University of Nottingham
Department of Mechanical Engineering

MM2MS3 Mechanics of Solids 3
Exercise Sheet 2 – Asymmetrical Bending Solutions

1. For the section shown in Figure Q1, determine:

   (a) The position of the Centroid, C
   (b) 2nd Moments of Area and Product Moment of Area about the x-y axes through C
   (c) The Principal 2nd Moments of Area
   (d) The directions of the Principal Axes

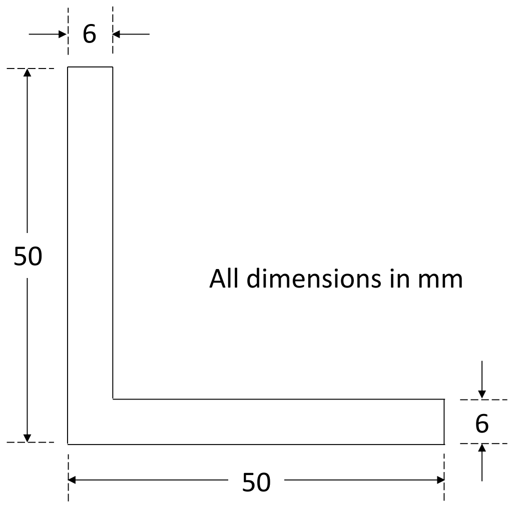

Fig Q1

[Ans: a) 14.7mm from bottom and left edges, b) I_x = 131,257.96mm^4, I_y = 131,257.96mm^4 & I_{xy} = -77,234.04mm^4, c) I_p = 208,491.1mm^4 & I_Q = 54,023.92mm^4, d) 45° anti-clockwise from x-y axes]

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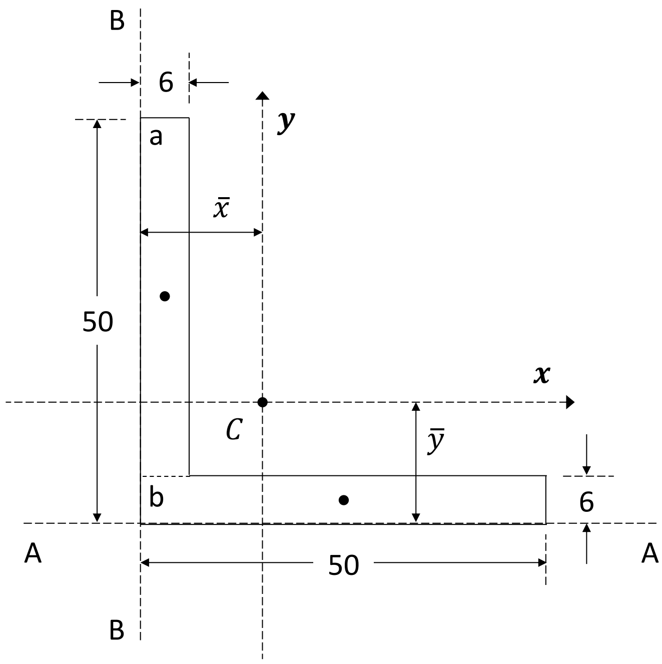
Solution 1

(a) **Position of Centroid, C**



$$Total\ Area, A = (6\times44)_a + (50\times6)_b = 564mm^4$$

Taking moments about AA:

$$\bar{y} = \frac{(6\times44\times28)_a + (50\times6\times3)_b}{564} = 14.7mm$$

Similarly, taking moments about BB:

$$\bar{x} = \frac{(44\times6\times3)_a + (6\times50\times25)_b}{564} = 14.7mm$$

(b) **2nd Moments of Area and Product Moment of Area about the x-y axes through C**

Therefore, using the Parallel Axis Theorem,

$$I_{x'} = (I_x + Ab^2)_a + (I_x + Ab^2)_b$$

$$= \left(\frac{6\times44^3}{12} + 6\times44\times(28-14.7)^2\right) + \left(\frac{50\times6^3}{12} + 50\times6\times(3-14.7)^2\right)$$

$$\therefore I_{x\prime} = 131{,}257.96mm^4$$

and,

$$I_{y\prime} = \left(I_y + Aa^2\right)_a + \left(I_y + Aa^2\right)_b$$

$$= \left(\frac{44\times6^3}{12} + 44\times6\times(3-14.7)^2\right) + \left(\frac{6\times50^3}{12} + 6\times50\times(25-14.7)^2\right)$$

$$\therefore I_{y\prime} = 131{,}257.96mm^4$$

Also,

$$I_{x\prime y\prime} = \left(I_{xy} + Aab\right)_a + \left(I_{xy} + Aab\right)_b$$

$$= \left(0 + 6\times44\times(3-14.7)\times(28-14.7)\right) + \left(0 + 50\times6\times(25-14.7)\times(3-14.7)\right)$$

$$\therefore I_{x\prime y\prime} = -77{,}234.04mm^4$$

(c) **Principal Second Moments of Area**

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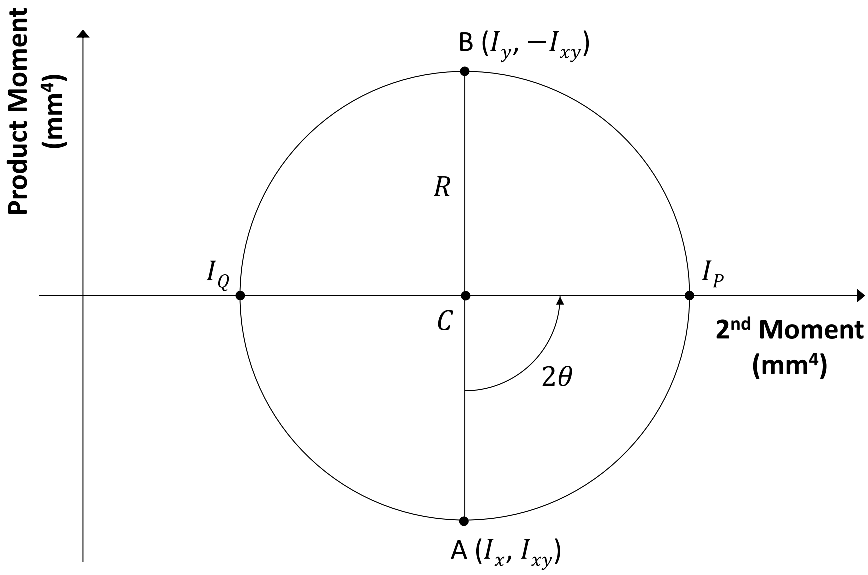
Mohr's Circle



$$Centre, C = \frac{I_{x'} + I_{y'}}{2} = 131{,}257.96mm^4$$

$$Radius, R = \sqrt{\left(\frac{I_{x'} - I_{y'}}{2}\right)^2 + {I_{x'y'}}^2} = 77{,}234.04mm^4$$

Therefore, the Principal 2nd Moments of Area are:

$$I_P = C + R = 131{,}257.96 + 77{,}234.04 = 208{,}491.1mm^4$$

and,

$$I_P = C - R = 131{,}257.96 - 77{,}234.04 = 54{,}023.92mm^4$$

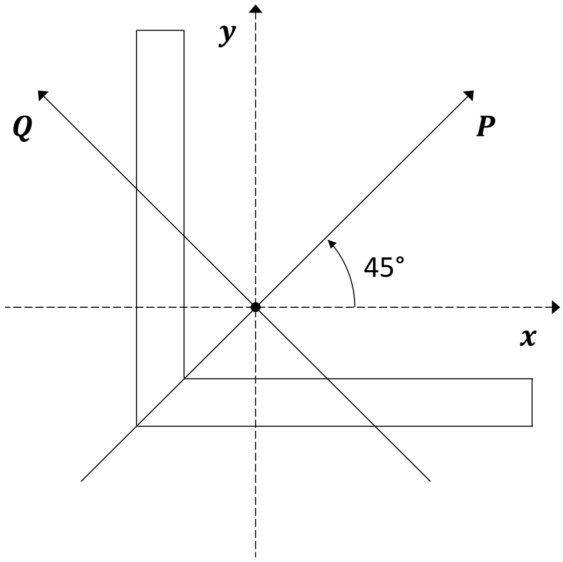
(d) **Directions of the Principal Axes**

Also,

$$2\theta = -90°$$

$$\therefore \theta = -45°$$

Therefore the Principal Axes are at 45° anti-clockwise from the x-y axes, as shown on the diagram below.



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2. Calculate (a) the Principal 2nd Moments of Area and (b) the directions of the Principal Axes for the section shown in Figure Q2.

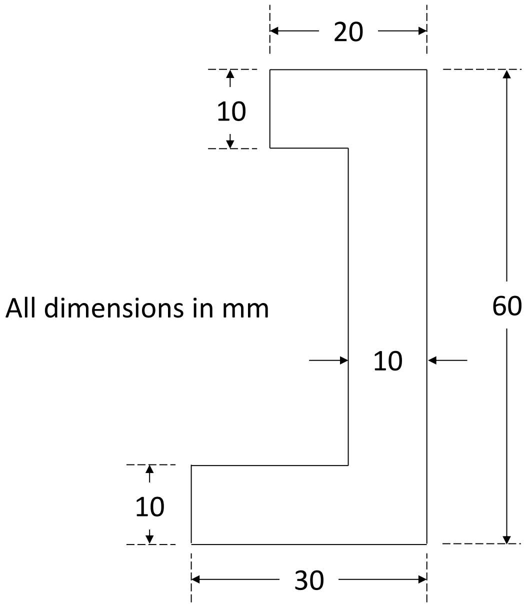

Fig Q2

[Ans: a) I_p = 367,810.05mm^4 & I_Q = 44,967.75mm^4, b) 6.97°]

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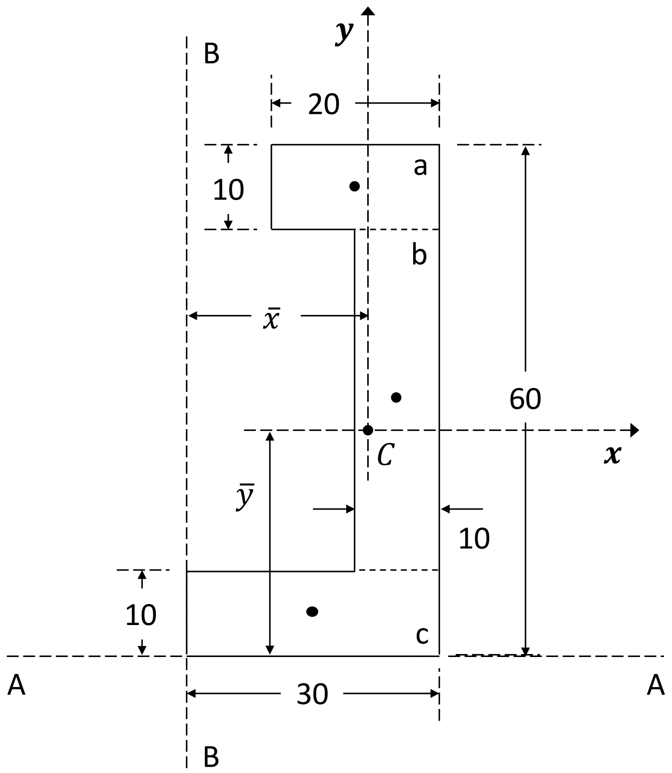
Exercise Sheet 2 – Asymmetrical Bending Solutions

Solution 2

(a)

Position of Centroid, C



$$Total\ Area, A = (20\times10)_a + (10\times40)_b + (30\times10)_c = 900mm^4$$

Taking moments about AA:

$$\bar{y} = \frac{(20\times10\times55)_a + (10\times40\times30)_b + (30\times10\times5)_c}{900} = 27.22mm$$

Similarly, taking moments about BB:

$$\bar{x} = \frac{(10\times20\times20)_a + (40\times10\times25)_b + (10\times30\times15)_c}{900} = 20.56mm$$

2nd Moments of Area and Product Moment of Area about the x-y axes through C

Therefore, using the Parallel Axis Theorem,

$$I_{x\prime} = (I_x + Ab^2)_a + (I_x + Ab^2)_b + (I_x + Ab^2)_c$$

$$= \left(\frac{20\times10^3}{12} + 20\times10\times(55-27.22)^2\right) + \left(\frac{10\times40^3}{12} + 10\times40\times(30-27.22)^2\right) + \left(\frac{30\times10^3}{12} + 30\times10\times(5-27.22)^2\right)$$

$$= 363{,}055.56mm^4$$

and,

$$I_{y\prime} = \left(I_y + Aa^2\right)_a + \left(I_y + Aa^2\right)_b + \left(I_y + Aa^2\right)_c$$

$$= \left(\frac{10\times20^3}{12} + 10\times20\times(20-20.56)^2\right) + \left(\frac{40\times10^3}{12} + 40\times10\times(25-20.56)^2\right) + \left(\frac{10\times30^3}{12} + 10\times30\times(15-20.56)^2\right)$$

$$= 49{,}722.24mm^4$$

Also,

$$I_{x\prime y\prime} = \left(I_{xy} + Aab\right)_a + \left(I_{xy} + Aab\right)_b + \left(I_{xy} + Aab\right)_c$$

$$= \left(0 + 20\times10\times(20-20.56)\times(55-27.22)\right) + \left(0 + 10\times40\times(25-20.56)\times(30-27.22)\right) + \left(0 + 30\times10\times(15-20.56)\times(5-27.22)\right)$$

$$= 38{,}888.88mm^4$$

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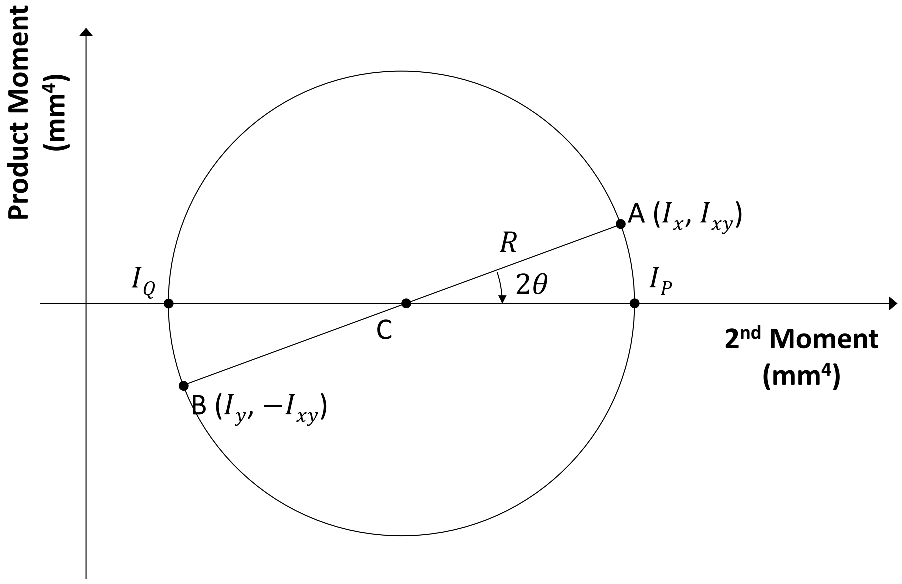
Exercise Sheet 2 – Asymmetrical Bending Solutions

Principal Second Moments of Area

Mohr's Circle



$$Centre, C = \frac{I_{x'} + I_{y'}}{2} = \frac{363{,}055.56 + 49{,}722.24}{2} = 206{,}388.9mm$$

$$Radius, R = \sqrt{\left(\frac{I_{x'} - I_{y'}}{2}\right)^2 + {I_{x'y'}}^2} = \sqrt{\left(\frac{363{,}055.56 - 49{,}722.24}{2}\right)^2 + 38{,}888.88^2} = 161{,}421.15mm$$

Therefore, the Principal 2nd Moments of Area are:

$$I_P = C + R = 206{,}388.9 + 161{,}421.15 = 367{,}810.05mm^4$$

and,

$$I_Q = C - R = 206{,}388.9 - 161{,}421.15 = 44{,}967.75mm^4$$

(b)

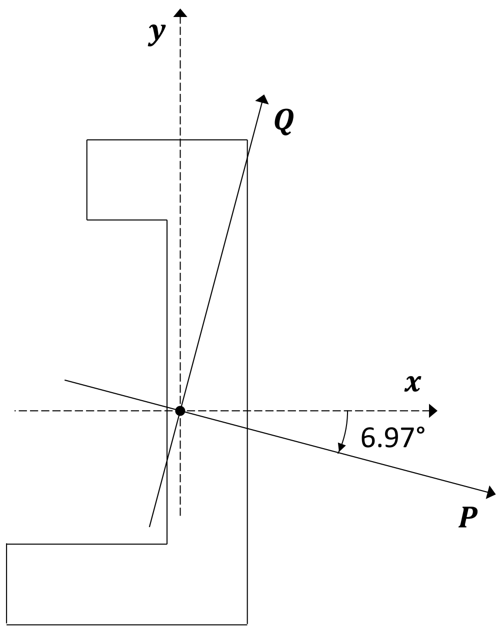
Directions of the Principal Axes

From the Mohr's Circle above:

$$sin2\theta = \frac{I_{xy}}{R} = \frac{38{,}888.88}{161{,}421.15}$$

$$\therefore \theta = 6.97°$$

Therefore the Principal Axes are at 6.97° (clockwise) from the x-y axes, as shown on the diagram below.



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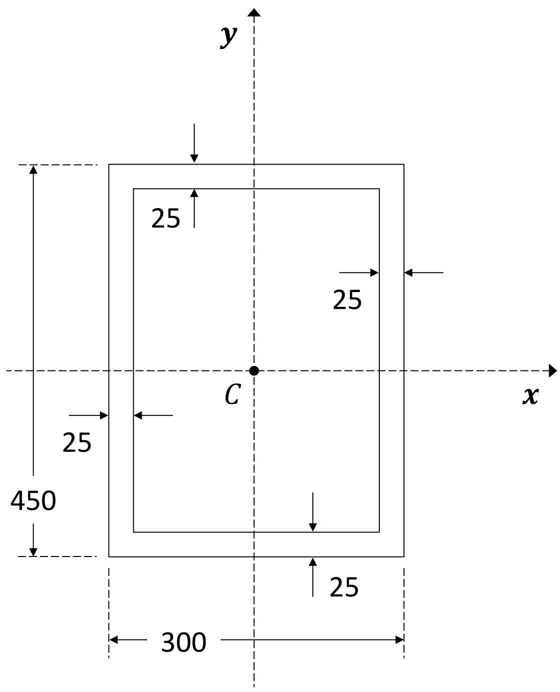
Exercise Sheet 2 – Asymmetrical Bending Solutions

3. A box section beam, 300mm wide, 450mm deep, with a uniform wall thickness of 25mm is subjected to a uniform bending moment, M. The plane of bending is inclined at an angle of 30° to the longer principal axis of the section. Determine the maximum permissible bending moment if the maximum stress in the beam is not to exceed 120MPa.

[Ans: 334.54kNm]

Solution 3

Principal 2nd Moments of Area



All dimensions in mm

Due to 2 planes of symmetry in the section, it can be seen that the Principal (P-Q) Axes lie on the x-y axes,

i.e.,

$$\theta = 0°$$

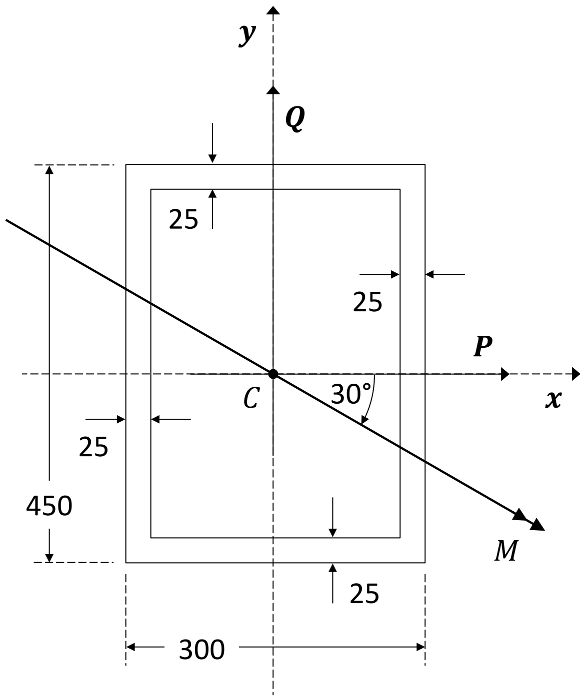
where θ is the angle between the x-y axes and the Principal (P-Q) Axes. Also,

$$I_P = I_x = \left(\frac{b_o {d_o}^3}{12} - \frac{b_i {d_i}^3}{12}\right)_x = \frac{300\times 450^3}{12} - \frac{250\times 400^3}{12} = 944{,}791{,}666.67mm^4$$

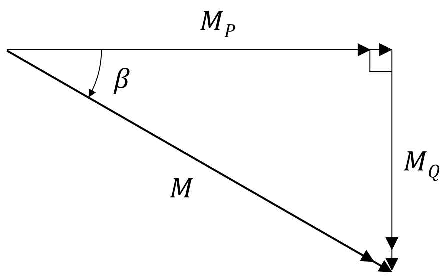
and,

$$I_Q = I_y = \left(\frac{b_o {d_o}^3}{12} - \frac{b_i {d_i}^3}{12}\right)_y = \frac{450\times300^3}{12} - \frac{400\times250^3}{12} = 491{,}666{,}666.67mm^4$$

Bending Moment is applied at 30° to the longer Principal Axis (i.e. the Q-axis) as shown below,



Resolve applied Bending Moment onto Principal Axes



Therefore,

$$M_P = Mcos\beta = Mcos30$$

and,

$$M_Q = -Msin\beta = -Msin30$$

(note negative sign as M_Q is in the negative y direction)

Calculation of position of Neutral Axis

$$\sigma_b = \frac{M_P Q}{I_P} - \frac{M_Q P}{I_Q}$$

At the Neutral Axis, σ_b = 0, therefore,

$$\frac{M_P Q}{I_P} - \frac{M_Q P}{I_Q} = 0$$

$$\therefore \frac{M_P Q}{I_P} = \frac{M_Q P}{I_Q}$$

$$\therefore \frac{Q}{P} = \frac{M_Q I_P}{M_P I_Q}$$

Therefore, α, the angle between the Neutral Axis and the Principal Axes can be defined as,

$$\alpha = \tan^{-1}\left(\frac{Q}{P}\right) = \tan^{-1}\left(\frac{M_Q I_P}{M_P I_Q}\right) = \tan^{-1}\left(\frac{-Msin30\times 944{,}791{,}666.67}{Mcos30\times 491{,}666{,}666.67}\right) = -47.97°$$

Therefore the Neutral Axis is at 47.97° (clockwise) from the Principal Axes as shown below,

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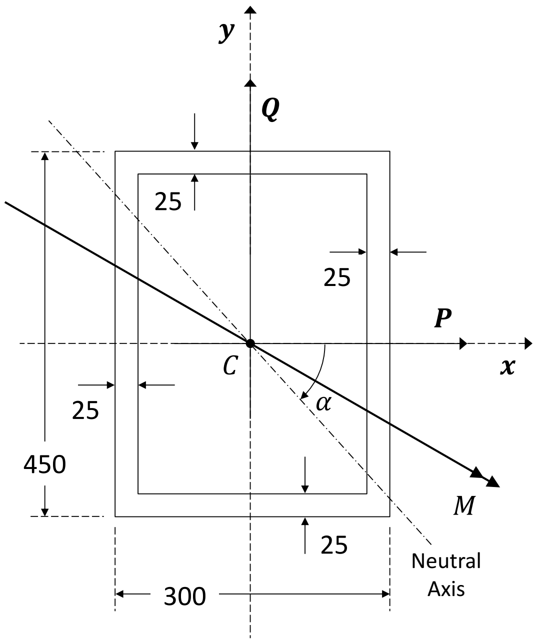
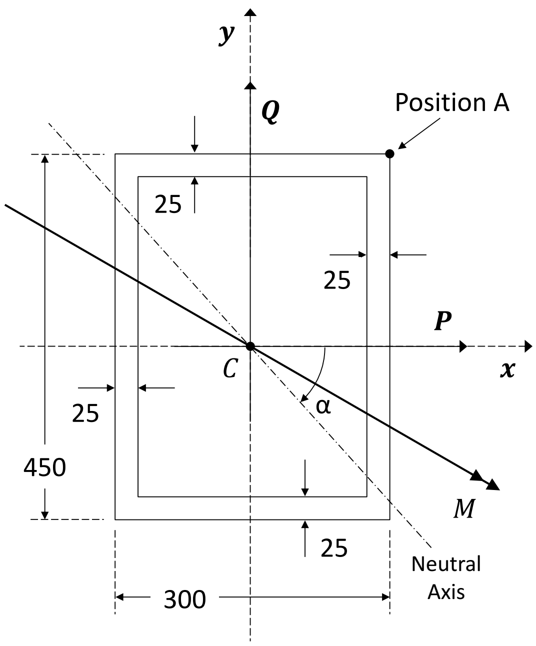


Maximum Tensile Stress in the section

It can be seen that the maximum (tensile) Bending Stress will be at position A, as shown below,



As above,

$$\sigma_b = \frac{M_P Q}{I_P} - \frac{M_Q P}{I_Q}$$

Therefore, the co-ordinates of point A on the P-Q axes are required. In this case, these are the same as the x-y co-ordinates and are:

$$P = 150mm$$

and,

$$Q = 225mm$$

These P-Q co-ordinates for position A can now be substituted into the equation for bending stress to give:

$$\sigma_b = \frac{M_P Q}{I_P} - \frac{M_Q P}{I_Q} = \frac{Mcos30\times225}{944{,}791{,}666.67} - \frac{-Msin30\times150}{491{,}666{,}666.67}$$

$$\therefore \sigma_b = M(2.062\times10^{-7} + 1.525\times10^{-7}) = 3.587\times10^{-7}\times M$$

As the maximum stress in the beam is not to exceed 120MPa:

$$120 = 3.587\times10^{-7}\times M$$

$$\boxed{\therefore M = 33.454Nmm = 334.54kNm}$$

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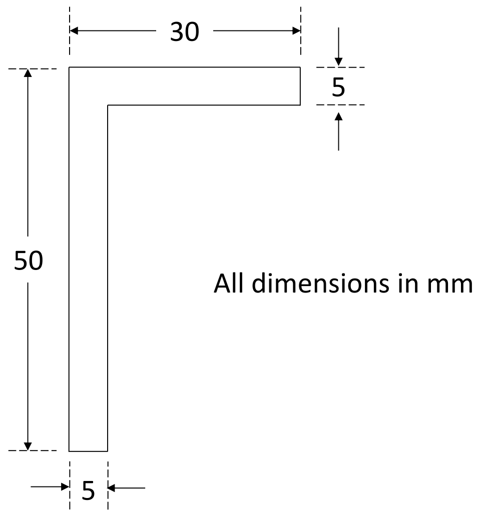
4. A 50mm by 30mm by 5mm angle is used as a cantilever of length 500mm, with the 30mm leg horizontal and uppermost. A vertical load of 1000N is applied at the free end. Determine (a) the position of the neutral axis and (b) the maximum tensile and compressive bending stresses.

[Ans: a) 86.79°, b) 201.18MPa & -94.38MPa]

Solution 4



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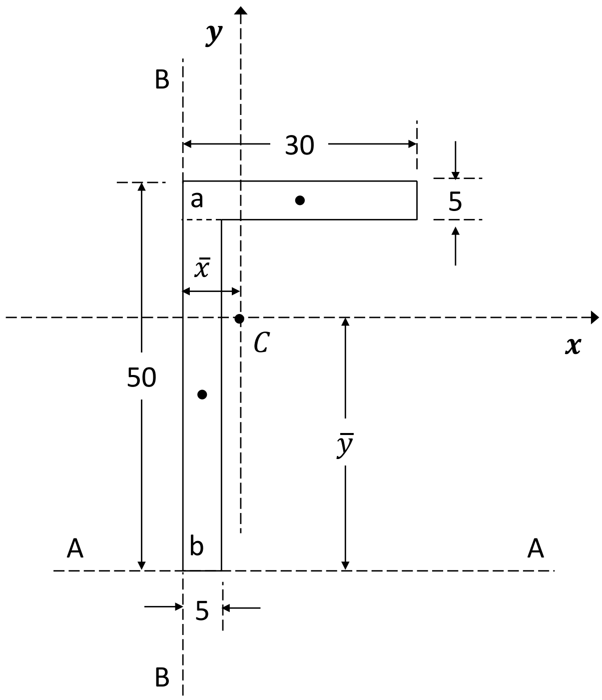
(a)

Position of Centroid, *C*



$$Total\ Area, A = (30\times5)_a + (5\times45)_b = 375mm^4$$

Taking moments about AA:

$$\bar{y} = \frac{(30\times5\times47.5)_a + (5\times45\times22.5)_b}{375} = 32.5mm$$

Similarly, taking moments about BB:

$$\bar{x} = \frac{(5\times30\times15)_a + (45\times5\times2.5)_b}{375} = 7.5mm$$

2nd Moments of Area and Product Moment of Area about the x-y axes through *C*

Therefore, using the Parallel Axis Theorem,

$$I_{x\prime} = (I_x + Ab^2)_a + (I_x + Ab^2)_b$$

$$= \left(\frac{30\times5^3}{12} + 30\times5\times(47.5-32.5)^2\right) + \left(\frac{5\times45^3}{12} + 5\times45\times(22.5-32.5)^2\right)$$

$$= 94{,}531.25mm^4$$

and,

$$I_{y\prime} = \left(I_y + Aa^2\right)_a + \left(I_y + Aa^2\right)_b$$

$$= \left(\frac{5\times30^3}{12} + 5\times30\times(15-7.5)^2\right) + \left(\frac{45\times5^3}{12} + 45\times5\times(2.5-7.5)^2\right)$$

$$= 25{,}781.25mm^4$$

Also,

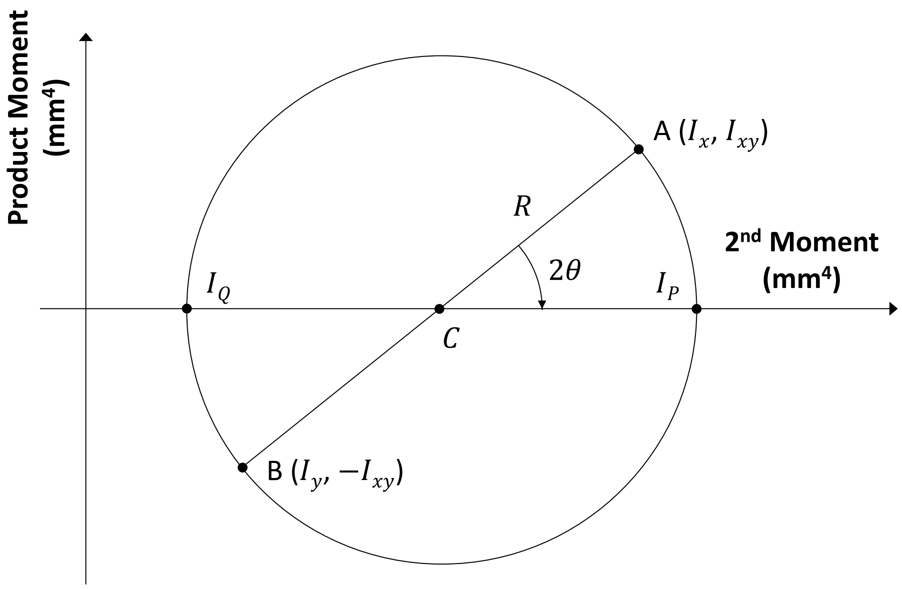
$$I_{x\prime y\prime} = \left(I_{xy} + Aab\right)_a + \left(I_{xy} + Aab\right)_b$$

$$= \left(0 + 30\times5\times(15-7.5)\times(47.5-32.5)\right) + \left(0 + 45\times5\times(2.5-7.5)\times(22.5-32.5)\right)$$

$$= 28{,}125mm^4$$

Principal Second Moments of Area

Mohr's Circle



$$Centre, C = \frac{I_{x'} + I_{y'}}{2} = \frac{94{,}531.25 + 25{,}781.25}{2} = 60{,}156.25mm$$

$$Radius, R = \sqrt{\left(\frac{I_{x'} - I_{y'}}{2}\right)^2 + I_{x'y'}{}^2} = \sqrt{\left(\frac{94{,}531.25 - 25{,}781.25}{2}\right)^2 + 28{,}125^2} = 44{,}414.6mm$$

Therefore, the Principal 2nd Moments of Area are:

$$I_P = C + R = 60{,}156.25 + 44{,}414.6 = 104{,}570.85mm^4$$

and,

$$I_Q = C - R = 60{,}156.25 - 44{,}414.6 = 15{,}741.65mm^4$$

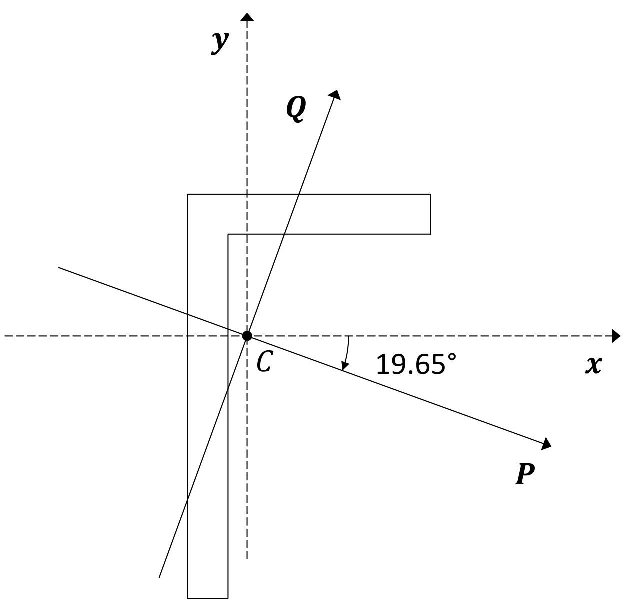
Directions of the Principal Axes

From the Mohr's circle above:

$$sin2\theta = \frac{I_{xy}}{R} = \frac{28{,}125}{44{,}414.6}$$

$$\therefore \theta = 19.65°$$

Therefore the Principal Axes are at 19.65° clockwise from the x-y axes, as shown on the diagram below.



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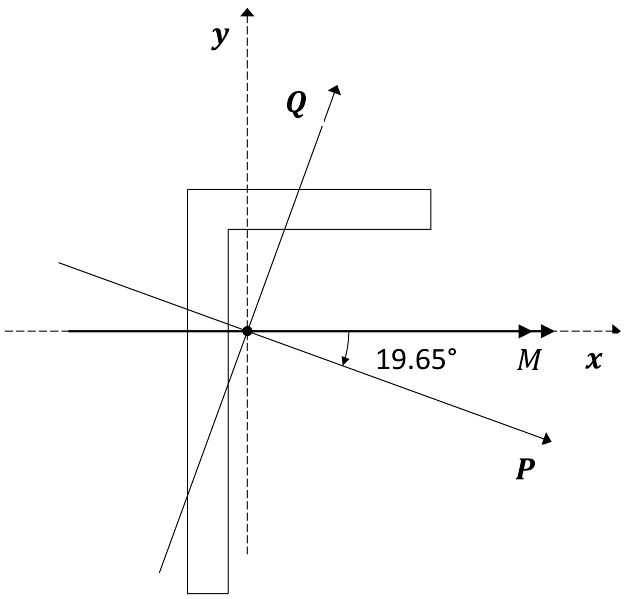
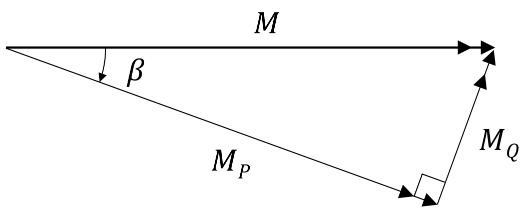
As this is a 500mm cantilever beam with a vertical load of 1000N applied to the end, it is the equivalent of having a 500,000Nmm ($M = P \times L$) Bending Moment applied about the x-axis as shown below,



Resolve applied Bending Moment onto Principal Axes



Therefore,

$$M_P = M cos\theta = 500{,}000 cos 19.65 = 470{,}882.18 Nmm$$

and,

$$M_Q = M sin\theta = 500{,}000 sin 19.65 = 168{,}136.77 Nmm$$

Calculation of position of Neutral Axis

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$$\sigma_b = \frac{M_P Q}{I_P} - \frac{M_Q P}{I_Q}$$

At the Neutral Axis, σ_b = 0, therefore,

$$\frac{M_P Q}{I_P} - \frac{M_Q P}{I_Q} = 0$$

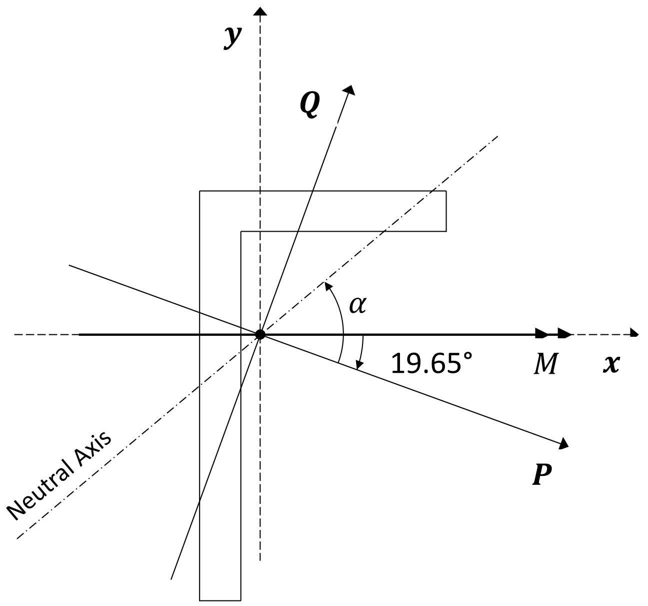
$$\therefore \frac{M_P Q}{I_P} = \frac{M_Q P}{I_Q}$$

$$\therefore \frac{Q}{P} = \frac{M_Q I_P}{M_P I_Q}$$

Therefore, α, the angle between the Neutral Axis and the Principal Axes can be defined as,

$$\alpha = \tan^{-1}\left(\frac{Q}{P}\right) = \tan^{-1}\left(\frac{M_Q I_P}{M_P I_Q}\right) = \tan^{-1}\left(\frac{168{,}136.77 \times 104{,}570.85}{470{,}882.18 \times 15{,}741.65}\right) = 67.14°$$

Therefore the Neutral Axis is at 67.14° (anti-clockwise) from the Principal Axes as shown below,

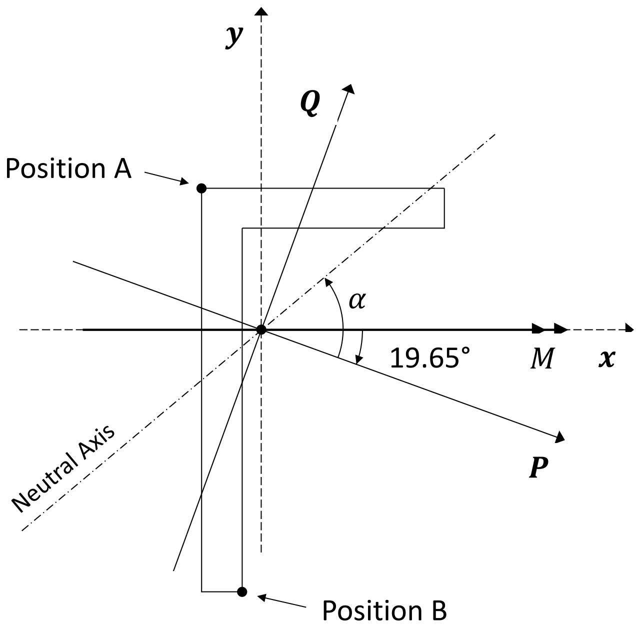


The Neutral Axis is therefore at (19.65° - 67.14° =) -47.49° (anti-clockwise) from the x-axis.

(b)

Maximum Tensile and Compressive Stresses in the section

By observation, it is considered that the maximum tensile and compressive stresses in the section will be at positions A and B, respectively, as shown below,



As above,

$$\sigma_b = \frac{M_P Q}{I_P} - \frac{M_Q P}{I_Q}$$

Therefore, the co-ordinates of point A on the P-Q axes are required. These are calculated as:

$$P = xcos\theta - ysin\theta$$

and,

$$Q = xsin\theta + ycos\theta$$

Where for point A, x = -7.5mm and y = 17.5mm. Therefore,

$$P = -7.5cos19.65 - 17.5sin19.65 = -12.95mm$$

and,

$$Q = -7.5sin19.65 + 17.5cos19.65 = 13.96mm$$

These P-Q co-ordinates for position A can now be substituted into the equation for bending stress to give:

$$\sigma_{bA} = \frac{M_P Q}{I_P} - \frac{M_Q P}{I_Q} = \frac{470{,}882.18 \times 13.96}{104{,}570.85} - \frac{168{,}136.77 \times -12.95}{15{,}741.65}$$

$$\therefore \sigma_{bA} = 201.18MPa$$

And for point B, x = -2.5mm and y = -32.5mm. Therefore,

$$P = -2.5cos19.65 + 32.5sin19.65 = 8.58mm$$

and,

$$Q = -2.5sin19.65 - 32.5cos19.65 = -31.45mm$$

These P-Q co-ordinates for position B can now be substituted into the equation for bending stress to give:

$$\sigma_{bB} = \frac{M_P Q}{I_P} - \frac{M_Q P}{I_Q} = \frac{470{,}882.18 \times -31.45}{104{,}570.85} - \frac{168{,}136.77 \times 8.58}{15{,}741.65} = -141.62 - 91.64$$

$$\therefore \sigma_{bB} = -233.26MPa$$

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5. Calculate (a) the position of the Neutral Axis and (b) the maximum tensile stress for the section shown in Figure Q5 when a Bending Moment of 225Nm is applied about the x-axis in the sense shown.

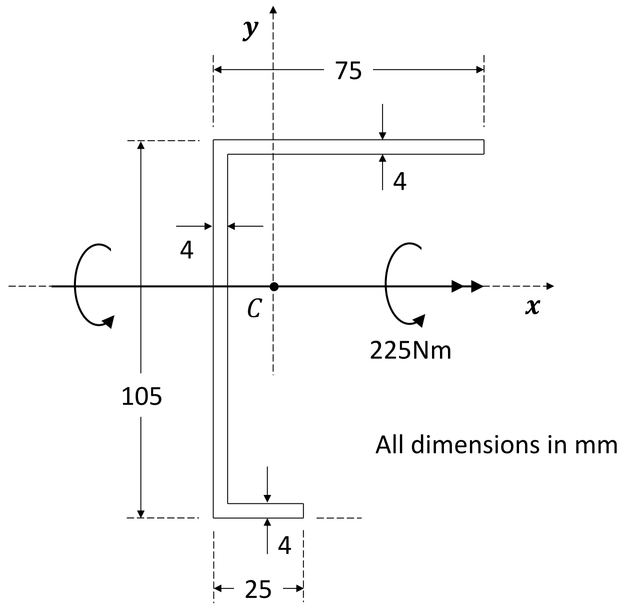

Fig Q5

[Ans: a) 42.82° (anti-clockwise) from the x-y axes, b) 14.22MPa]

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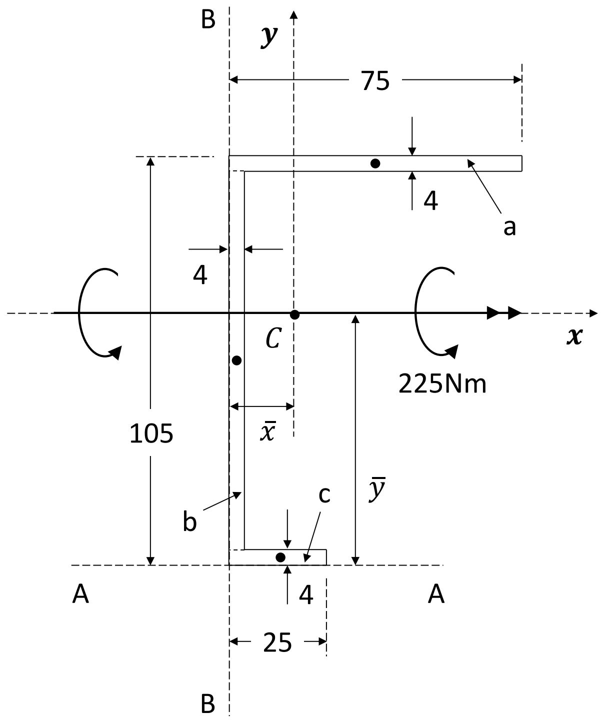
Solution 5

(a)

Position of Centroid, C



$$Total\ Area, A = (75\times4)_a + (4\times97)_b + (25\times4)_c = 788mm^4$$

Taking moments about AA:

$$\bar{y} = \frac{(75\times4\times103)_a + (4\times97\times52.5)_b + (25\times4\times2)_c}{788} = 65.32mm$$

Similarly, taking moments about BB:

$$\bar{x} = \frac{(4\times75\times37.5)_a + (97\times4\times2)_b + (4\times25\times12.5)_c}{788} = 16.85mm$$

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2nd Moments of Area and Product Moment of Area about the x-y axes through C

Therefore, using the Parallel Axis Theorem,

$$I_{x'} = (I_x + Ab^2)_a + (I_x + Ab^2)_b + (I_x + Ab^2)_c$$

$$= \left(\frac{75\times4^3}{12} + 75\times4\times(103 - 65.32)^2\right) + \left(\frac{4\times97^3}{12} + 4\times97\times(52.5 - 65.32)^2\right)$$

$$+ \left(\frac{25\times4^3}{12} + 25\times4\times(2 - 65.32)^2\right)$$

$$= 1{,}195{,}403.35 mm^4$$

and,

$$I_{y'} = \left(I_y + Aa^2\right)_a + \left(I_y + Aa^2\right)_b + \left(I_y + Aa^2\right)_c$$

$$= \left(\frac{4\times75^3}{12} + 4\times75\times(37.5 - 16.85)^2\right) + \left(\frac{97\times4^3}{12} + 97\times4\times(2 - 16.85)^2\right)$$

$$+ \left(\frac{4\times25^3}{12} + 4\times25\times(12.5 - 16.85)^2\right)$$

$$= 361{,}732.39 mm^4$$

Also,

$$I_{x'y'} = \left(I_{xy} + Aab\right)_a + \left(I_{xy} + Aab\right)_b + \left(I_{xy} + Aab\right)_c$$

$$= \left(0 + 75\times4\times(37.5 - 16.85)\times(103 - 65.32)\right) + \left(0 + 4\times97\times(2 - 16.85)\times(52.5 - 65.32)\right)$$

$$+\left(0 + 25\times4\times(12.5 - 16.85)\times(2 - 65.32)\right)$$

$$= 334{,}838.08 mm^4$$

Principal 2nd Moments of Area

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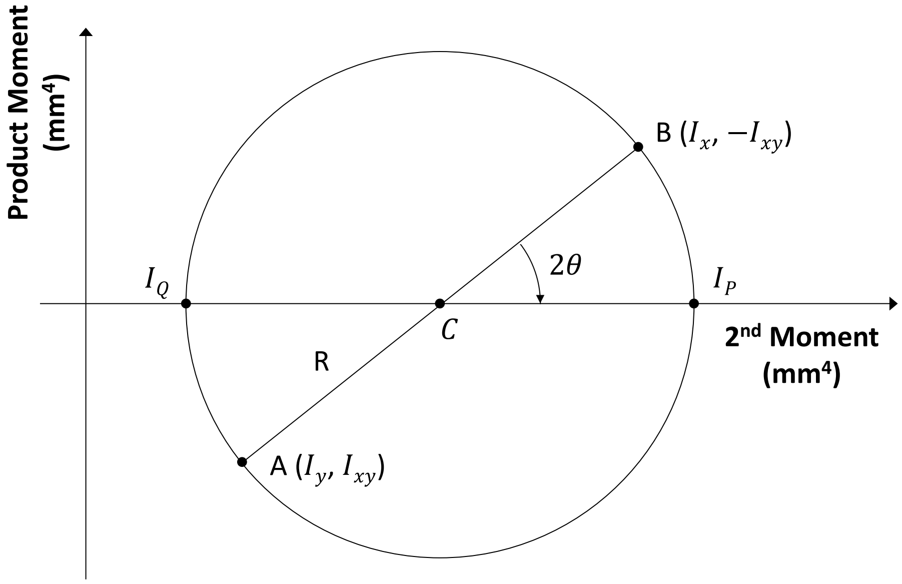
Mohr's Circle



$$Centre, C = \frac{I_{x\prime} + I_{y\prime}}{2} = \frac{1{,}195{,}403.35 + 361{,}732.39}{2} = 778{,}567.87mm$$

$$Radius, R = \sqrt{\left(\frac{I_{x'} - I_{y'}}{2}\right)^2 + {I_{x'y'}}^2} = \sqrt{\left(\frac{1{,}195{,}403.35 - 361{,}732.39}{2}\right)^2 + 334{,}838.08^2}$$

$$= 534{,}666.59mm$$

Therefore, the Principal 2nd Moments of Area are:

$$I_P = C + R = 778{,}567.87 + 534{,}666.59 = 1{,}313{,}234.45mm^4$$

and,

$$I_Q = C - R = 778{,}567.87 - 534{,}666.59 = 243{,}901.27mm^4$$

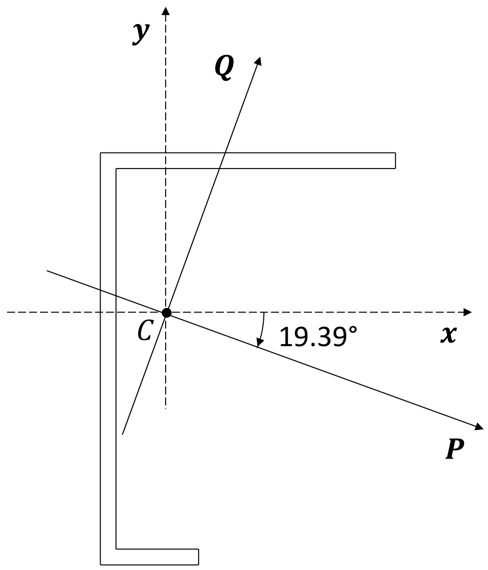
Directions of the Principal Axes

From the Mohr's circle above:

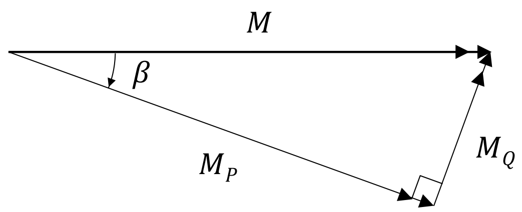
$$sin2\theta = \frac{I_{xy}}{R} = \frac{334{,}838.08}{534{,}666.59}$$

$$\therefore \theta = 19.39°$$

Therefore the Principal Axes are at 19.39° clockwise from the x-y axes, as shown on the diagram below.



Resolve applied bending moment onto Principal Axes



Therefore,

$$M_P = Mcos\theta = 225cos19.39 = 212.24Nm = 212.24\times10^3 Nmm$$

and,

$$M_Q = Msin\theta = 225sin19.39 = 74.7Nm = 74.7\times10^3 Nmm$$

Calculation of position of Neutral Axis

$$\sigma_b = \frac{M_P Q}{I_P} - \frac{M_Q P}{I_Q}$$

At the Neutral Axis, σ_b = 0, therefore,

$$\frac{M_P Q}{I_P} - \frac{M_Q P}{I_Q} = 0$$

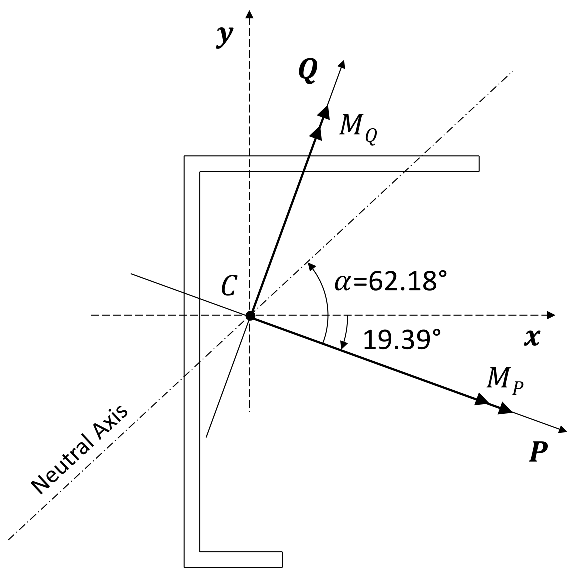
$$\therefore \frac{M_P Q}{I_P} = \frac{M_Q P}{I_Q}$$

$$\therefore \frac{Q}{P} = \frac{M_Q I_P}{M_P I_Q}$$

Therefore, α, the angle between the Neutral Axis and the Principal Axes can be defined as,

$$\alpha = \tan^{-1}\left(\frac{Q}{P}\right) = \tan^{-1}\left(\frac{M_Q I_P}{M_P I_Q}\right) = \tan^{-1}\left(\frac{74.7\times10^3\times1{,}313{,}234.45}{212.24\times10^3\times243{,}901.27}\right) = 62.18°$$

Therefore the Neutral Axis is at 62.18° (anti-clockwise) from the Principal Axes as shown below,



The Neutral Axis is therefore at (19.82° − 62.64°) -42.82° (anti-clockwise) from the x-axis.

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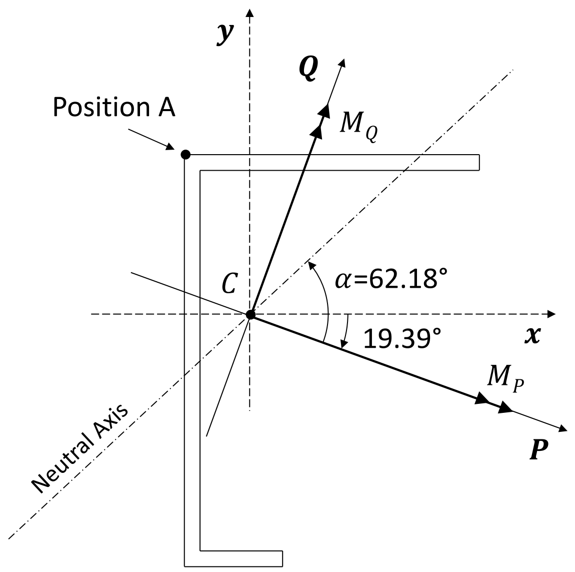
(b)

Maximum Tensile Stress in the section

By observation, it is considered that the maximum tensile stress will be at position A, as shown below,



As above,

$$\sigma_b = \frac{M_P Q}{I_P} - \frac{M_Q P}{I_Q}$$

Therefore, the co-ordinates of point A on the P-Q axes are required. These are calculated as:

$$P = xcos\theta - ysin\theta$$

and,

$$Q = xsin\theta + ycos\theta$$

Where for point A, x = -16.85mm and y = 39.68mm. Therefore,

$$P = -16.85cos19.39 - 39.68sin19.39 = -29.06mm$$

and,

$$Q = -16.85sin19.39 + 39.68cos19.39 = 31.84mm$$

These P-Q co-ordinates for position A can now be substituted into the equation for bending stress to give:

$$\sigma_b = \frac{M_P Q}{I_P} - \frac{M_Q P}{I_Q} = \frac{212.24\times10^3\times31.84}{1{,}313{,}234.45} - \frac{74.7\times10^3\times-29.06}{243{,}901.27}$$

$$\therefore \sigma_b = 14.39MPa$$