University of Nottingham
SCHOOL OF MATHEMATICAL SCIENCES

MTHS2007 Advanced Mathematics and Statistics for Mechanical Engineers

Examination Formula Sheet

Fourier Series
The Fourier series for a periodic function $f(t)$ with period 2ℓ is

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{\ell} + b_n \sin\frac{n\pi x}{\ell}\right),$$

where

$$a_n = \frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\cos\frac{n\pi x}{\ell}\mathrm{d}x, \quad b_n = \frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\sin\frac{n\pi x}{\ell}\mathrm{d}x.$$

Properties of Trigonometric Functions

$$\tan x = \frac{\sin x}{\cos x}, \qquad \cot x = \frac{\cos x}{\sin x}, \qquad \sec x = \frac{1}{\cos x}, \qquad \mathrm{cosec} x = \frac{1}{\sin x},$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A \sin B, \qquad \cos(A \pm B) = \cos A\cos B \mp \sin A \sin B,$$

$$\sin 2A = 2\sin A\cos A, \qquad \cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A,$$

$$\sin A + \sin B = 2\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B), \quad \sin A - \sin B = 2\sin\frac{1}{2}(A-B)\cos\frac{1}{2}(A+B),$$

$$\cos A + \cos B = 2\cos\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B), \quad \cos A - \cos B = 2\sin\frac{1}{2}(A+B)\sin\frac{1}{2}(B-A).$$

Statistics and Probability Sum rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
Conditional Probability: $P(A|B) = P(A \cap B)/P(B)$.

$X \sim \mathsf{Bin}(n,p)$ means $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$ $(k = 0, \ldots, n)$,
$E(X) = np$, $V(X) = np(1-p)$.

$X \sim \mathsf{Poi}(\lambda)$ means $P(X = k) = \dfrac{e^{-\lambda}\lambda^k}{k!}$ $(k = 0, 1, \ldots)$, $E(X) = \lambda$, $V(X) = \lambda$.

$X \sim \mathsf{N}(\mu, \sigma^2)$ means $P(X \le x) = \displaystyle\int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} \mathrm{e}^{-\frac{1}{2}(\frac{t-\mu}{\sigma})^2}\,\mathrm{d}t$ $(x \in \mathbb{R})$, $E(X) = \mu$, $V(X) = \sigma^2$.

If $X \sim \mathsf{N}(\mu, \sigma^2)$ then $Z = \frac{X-\mu}{\sigma} \sim \mathsf{N}(0,1)$.
$E(X \pm Y) = E(X) \pm E(Y)$ for any random variables X and Y.
$V(X \pm Y) = V(X) + V(Y)$ for independent random variables X and Y.
Sample mean $\bar{x} = \frac{1}{n}\sum_i x_i$, sample variance $s^2 = \frac{1}{n-1}\sum_i (x_i - \bar{x})^2 = \frac{1}{n-1}(\sum_i x_i^2 - n\bar{x}^2)$.
If observations are normally distributed or sample size is large then the endpoints of a $100(1-\alpha)\%$ CI for the mean are $\bar{x} \pm z_{\alpha/2}\sigma/\sqrt{n}$, where $P(Z > z_{\alpha/2}) = \alpha/2$. Valid to replace σ with s so long as sample size is large.

A test of the null hypothesis $\mu = \mu_0$ uses the test statistic $Z = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim \mathsf{N}(0,1)$. The same conditions apply and the same extensions are valid as for the CI calculation above.

Table of Laplace Transforms

	$f(t)$	$\bar{f}(s)$	
1	1	$\frac{1}{s}, \quad s > 0$	
2	t	$\frac{1}{s^2}, \quad s > 0$	
3	$t^n \quad n = 0, 1, 2, \ldots$	$\frac{n!}{s^{n+1}}, s > 0$	
4	e^{at}	$\frac{1}{s-a}, \quad s > a$	
5	$\sin at$	$\frac{a}{s^2+a^2}, \quad s > 0$	
6	$\cos at$	$\frac{s}{s^2+a^2}, \quad s > 0$	
7	$\sinh at$	$\frac{a}{s^2-a^2}, \quad s > \lvert a \rvert$	
8	$\cosh at$	$\frac{s}{s^2-a^2}, \quad s > \lvert a \rvert$	
9	$t \sin kt$	$\frac{2ks}{(s^2+k^2)^2}$	
10	$t \cos kt$	$\frac{s^2-k^2}{(s^2+k^2)^2}$	
11	$H(t-a)$ unit step function (Heaviside step function)	$\frac{\mathrm{e}^{-as}}{s}$	
12	$\delta(t-a)$ unit impulse function (Dirac delta function)	e^{-as}	
13	$\mathrm{e}^{at} f(t)$	$\bar{f}(s-a)$	First shifting theorem
14	$f(t-a)H(t-a)$	$\mathrm{e}^{-as}\bar{f}(s)$	Second shifting theorem
15	$\int_0^t f(u)g(t-u)\mathrm{d}u$	$\bar{f}(s)\bar{g}(s)$	Convolution theorem
16	$f'(t)$	$s\bar{f}(s) - f(0)$	
17	$f''(t)$	$s^2\bar{f}(s) - sf(0) - f'(0)$	

Table of Normal Distribution

Z is the *standardised* normal variate $N(0,1)$

$f(z)$ is the probability density function $\frac{1}{\sqrt{2\pi}}\mathrm{e}^{-\frac{1}{2}z^2}$

$F(z)$ is the cumulative distribution function $\int_{-\infty}^{z}\frac{1}{\sqrt{2\pi}}\mathrm{e}^{-\frac{1}{2}t^2}\mathrm{d}t$

$f(-z)=f(z), \quad F(-z)=1-F(z)$

		F(z)									
f(z)	z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.399	0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.397	0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.391	0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.381	0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.368	0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.352	0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.333	0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.312	0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.290	0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.266	0.9	0.8159	0.8168	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
0.242	1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
0.218	1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
0.194	1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
0.171	1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
0.150	1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
0.130	1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
0.111	1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
0.094	1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
0.079	1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
0.066	1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
0.054	2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
0.044	2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
0.035	2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
0.028	2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
0.022	2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
0.018	2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
0.014	2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
0.010	2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
0.008	2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
0.006	2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

z	0.675	1.282	1.645	1.960	2.326	2.576	3.090
F(z)	0.750	0.900	0.950	0.975	0.990	0.995	0.999