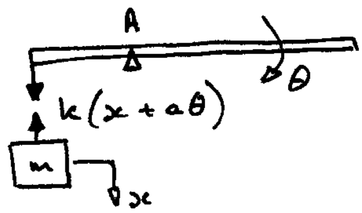
STRUCTURAL VIBRATION 1 SHEET 2

Q1 If you have difficulty with (a) or (b), please go to the next Subject Tutorial.

For (b), note that for springs in PARALLEL, $K = k_1 + k_2 + \ldots$

For springs in SERIES, $\dfrac{1}{K} = \dfrac{1}{k_1} + \dfrac{1}{k_2} + \ldots$

(c)



Equations of motion

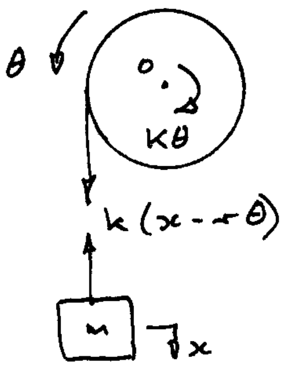
Bar: A $\ddot{\theta}$

$$-k(x + a\theta)a = I\ddot{\theta}$$

Mass: $\ddot{x}$

$$-k(x + a\theta) = m\ddot{x}$$

(d)



Equations of motion

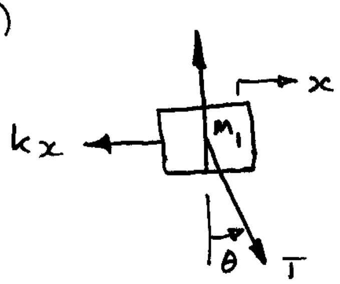
Disc: O $\ddot{\theta}$

$$k(x - r\theta)r - K\theta = I\ddot{\theta}$$

Mass: $\ddot{x}$

$$-k(x - r\theta) = m\ddot{x}$$

(e)



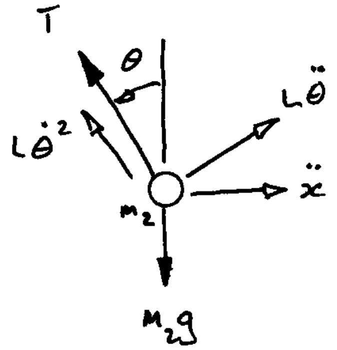
Equation of motion

Mass m_1: $\ddot{x}$

$$T\sin\theta - kx = m_1\ddot{x}$$

For small θ,

$$T\theta - kx = m_1\ddot{x} \qquad (1)$$



Linear equations of motion involve the <u>absolute</u> acceleration of the centre of mass. Here, mass m_2 has components of $L\dot{\theta}^2$ and $L\ddot{\theta}$ <u>relative to</u> mass m_1, which itself has an absolute acceleration $\ddot{x}$. Equations for horizontal an vertical translation are

$$\rightarrow: \quad -T\sin\theta = m_2\left[\ddot{x} + L\ddot{\theta}\cos\theta - L\dot{\theta}^2\sin\theta\right]$$

$$\uparrow: \quad T\cos\theta - m_2 g = m_2\left[L\ddot{\theta}\sin\theta + L\dot{\theta}^2\cos\theta\right]$$

For small θ, these become

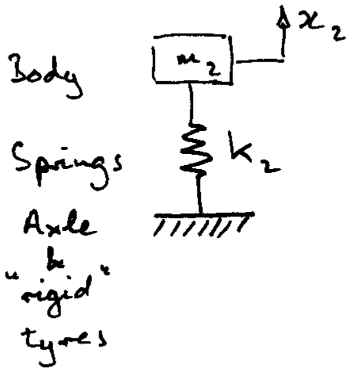
$$-T\theta = m_2\left[\ddot{x} + L\ddot{\theta} - L\dot{\theta}^2\theta\right] \qquad (2)$$

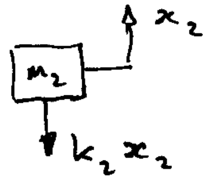
$$T - m_2 g = m_2\left[L\ddot{\theta}\theta + L\dot{\theta}^2\right] \qquad (3)$$

Neglect the products of small quantities so that

$\dot{\theta}^2\theta = \ddot{\theta}\theta = \dot{\theta}^2 = 0$ and then eliminate T from equations (1) - (3).

2(a) 1 degree-of-freedom model





$$-k_2 x_2 = m_2\ddot{x}_2$$

$$\text{or} \quad m_2\ddot{x}_2 + k_2 x_2 = 0$$

$$\omega_n = \sqrt{\frac{k_2}{m_2}} = \sqrt{\frac{40{,}000}{350}} = 1.70\ \text{Hz}$$

2(b) 2 degree-of-freedom model - see lecture notes

3. Equations of motion derived in Q1(a)

Put $x_1 = X_1 \cos \omega t$, etc and substitute for mass and stiffness values

$$\begin{bmatrix} 10^4 - 5\omega^2 & -10^4 & 0 \\ -10^4 & 4\times10^4 - 5\omega^2 & -3\times10^4 \\ 0 & -3\times10^4 & 3\times10^4 - 10\omega^2 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

Put $\phi = \dfrac{5\omega^2}{10^4}$ to simplify subsequent algebra. Hence

$$\begin{bmatrix} 1-\phi & -1 & 0 \\ -1 & 4-\phi & -3 \\ 0 & -3 & 3-2\phi \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad \begin{matrix} (i) \\ (ii) \\ (iii) \end{matrix}$$

The frequency equation is

$$2\phi^3 - 13\phi^2 + 12\phi = 0$$

Roots give the natural frequencies.

Mode shapes

From (i), $(1-\phi)X_1 - X_2 = 0 \quad \therefore \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 1/(1-\phi) \\ 1 \end{Bmatrix}$

From (iii), $-3X_2 + (3-2\phi)X_3 = 0 \quad \therefore \begin{Bmatrix} X_3 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 3/(3-2\phi) \\ 1 \end{Bmatrix}$

Combining the two, we get,

$$\begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} 1/(1-\phi) \\ 1 \\ 3/(3-2\phi) \end{Bmatrix}$$