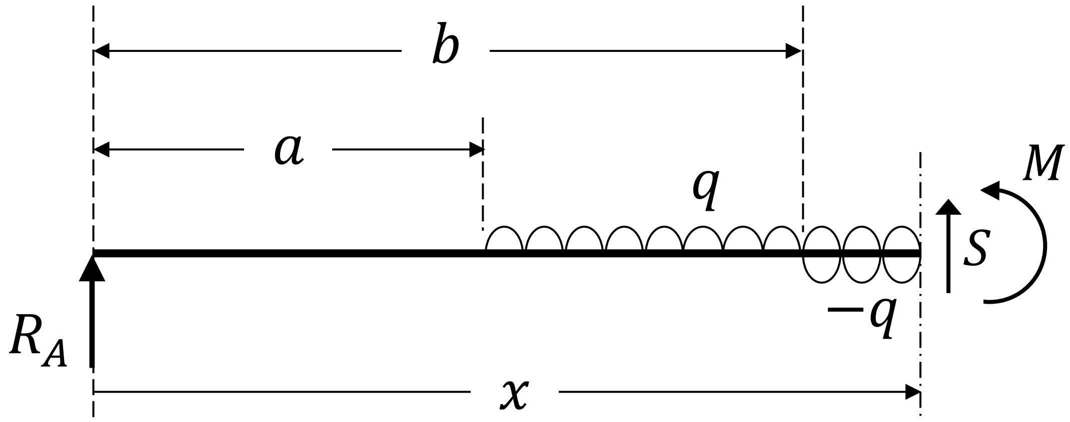
In this case, the UDL, q, runs from distance a from the origin (left-hand end) of the beam, up to distance b from the origin. Discontinuities therefore occur both at the position where the UDL commences, and at the position where it ends.

In order to progress towards a general bending moment expression for the beam, the applied discontinuous UDL, q, is extended to the end of the beam and an additional, negative, counterbalancing UDL superimposed over the newly extended part.



The extended applied UDL, q, and the added counterbalancing UDL, $-q$, mathematically cancel each other out and therefore this gives a statically equivalent system to the original partially extended (discontinuous) UDL.

Drawing a free body diagram of the beam sectioned after the discontinuity:

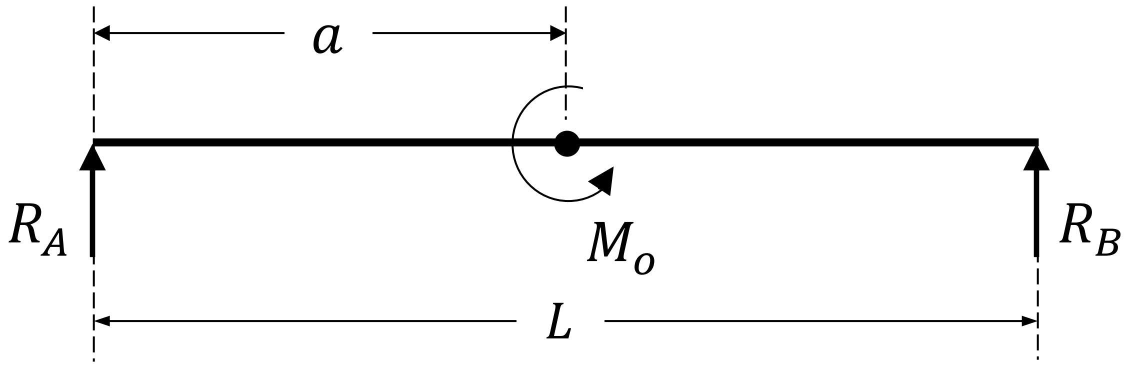


Taking moments about the section position in order to determine an expression for the bending moment, M:

$$M = R_A x + \frac{q\langle x-b\rangle^2}{2} - \frac{q\langle x-a\rangle^2}{2}$$

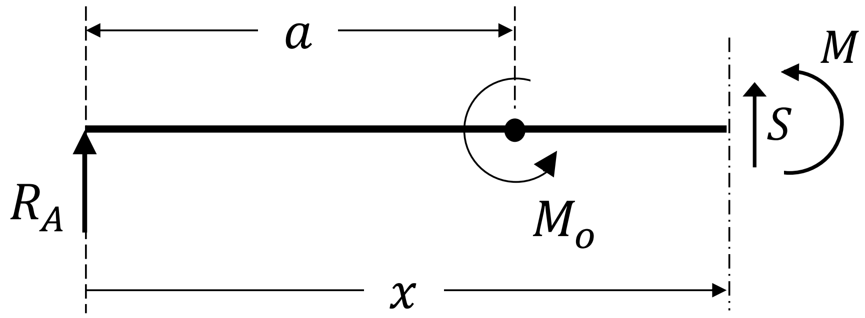
Point Bending Moment

Consider a point bending moment, M_o Nm, acting at a distance a from the left-hand side of a beam.



This point bending moment gives rise to a discontinuity in the bending moment expression.

Drawing a free body diagram of the beam sectioned after the discontinuity:



Taking moments about the section position in order to determine an expression for the bending moment, M:

$$M = R_A x - M_o\langle x - a\rangle^0$$

Note that the form of the discontinuity function for the point bending moment is the same as for a point load, except that the bracketed length is raised to the power zero. This is simply a mathematical convenience for facilitating Macaulay's method whilst maintaining the correct units of the moment, M_o. I.e. if a position in the beam where $x < a$ is considered for evaluation, then the contents of the Macaulay brackets is negative and the entire term set to zero. However, if a position in the beam where $x > a$ is considered for evaluation, then the contents of the Macaulay brackets is positive, and the term is included – but as the length term, $\langle x - a\rangle^0$, is raised to the power of zero, it becomes 1, and so the term simplifies to M_o.

Summary of the Discontinuity Functions

When developing the bending moment expression for a beam with load discontinuities, the singularity functions used for each different type of load are:

Load Type	Singularity Function
Point Load, P	$P\langle x-a\rangle$
Continuous UDL, w – single discontinuity	$\frac{w\langle x-a\rangle^2}{2}$
Discontinuous UDL, q – double discontinuity	$\frac{q\langle x-b\rangle^2}{2}-\frac{q\langle x-a\rangle^2}{2}$
Point Bending Moment, M_o	$M_o\langle x-a\rangle^0$

Note that in these singularity functions, the exponent is 1 for a point load, 2 for a UDL and 0 for a point bending moment.

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