

Probability and Statistics

Problem Sheet 6

- Components are made by one of four machines: A, B, C or D. Machine A makes 30% of the components, machine B makes 17%, machine C makes 21% and machine D makes the rest. Calculate the probability that a component is made by
 - machine A or machine B;
 - a machine other than machine C;
 - machine B or machine D.
- A box contains 10 red, 12 blue and 8 yellow blocks.
 - A block is picked at random. What is the probability that it is blue?
 - A block is picked at random, its colour noted and then it is replaced. This is then repeated. Calculate the probability that
 - both blocks are yellow;
 - one block is yellow and one is red;
 - both blocks are the same colour;
 - the two block are different colours.
 - Repeat the questions in (b) above assuming that the first block is *not* replaced.
- Students sit for two tests. The pass rate for the first test is 75% and for the second test 60%; and 50% of students pass both tests. Determine the probability that
 - a student passes at least one of the tests;
 - a student fails both tests;
 - a student passes the second test, given that they passed the first;
 - a student passes the second test, given that they failed the first.
- Machines X and Y manufacture spark plugs. Machine X makes 65% of spark plugs and machine Y makes the rest. Spark plugs made by machine X are acceptable 98% of the time; those made by machine Y are acceptable 95% of the time. A spark plug is chosen at random. Determine the probability that it is
 - acceptable, given that it is made by machine X;
 - not acceptable, given it is made by machine Y;
 - acceptable;
 - made by machine X, given it is acceptable.
- The probability that each component produced by a particular process is acceptable is 0.92. If ten components are selected at random, determine the probability that

*erroneously written as B
in earlier version*

- (a) all ten are acceptable;
 - (b) eight are acceptable;
 - (c) at least eight are acceptable.
 - (d) What is the mean number that are acceptable?
6. The foreman of the casting section in a certain factory finds that on average 1 in 5 of the castings are defective. If the section makes 8 castings in a day, what is the probability that 2 of these will be defective? What is the probability that 6 or more defective castings are made in a day?
7. The mean and variance of a binomial distribution are 2 and 1.5 respectively. What is the probability that this binomial distribution takes the value 1?
8. A machine that produces screws produces on average 2 defective screws out of 100; and the screws are packed in boxes of 250. Use the Poisson distribution to determine the probability that a box contains
- (a) no defective screws;
 - (b) two defective screws;
 - (c) more than one defective screw.
9. A particular plant using chlorine gas releases a detectable amount twice a month on average. Assuming the number of releases follows a Poisson distribution:
- (a) Find the probability that there will be no more than 4 releases during a month.
 - (b) What is the expected number of releases during a 3 month period?
10. If the random variable X has a normal distribution with mean $\mu = 12$ and standard deviation $\sigma = 2$ then find the following probabilities:
- (a) $P(X > 10)$;
 - (b) $P(11.4 < X < 14.2)$;
 - (c) $P(9.9 < X < 13.8)$.
11. A random variable Z follows the standard normal distribution, $Z \sim N(0, 1)$. Find the value of the constant c such that
- (a) $P(Z \leq c) = 0.95$;
 - (b) $P(Z \geq c) = 0.90$;
 - (c) $P(-c \leq Z \leq c) = 0.99$.
12. Sacks of grain packed by an automatic loader have an average weight of 51.8kg. It is found that 10% of the sacks are over 52.7kg. Assuming that the weights follow a normal distribution, find the standard deviation of the weights of the sacks.
13. 500 candidates are to sit an examination, in which individual marks are known to be normally distributed with mean 60 and standard deviation 12. Calculate

- (a) How many candidates are expected to pass if 50 is the pass mark;
- (b) How many are expected to score above 80;
- (c) What the pass mark should be changed to in order that 450 candidates are expected to pass.

14. A firm produces industrial electrical motors according to two specifications, a 'standard' motor and a 'long-life' motor. The long-life motors are found to have mean lives of 1700 hours and standard deviation 320 hours. 'Standard' motors are found to have a mean life of 1400 hours and a standard deviation of 240 hours.

If in each case the lifetime is assumed to follow a normal distribution and a motor of each type is chosen at random find


- (a) the probability that a standard motor will last more than 1000 hours;
- (b) the probability that the standard motor will last longer than the long-life motor;
- (c) the probability that the long-life motor will outlast the standard motor by at least 800 hours.

If delivery is taken of four of each type, what is the probability that the total life of the standard motors will exceed the total life of the long-life motors?

15. A random variable is assumed to follow a Normal distribution with unknown mean μ and variance 150. A sample of size 10 gives the following data summaries:

$$\sum_{i=1}^{10} x_i = 170.8, \quad \sum_{i=1}^{10} x_i^2 = 2041.9.$$

erroneously written as 2.0 in previous version

- (a) Obtain an estimate of the unknown mean μ .
- (b) Construct a 90% confidence interval for μ .
- (c) Suppose it is desired to obtain a 90% confidence interval for μ with a width of 5.0.  What sample size is necessary to achieve this level of precision?

16. The times (in seconds) necessary to fill a container of given capacity were recorded. Thirty five experiments were carried out using a new design of filling machine, with the mean time being 14.913 seconds and standard deviation 0.327.

- (a) Obtain a 95% confidence interval for the mean filling time for this machine.
- (b) What assumptions do you need to make for your calculation to be valid?
- (c) What does the confidence interval suggest about the manufacturer's claim that the mean filling time of the new machine is less than 15 seconds?