MTHS2007 Advanced Mathematics and Statistics for Mechanical Engineers

Chapter 1: revision

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UNITED KINGDOM · CHINA · MALAYSIA

1 Brief Revision

Introduction

In case you've forgotten anything from your mathematical studies in your first year......

1.1 Complex numbers

1.2 Some trigonometry

1.3 Partial derivatives

- Complex numbers arise naturally when solving quadratic equations (and other polynomial equations).
- If $ax^2 + bx + c = 0$ with $a \neq 0$, we know that the solutions are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

- If $b^2 < 4ac$, this expression involves the square root of a negative number. No such real number exists.

- For example, $x^2 - 2x + 2 = 0$ has the two solutions

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4.1.2}}{2.1} = \frac{2 \pm \sqrt{-4}}{2} = 1 \pm \sqrt{-1}.$$

- We define $\mathrm{i} = \sqrt{-1}$, so that $\mathrm{i}^2 = -1$.
- i is an example of an *imaginary* number.
- $1 + \mathrm{i}$ is an example of a *complex* number.
- **Make sure that you can remember how to solve quadratic equations!**

Some Taylor series

$$\exp x = e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \dots,$$

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \dots,$$

$$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + \dots.$$

x **must be measured in radians, not degrees!**

- These Taylor series show that

$$e^{ix} = 1 + ix + \frac{1}{2!}(ix)^2 + \frac{1}{3!}(ix)^3 + \frac{1}{4!}(ix)^4 + \dots$$

$$= 1 + ix - \frac{1}{2!}x^2 - \frac{1}{3!}ix^3 + \frac{1}{4!}x^4 + \dots$$

$$= 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + \dots$$

$$+ix - \frac{1}{3!}ix^3 + \dots$$

$$\Rightarrow e^{ix} = \cos x + i \sin x$$

- This is **Euler's formula**.
- We will need to remember Euler's formula when we study constant coefficient ordinary differential equations (Chapter 2).

1.2 Some Trigonometry

- The standard formula sheet contains the identities

$$\sin\,(A \pm B) = \sin A \cos B \pm \cos A \sin B,$$

$$\cos\,(A \pm B) = \cos A \cos B \mp \sin A \sin B.$$

- These come from Euler's formula, because

$$e^{i(A \pm B)} = e^{iA} e^{\pm iB},$$

$$\Rightarrow \cos\,(A \pm B) + i \sin\,(A \pm B) = (\cos A + i \sin A)(\cos B \pm i \sin B)$$

$$= \cos A \cos B \mp \sin A \sin B + i \sin A \cos B \pm i \cos A \sin B.$$

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$$\sin\,(A \pm B) = \sin A \cos B \pm \cos A \sin B,$$

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- Now, we can see that

$$\sin\,(A - B) + \sin\,(A + B)$$

$$= (\sin A \cos B - \cos A \sin B) + (\sin A \cos B + \cos A \sin B)$$

$$= 2 \sin A \cos B$$

and therefore

$$\sin A \cos B = \frac{1}{2}\{\sin\,(A - B) + \sin\,(A + B)\}.$$

- The formulas

$$\sin A \cos B = \frac{1}{2}\left\{ \sin\,(A-B) + \sin\,(A+B) \right\},$$

$$\sin A \sin B = \frac{1}{2}\left\{ \cos\,(A-B) - \cos\,(A+B) \right\},$$

$$\cos A \cos B = \frac{1}{2}\left\{ \cos\,(A-B) + \cos\,(A+B) \right\},$$

  will be crucial when we come to study **Fourier Series** in Chapters 3 and 4, but are not on your Formula Sheet.

1.3 Partial Derivatives

- **Partial Differential Equations** (Chapter 6) involve partial derivatives.
- For example, if $f(x,y) = e^{-x}\cos \pi y$,

$$\frac{\partial f}{\partial x} = -e^{-x}\cos \pi y, \quad \frac{\partial f}{\partial y} = -\pi e^{-x}\sin \pi y,$$

$$\frac{\partial^2 f}{\partial x^2} = e^{-x}\cos \pi y, \quad \frac{\partial^2 f}{\partial y^2} = -\pi^2 e^{-x}\cos \pi y, \quad \frac{\partial^2 f}{\partial x \partial y} = \pi e^{-x}\sin \pi y.$$

- **Make sure that you are can confidently differentiate simple functions of more than one variable.**