Elastic Instability (Buckling)

Lecture 2 – Yielding vs Buckling

Department of Mechanical, Materials & Manufacturing Engineering

MMME2053 – Mechanics of Solids

University of Nottingham
UK | CHINA | MALAYSIA

Elastic Instability (Buckling) Methods

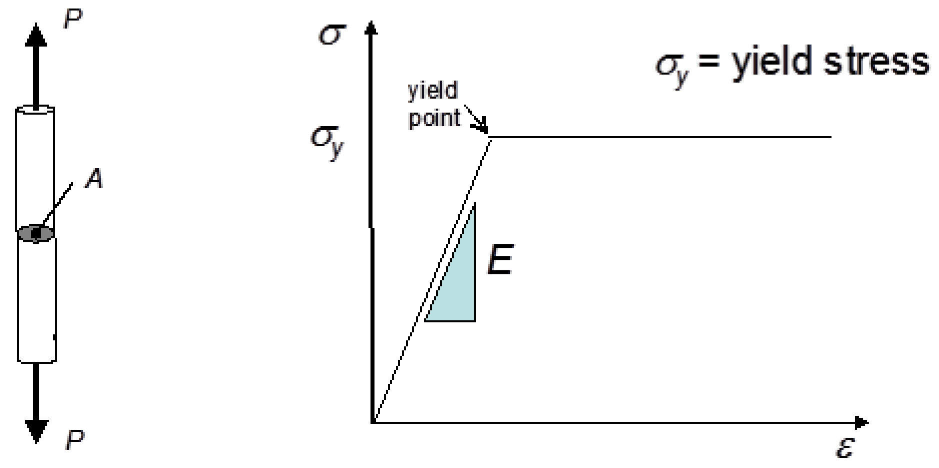
Learning Outcomes

1. Know the meanings of and the differences between stable, unstable and neutral equilibria (knowledge);
2. Be able to apply Macaulay's method for determining beam deflection in situations with axial loading (application);
3. Be able to determine the buckling loads for ideal struts (application);
4. Be able to include the interaction of yield behaviour with buckling and how to represent this interaction graphically (knowledge/application).

Struts

Compression of Rods/Columns

If we assume that a rod is loaded perfectly axially, and the material can be represented by an elastic-perfectly plastic stress-strain curve (as shown in the figure below), then the plastic collapse failure would occur in compression if $\sigma = -\frac{P}{A}$ reaches $-\sigma_{\mathrm{y}}$ before the buckling load is reached.



Assuming that the rod behaves as a hinged-hinged rod, then:

$$P_c = \frac{\pi^2 EI}{\mathrm{L}^2}$$

And defining the second moment of area, I:

$$I = Ak^2$$

where k is the radius of gyration, therefore:

$$P_c = \frac{\pi^2 EAk^2}{\mathrm{L}^2}$$

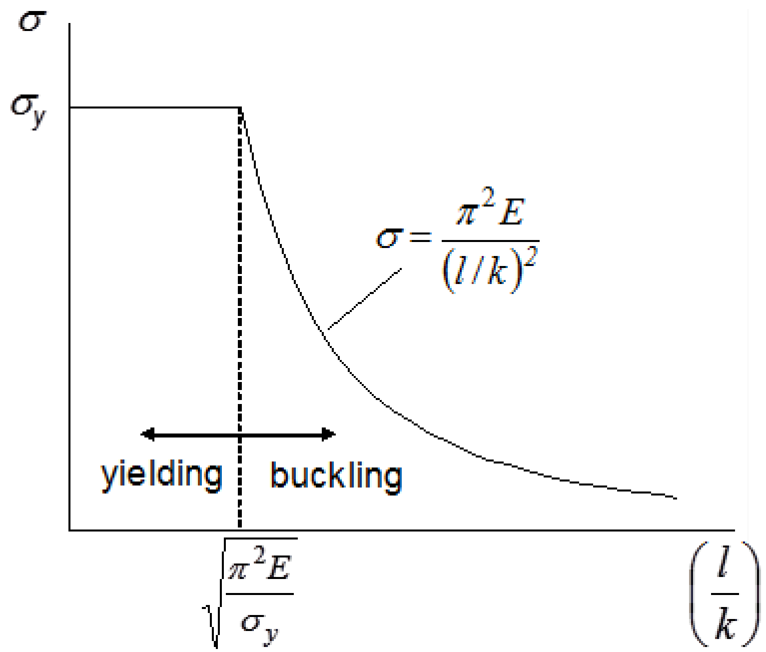
Substituting $\sigma = \frac{P_c}{A}$ into this:

$$\sigma = \frac{\pi^2 E}{\left(\frac{L}{k}\right)^2}$$

where $\frac{L}{k}$ is defined as the slenderness ratio.

Therefore, buckling will occur if $\sigma = \frac{\pi^2 E}{\left(\frac{L^2}{k^2}\right)}$, whereas plastic collapse will occur if $\sigma = \sigma_y$

This can be represented diagrammatically as shown in the Figure below.



Struts

Some Other Notes

In contrast to the classical cases considered here, actual compression members are seldom truly pinned or completely fixed against rotation at the ends. Because of this uncertainty regarding the fixity of the ends, struts or columns are often assumed to be pin-ended. This procedure is conservative.

The above equations are not applicable in the inelastic range, i.e. for $\sigma > \sigma_y$, and must be modified.

The critical load formulae for struts or columns are remarkable in that they do not contain any strength property of the material and yet they determine the load carrying capacity of the member. The only material property required is the elastic modulus, E, which is a measure of the stiffness of the strut.

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