Department of Mechanical, Materials & Manufacturing Engineering

DYNAMICS (VIBRATION)

SHEET 5 : HARMONIC EXCITATION

1. A circular steel shaft (length 1 m, diameter 40 mm) is clamped at one end and carries a flywheel with a moment of inertia of 2 kgm^2 at the other end. The torsion formula is $\frac{2\tau_{MAX}}{d} = \frac{32\,T}{\pi\,d^4} = \frac{G\,\theta}{L}$, and assume that the shear modulus, G, is 80 GPa.

   i) Use the torsion formula to find the maximum shear stress in the shaft due to a ***static*** torque of 800 Nm applied to the flywheel.

   ii) Calculate the undamped natural frequency for torsional vibration.

   iii) If, instead of the static torque, a ***sinusoidally alternating*** torque with amplitude 800 Nm and frequency 12 Hz is applied to the flywheel, solve the equation of motion to find the *steady-state* amplitude of the twist in the shaft, neglecting damping. Hence find the corresponding maximum shear stress in the shaft.

   iv) A torsional damper with damping coefficient 100 Nms/rad is now connected between the flywheel and ground. Re-solve the equation of motion for the sinusoidal excitation case in part (*iii*) to obtain the new steady-state maximum shear stress.

   v) Calculate the phase angle between the angular displacement of the flywheel and the applied torque for the problem in part (*iv*).

   (i) 63.66 MPa; (*ii*) *15.96 Hz;* (*iii*) *0.09153 rad; 146.4 MPa;*
   (*iv*) *0.06931 rad; 110.9 MPa;* (*v*) *-40.78º*

2. Derive the frequency response function $\left(= \frac{\Theta^*}{P} \right)$ for the system in Figure Q2.

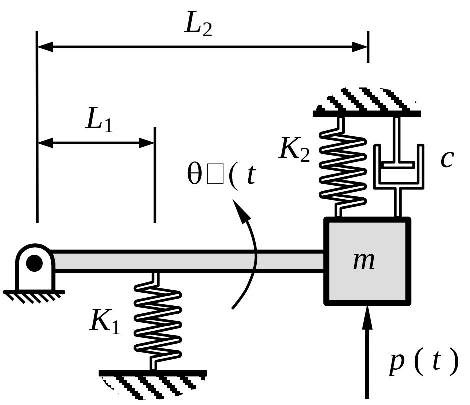

Figure Q2

$$H(\omega) = \frac{L_2}{\left(K_1 L_1^2 + K_2 L_2^2 - m L_2^2 \omega^2\right) + \mathrm{i}\omega c L_2^2}$$

3. Figure Q3 shows a hopper used to de-aerate powder prior to bagging. The filled hopper has a moment of inertia I_O about the pivot at O and is supported by a resilient mount at A that is equivalent to a spring of stiffness k_1 in parallel with a viscous damper with coefficient c_1. De-aeration is achieved by vibrating the hopper using a cam to impart a sinusoidal displacement of amplitude Y and frequency ω to the cam follower. The follower is separated from the hopper by a second resilient mount; equivalent to a spring of stiffness k_2 in parallel with a viscous damper with coefficient c_2.

Show that the steady-state angular displacement of the hopper is

$$\frac{L_2\left(k_2 + \mathbf{i}\,\omega\, c_2\right)Y}{\left(k_1 L_1^2 + k_2 L_2^2 - I_O\,\omega^2\right) + \mathbf{i}\,\omega\left(c_1 L_1^2 + c_2 L_2^2\right)}$$

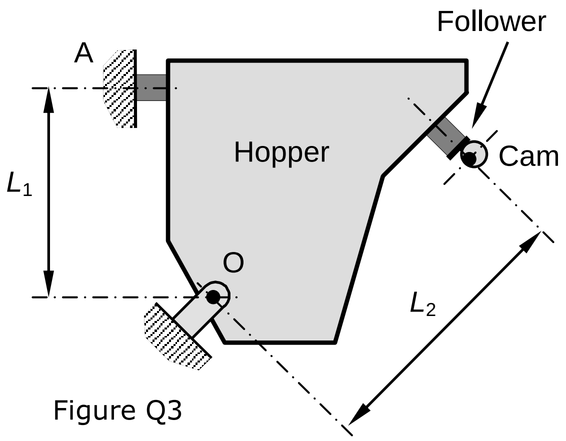

Figure Q3