Deflection of Beams

Lecture 4 – Statically Indeterminate Problems

Department of Mechanical, Materials & Manufacturing Engineering

MMME2053 – Mechanics of Solids

University of Nottingham
UK | CHINA | MALAYSIA

Deflection of Beams

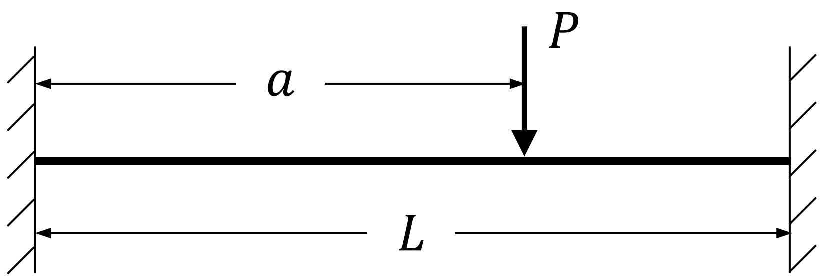
Learning Outcomes

1. Know how to derive the differential equation of the elastic line (i.e. deflection curve) of a beam (synthesis);
2. Employ Macaulay's method, also called the method of singularities, to determine bending moment expressions for beams where there are discontinuities in the bending moment distribution arising from discontinuous loading (application);
3. Be able to solve this equation by successive integration in order to yield the slope, $\frac{\mathrm{d}y}{\mathrm{d}x}$, and the deflection, y, of a beam at any position, x, along its span (application);
4. Recognise and use different singularity functions in the bending moment expression, relating to different loading conditions, including point loads, uniformly distributed loads and point bending moments (comprehension);
5. Employ Macaulay's method for statically indeterminate beam problems (application).

Statically Indeterminate Problems

Macaulay's method can also be used to solve for the slopes and deflections of statically indeterminate problems.

A beam is statically indeterminate if the reaction forces and/or bending moments cannot be determined by the equations of statics alone.



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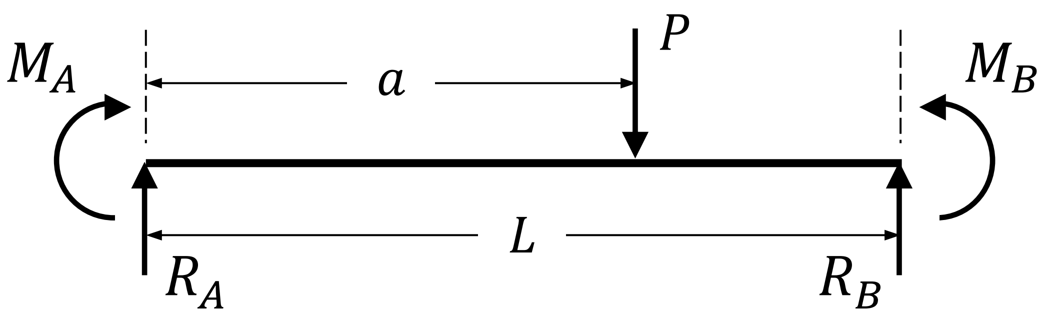
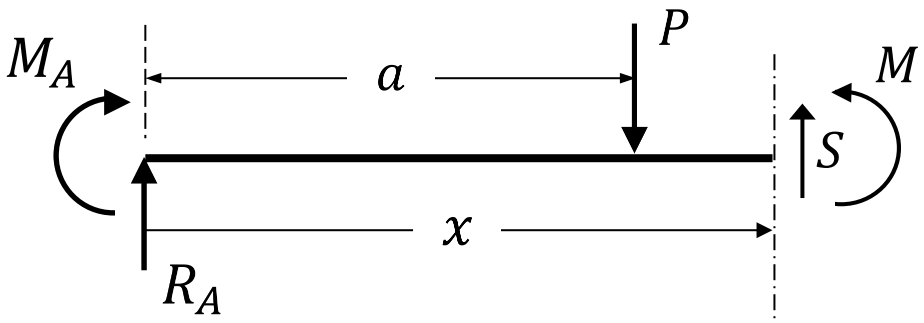


Taking the left-hand end as the origin and drawing a free body diagram of the beam sectioned after the discontinuity:



Taking moments about the section position in order to determine an expression for the bending moment, M:

$$M = R_A x + M_A - P\langle x - a\rangle$$

Substituting this into the 2nd order differential equation of the elastic line (equation (8) from lecture 1):

$$EI\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = R_A x + M_A - P\langle x - a\rangle$$

This 2nd order differential expression for the clamped-clamped beam can then be integrated with respect to x to give the slope, $\frac{\mathrm{d}y}{\mathrm{d}x}$, and integrated again to give the deflection, y, at any position x, along the length of the beam.

Integrating with respect to x:

$$EI\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{R_A x^2}{2} + M_A x - \frac{P\langle x-a\rangle^2}{2} + \mathrm{A}$$

Integrating with respect to x again:

$$EIy = \frac{R_A x^3}{6} + \frac{M_A x^2}{2} - \frac{P\langle x-a\rangle^3}{6} + \mathrm{A}x + \mathrm{B}$$

These expressions for slope and deflection contain four unknowns, namely the reactions M_A and R_A and the integration constants A and B. In this case, we can apply the four boundary conditions to these expressions to solve for these four unknowns.

BC1: at x = 0, y = 0

BC2: at x = 0, $\frac{\mathrm{d}y}{\mathrm{d}x}$ = 0

BC3: at x = L, y = 0

BC4: at x = L, $\frac{\mathrm{d}y}{\mathrm{d}x}$ = 0

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