Department of Mechanical, Materials & Manufacturing Engineering

DYNAMICS (VIBRATION)

SHEET 4: FREE VIBRATION RESPONSE

1. The mass-spring-damper system is shown in Figure Q1, with k = 500 N/m, c= 2 N.s/m and m=2 kg. An initial velocity of v is given to the mass at its equilibrium position ($x(0)=0$). Determine the expression for displacement of the mass, $x(t)$.

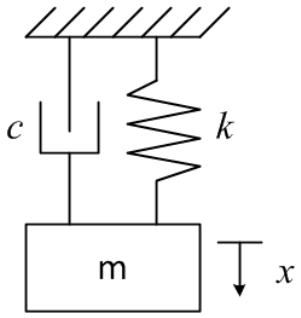

Figure Q1

$$x(t) = \frac{v}{\omega_d} e^{-\xi\omega_n t} \sin(\omega_d t);\ \omega_d = \omega_n\sqrt{1-\xi^2}$$

2. The system in Figure Q2 is initially at rest. The mass is displaced from its equilibrium position by 0.02 m. Find the expression for velocity of the mass, $\dot{x}$(t), if c = 63.24 N.s/m, k=250 N/m and m= 2 kg.

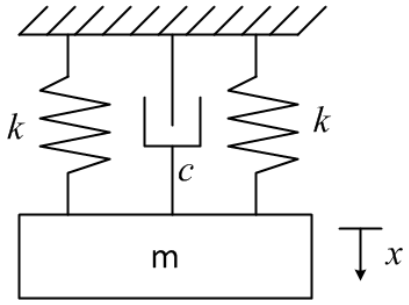

Figure Q2

$$\dot{x}(t) = -4.995te^{-15.81t}\ [m/s]$$

3. If the same damper must be used for system in Q2, what parameters can be changed to ensure the system is lightly damped? What is the criteria for changing these parameters?

   Either k or m can be increased; km>9.998e2 [N.kg/m]

4. A mass of 2 kg is suspended from a spring of stiffness 1 kN/m in parallel with a viscous damper. When the mass is lifted and released, it is observed that in the ensuing oscillations the first downward displacement from the equilibrium position is four times as large as the fourth downward displacement. Find the damping ratio and the damping coefficient for the system by assuming small damping. Perform the same calculations without the small damping assumption and consider the difference.

   0.0735; 6.57 Ns/m

5. The rigid beam shown in Fig. Q5 has a moment of inertia of 10 kgm2 about the pivot at A. End C is displaced downwards by 10 mm from its equilibrium position and then released from rest. Find the maximum upward displacement of C from its equilibrium position and the elapsed time at which this occurs.

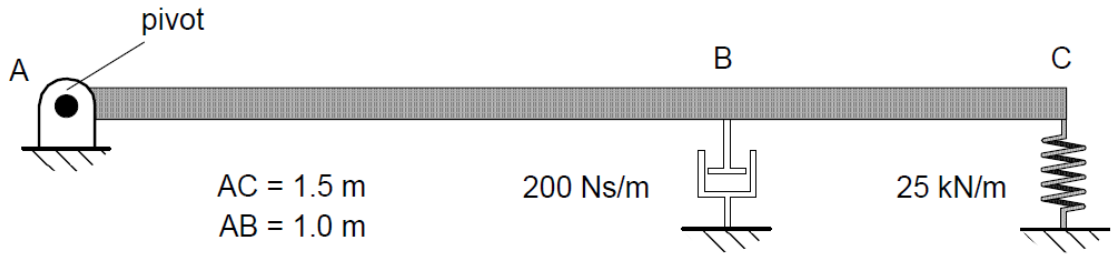

Figure Q5

   6.55 mm; 42.3 ms

6. In the rack and pinion gear system shown diagrammatically in Fig. Q6, the pinion has a pitch circle diameter of 200 mm and a moment of inertia of 0.005 kgm2 and is connected to ground by a shaft with a torsional stiffness of 80 Nm/rad. The rack has a mass of 2 kg and is connected to ground on each side by springs with stiffness 25 kN/m and viscous dampers with damping coefficients of 50 Ns/m.

   (i) Find the undamped natural frequency and the damping ratio of the system
   (ii) If an impulse is applied to the rack when it is at rest such that it acquires an instantaneous velocity of 1 m/s, find the maximum displacement of the ensuing vibration.

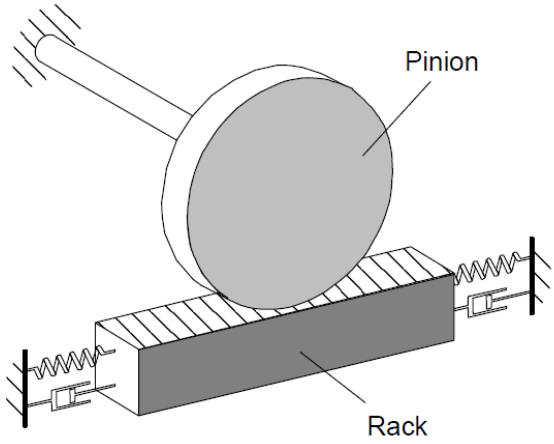

Figure Q6

(*i*) *24.2 Hz; 0.131;* (*ii*) *5.43 mm*