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LECTURE 6B

Operational Amplifier

Electromechanical Devices

MMME2051

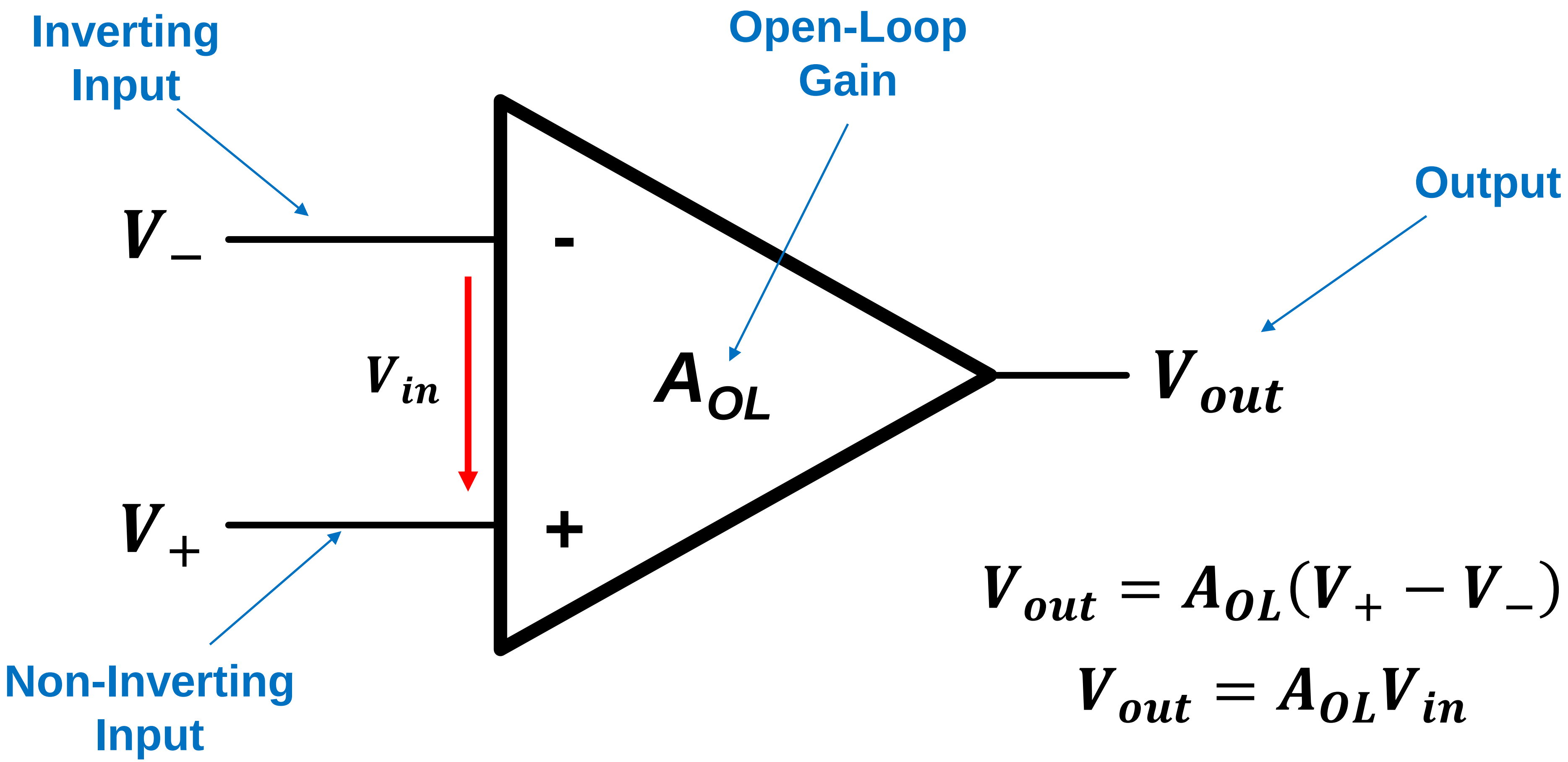
Module Convenor – Surojit Sen

Learning Outcomes

- **Operational Amplifier** (Op-Amp)
- Applications of Digital Circuit
  - **Voltage Follower**
  - **Inverting** Amplifier
  - **Non-Inverting** Amplifier
  - **Summing** Amplifier
  - **Piezoelectric** properties of Quartz
  - **Integrating** Amplifier
  - **Differencing** Amplifier

Operational Amplifier

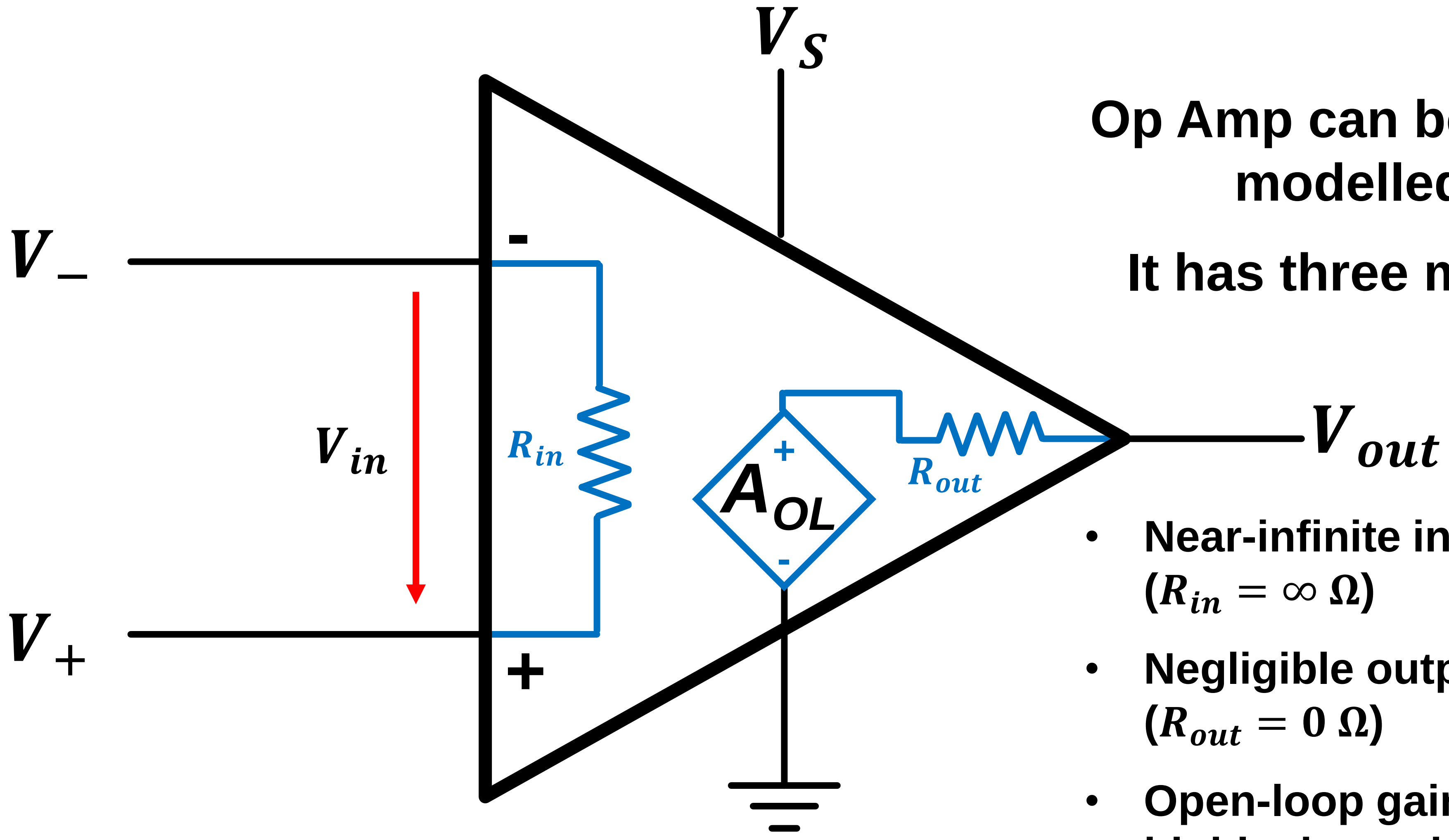
Amplifier intended for mathematical operations



$$V_{out} = A_{OL}(V_+ - V_-)$$

$$V_{out} = A_{OL}V_{in}$$

Operational Amplifier

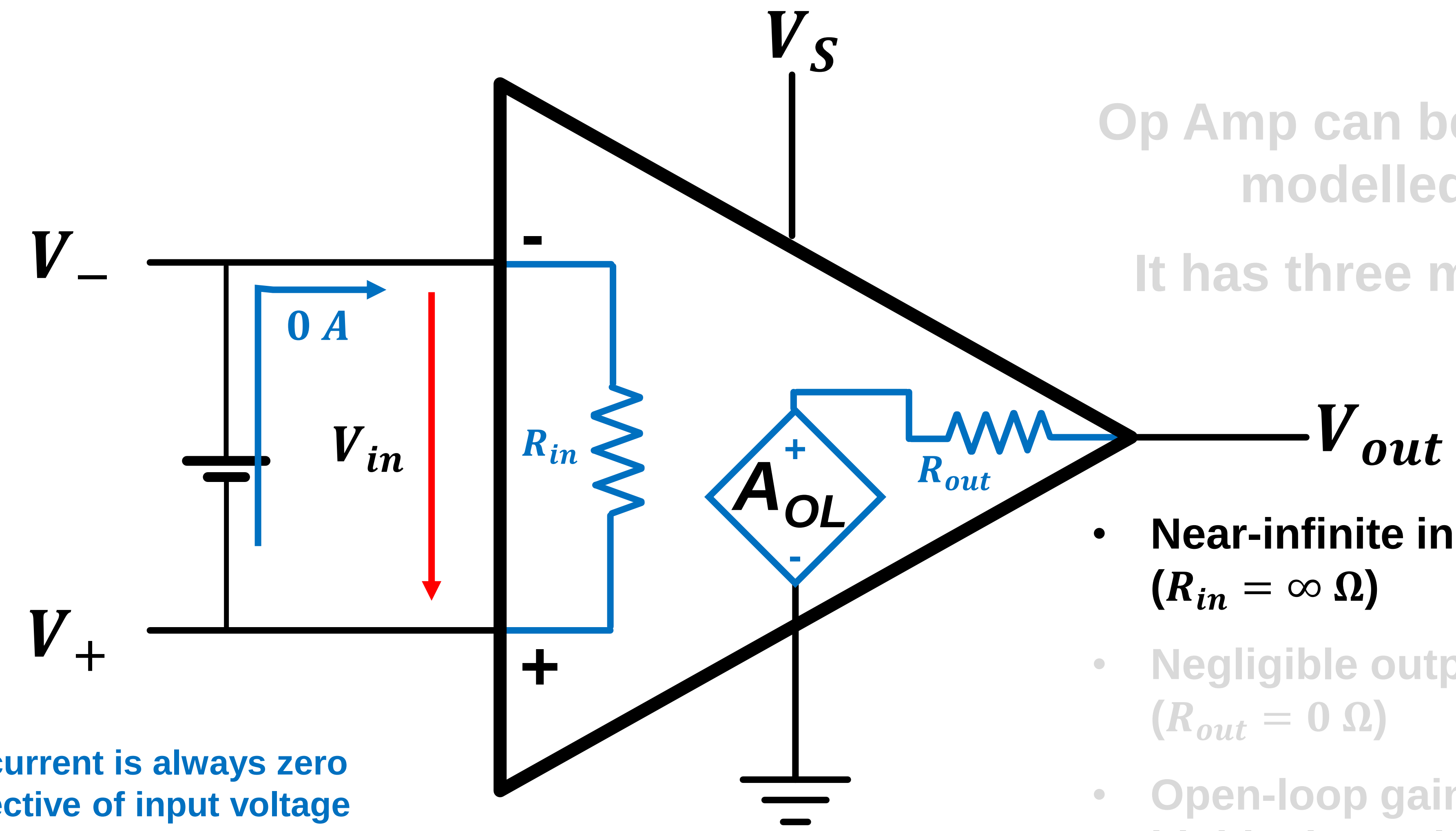


Op Amp can be simplistically modelled like this

It has three main features:

- **Near-infinite input impedance ($R_{in} = \infty\ \Omega$)**
- **Negligible output impedance ($R_{out} = 0\ \Omega$)**
- **Open-loop gain is very high and highly dependent on frequency**

Operational Amplifier



Input current is always zero irrespective of input voltage

Op Amp can be simplistically modelled like this

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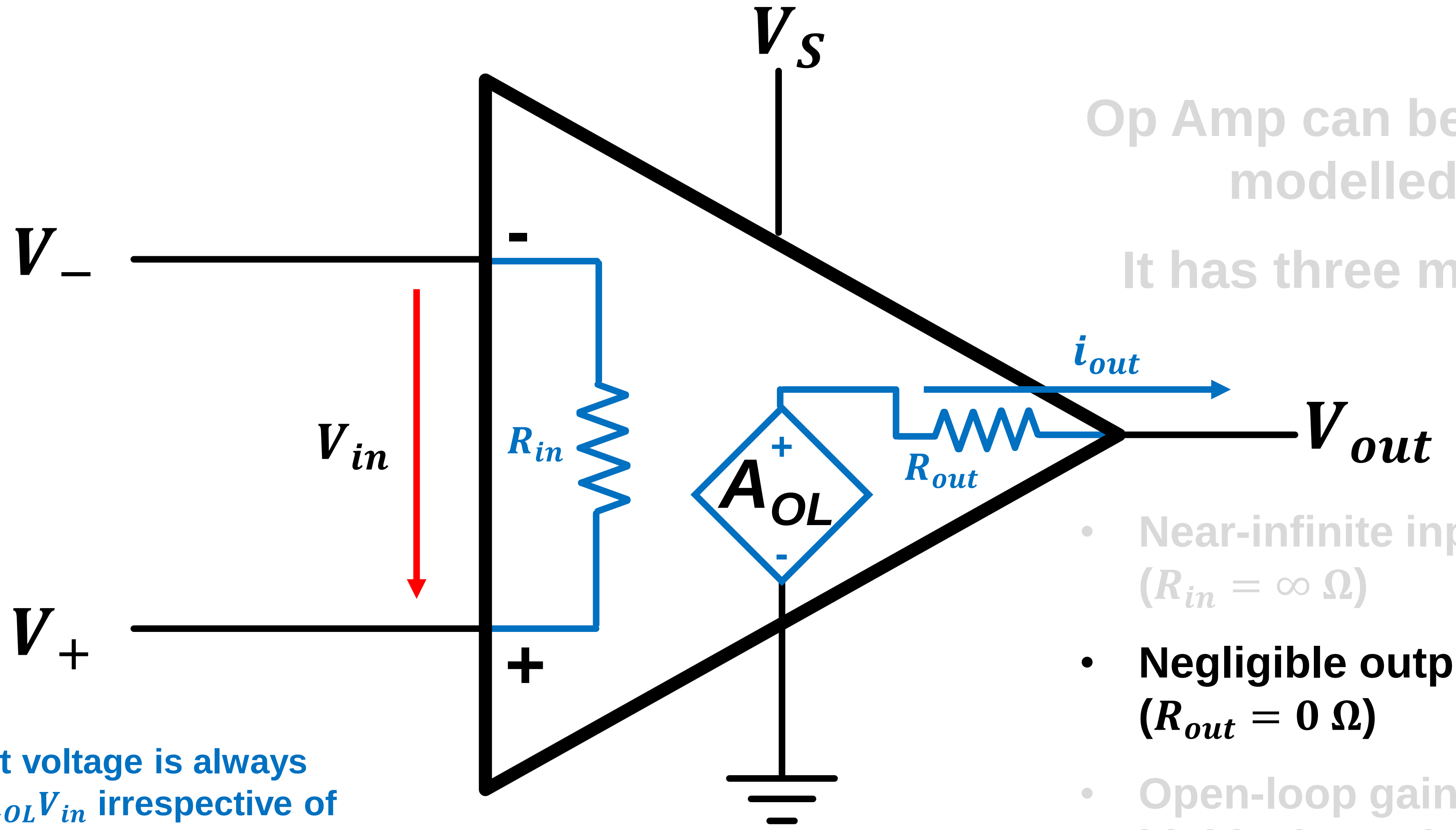
Operational Amplifier



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It has three main features:

- Near-infinite input impedance ($R_{in} = \infty\ \Omega$)
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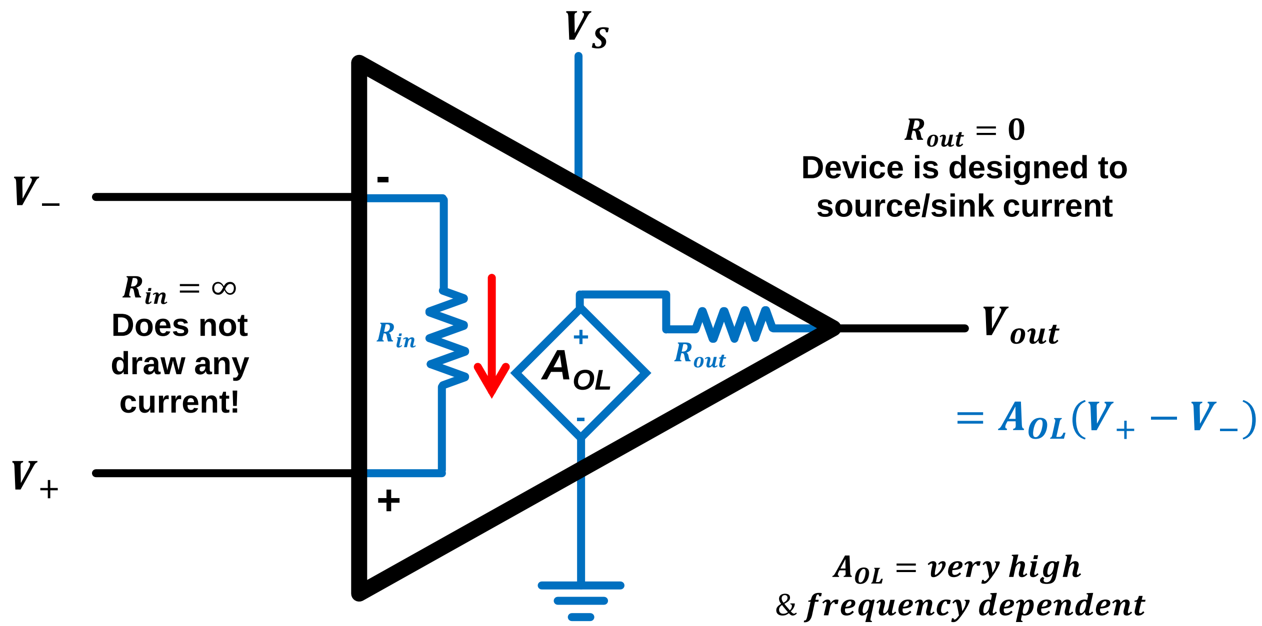
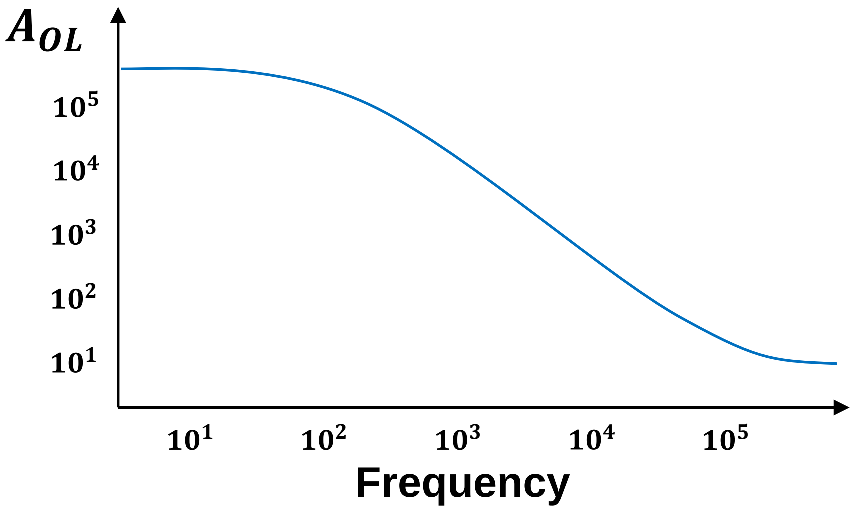
- Open-loop gain is very high and highly dependent on frequency

Output voltage is always $V_{out} = A_{OL}V_{in}$ irrespective of output current

Operational Amplifier

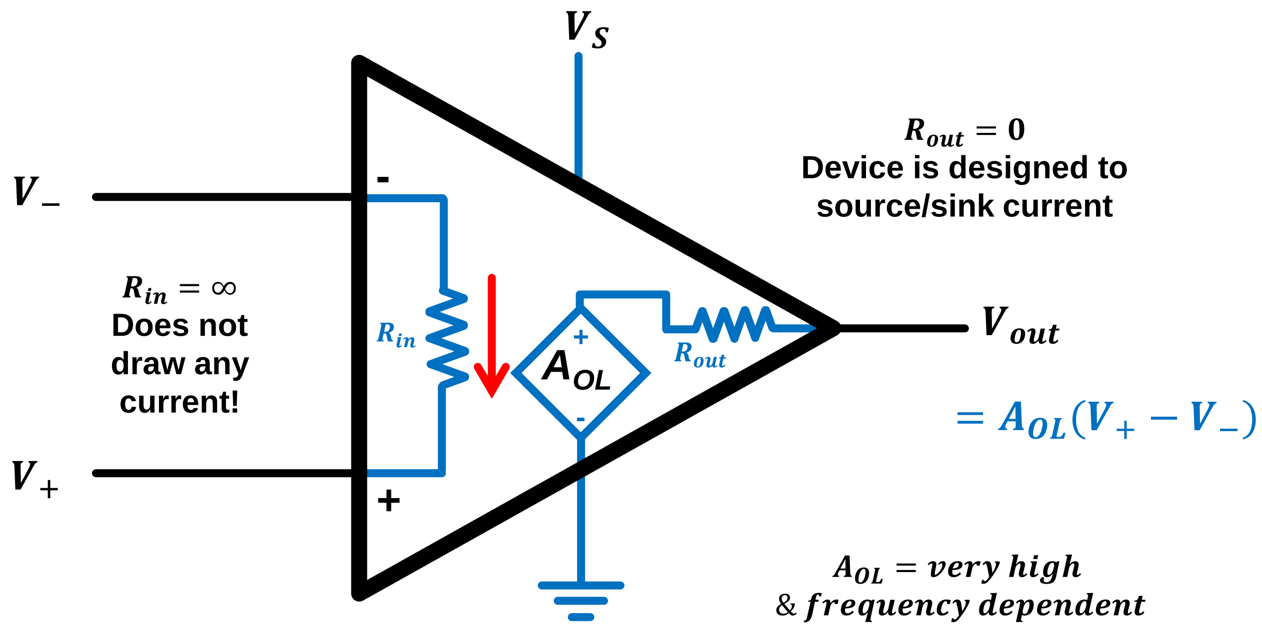


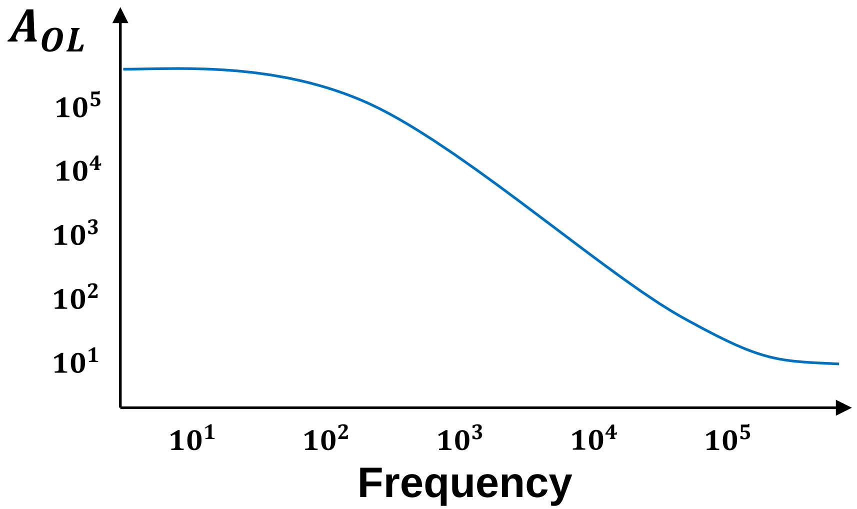


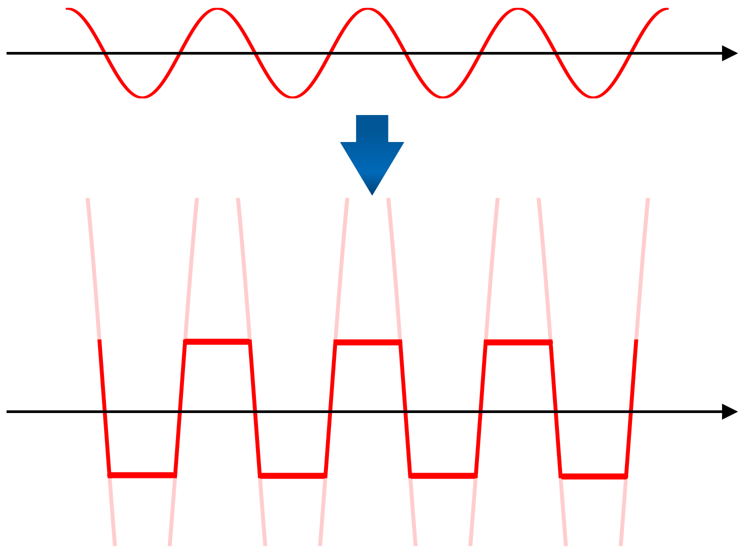
Open Loop gain is highly dependent on the frequency of input voltage

Almost never you see signal processing of a single frequency signal – operating the op-amp open-loop is not a good idea

Operational Amplifier







The other problem is the gain is so high (order of 1 million) that unless the input voltage is of the order of microvolts, output voltage will get clipped

Op amps are low voltage devices – they would be running of 5V most likely

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Operational Amplifier in Closed Loop

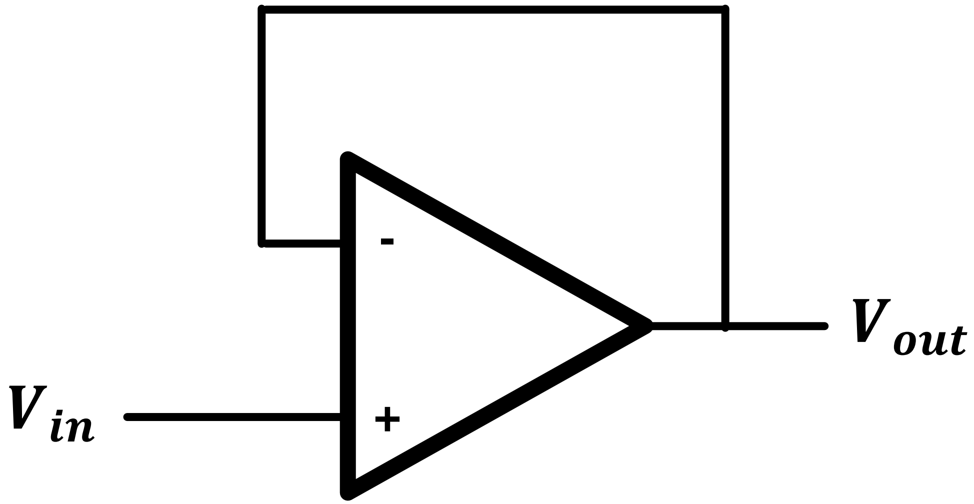
It is quite clear we cannot effectively use the op amp in open loop configuration

One smart way to deal with the issue is to offer up the output voltage (with all its issues) as feedback into the inverting input

This way you can eliminate the effect of A_{OL} to a partial extent

If we directly feed back output to its inverting input, we have a

Voltage Follower



Voltage Follower

$$V_{out} = A_{OL}(V_+ - V_-)$$

$$V_{out} = A_{OL}(V_{in} - V_{out})$$

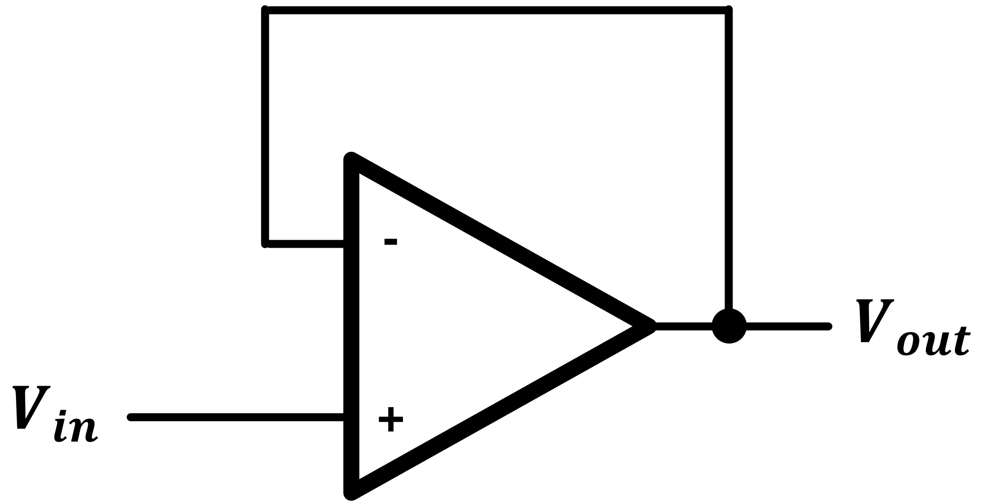
$$V_{out}(1 + A_{OL}) = A_{OL}V_{in}$$

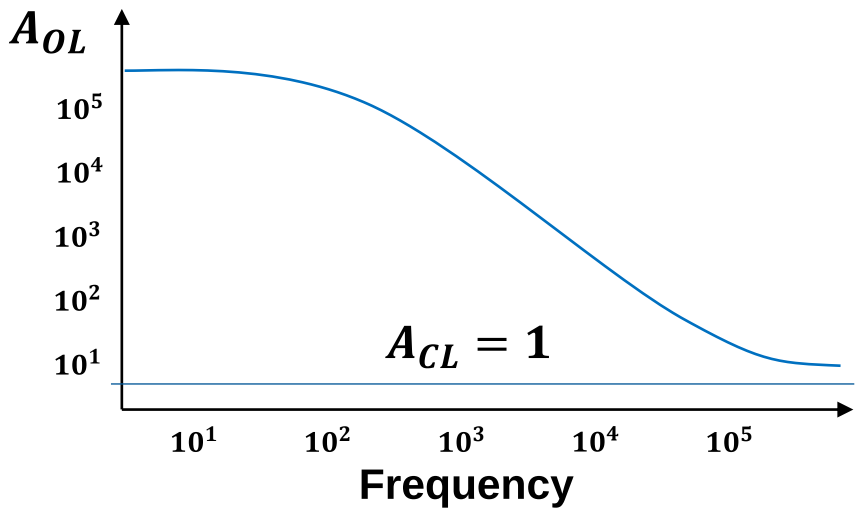
$$V_{out} = \frac{A_{OL}}{1 + A_{OL}}V_{in}$$

Safe to assume $A_{OL} \gg 1$

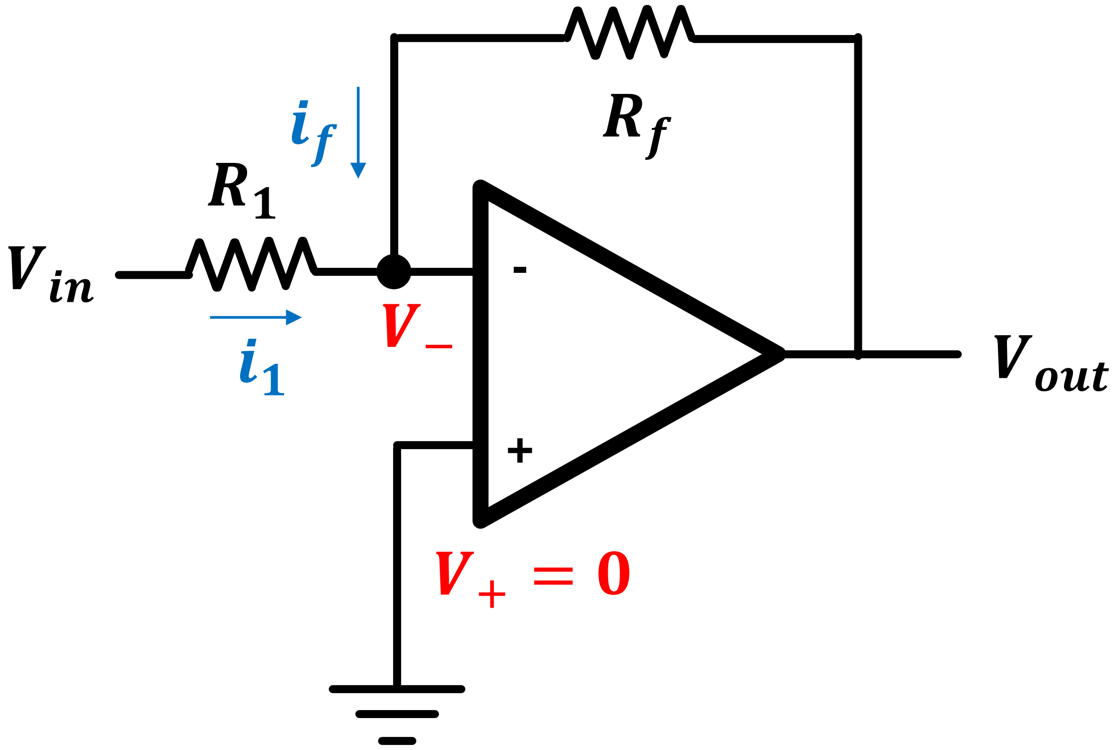
$$V_{out} = \frac{A_{OL}}{A_{OL}}V_{in}$$

$$\frac{V_{out}}{V_{in}} = A_{CL} = 1$$





Inverting Amplifier



$V_{out} = A_{OL}(V_+ - V_-)$

$i_f = -i_1$

$V_{out} = A_{OL}(0 - V_-)$

$V_{out} = -A_{OL}V_-$

$V_{in} - V_- = i_1 R_1$ and $V_{out} - V_- = i_f R_f$

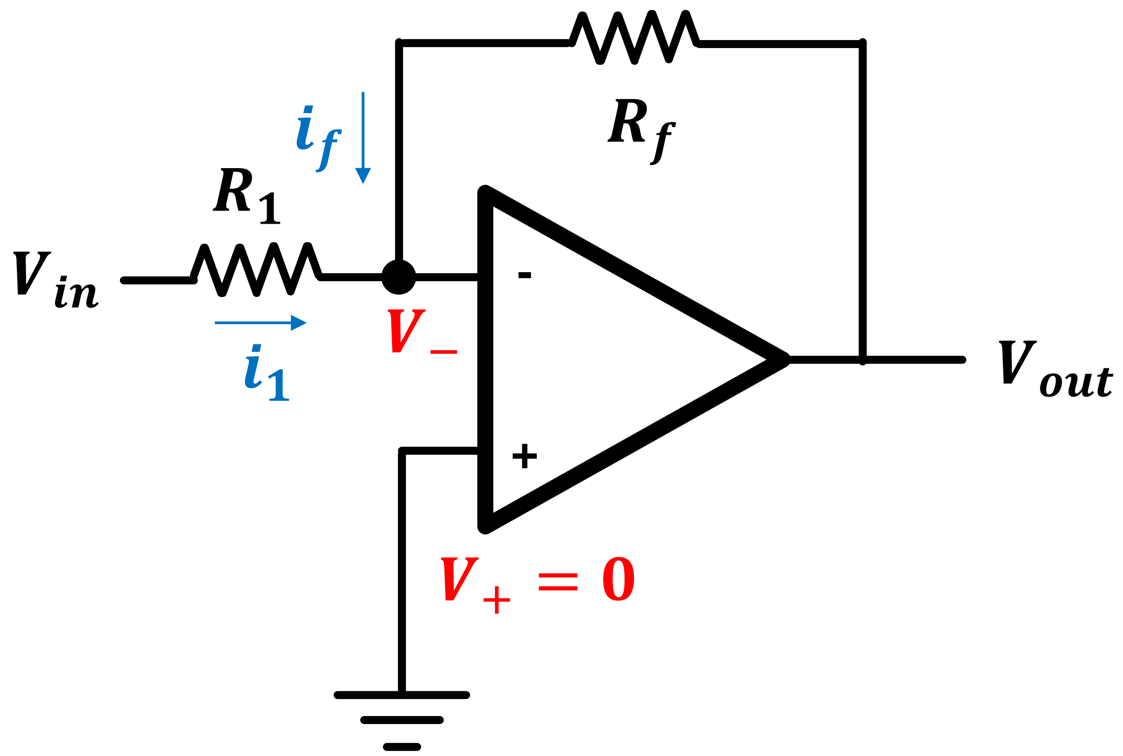
$$V_{in} - V_- = \frac{V_- - V_{out}}{R_f} R_1$$

$V_{in}R_f - V_-R_f = V_-R_1 - V_{out}R_1$

$V_{in}R_f + V_{out}R_1 = V_-R_1 + V_-R_f$

$V_{in}R_f + V_{out}R_1 = V_-(R_1 + R_f)$

Inverting Amplifier



$$V_{out} = A_{OL}(V_+ - V_-)$$

$$i_f = -i_1$$

$$V_{out} = A_{OL}(0 - V_-)$$

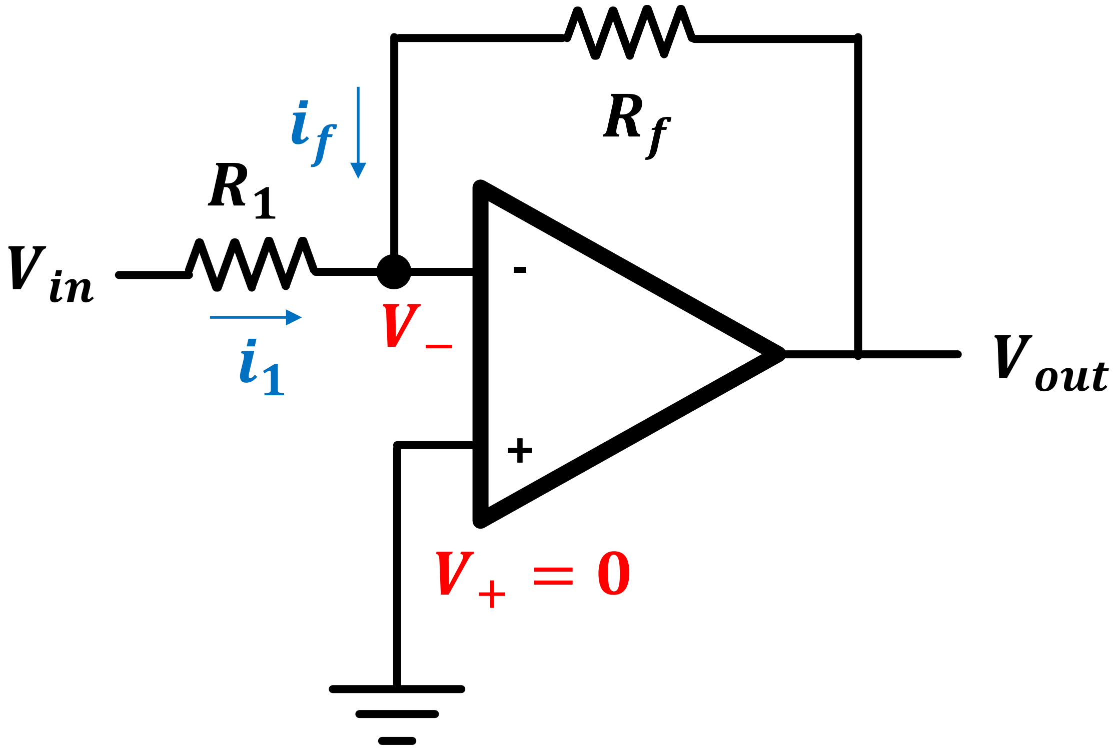
$$V_{out} = -A_{OL}V_-$$

$$V_{out} = -A_{OL}\frac{V_{in}R_f + V_{out}R_1}{R_1 + R_f}$$

$$V_{out}(1 + \frac{A_{OL}R_1}{R_1 + R_f}) = -A_{OL}\frac{V_{in}R_f}{R_1 + R_f}$$

$$V_{out}(\frac{A_{OL}R_1 + R_1 + R_f}{R_1 + R_f}) = -A_{OL}V_{in}\frac{R_f}{R_1 + R_f}$$

Inverting Amplifier



$$\frac{V_{out}}{V_{in}}\left(\frac{A_{OL}R_1 + R_1 + R_f}{R_1 + R_f}\right) = -A_{OL}\frac{R_f}{R_1 + R_f}$$

$$\frac{V_{out}}{V_{in}} = -A_{OL}\frac{R_f}{A_{OL}R_1 + R_1 + R_f}$$

$$\frac{V_{out}}{V_{in}} = \frac{-1}{\left(1 + \frac{1}{A_{OL}}\right)\frac{R_1}{R_f} + \left(\frac{1}{A_{OL}}\right)\frac{R_f}{R_f}}$$

$$\frac{V_{out}}{V_{in}} = \frac{-1}{(1 + 0)\frac{R_1}{R_f} + (0)1}$$

$$\frac{V_{out}}{V_{in}} = A_{CL} = -\frac{R_f}{R_1}$$

Non-Inverting Amplifier

$$V_{out} = A_{OL}(V_+ - V_-)$$
$$V_{out} = A_{OL}(V_{in} - V_-)$$

$$V_- = \frac{R_1}{R_1 + R_f} V_{out}$$

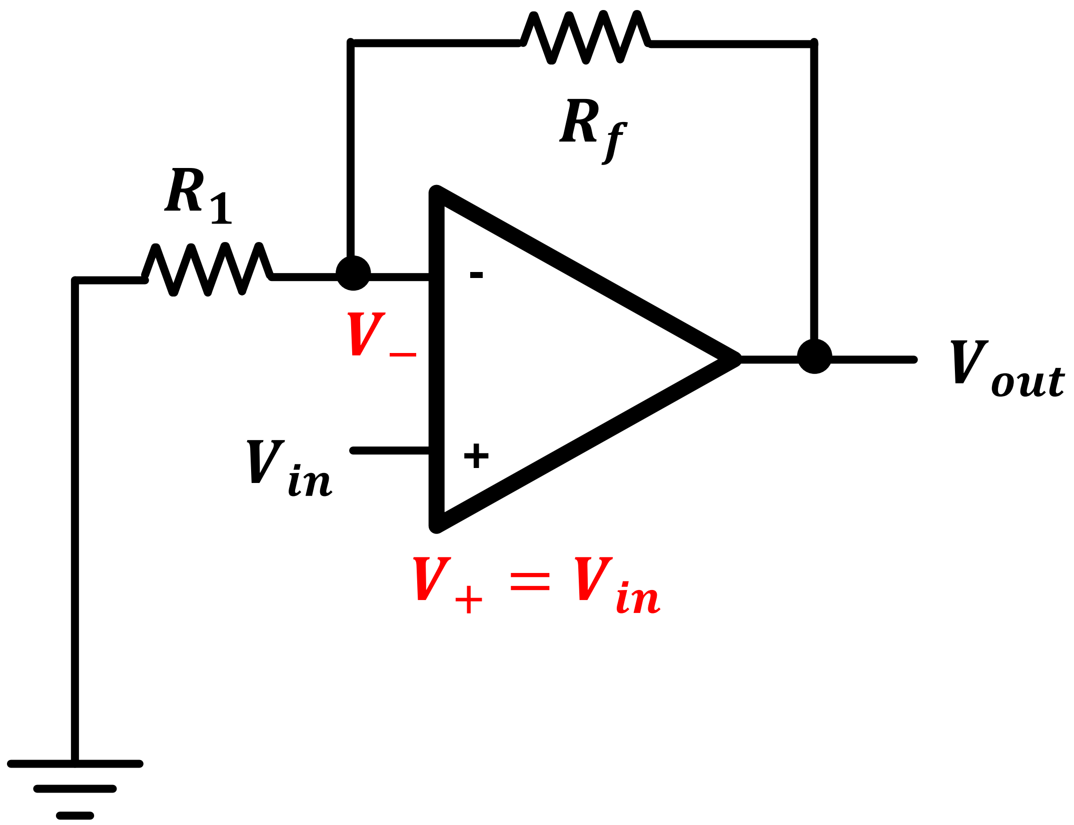
$$V_{out} = A_{OL}\left(V_{in} - \frac{R_1}{R_1 + R_f} V_{out}\right)$$

$$V_{out}(1 + A_{OL}\frac{R_1}{R_1 + R_f}) = A_{OL} V_{in}$$

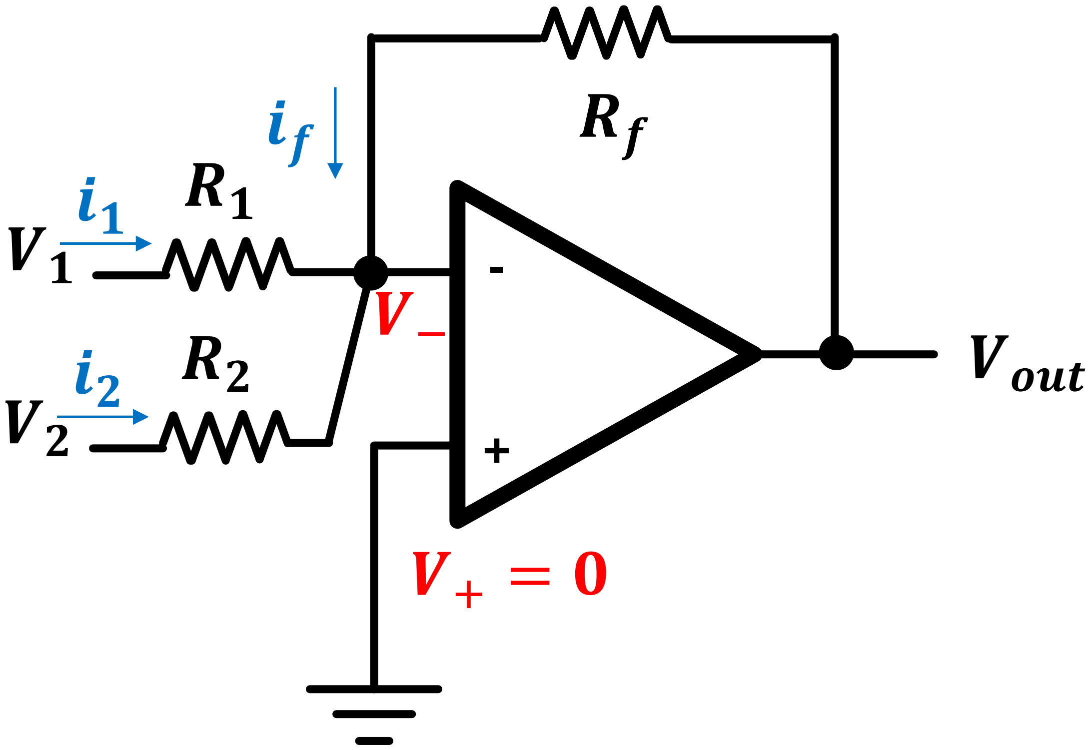
$$\frac{V_{out}}{V_{in}}(\frac{1}{A_{OL}} + \frac{R_1}{R_1 + R_f}) = 1$$

$$\frac{V_{out}}{V_{in}}(0 + \frac{R_1}{R_1 + R_f}) = 1$$

$$\frac{V_{out}}{V_{in}} = A_{CL} = 1 + \frac{R_f}{R_1}$$



Summing Amplifier

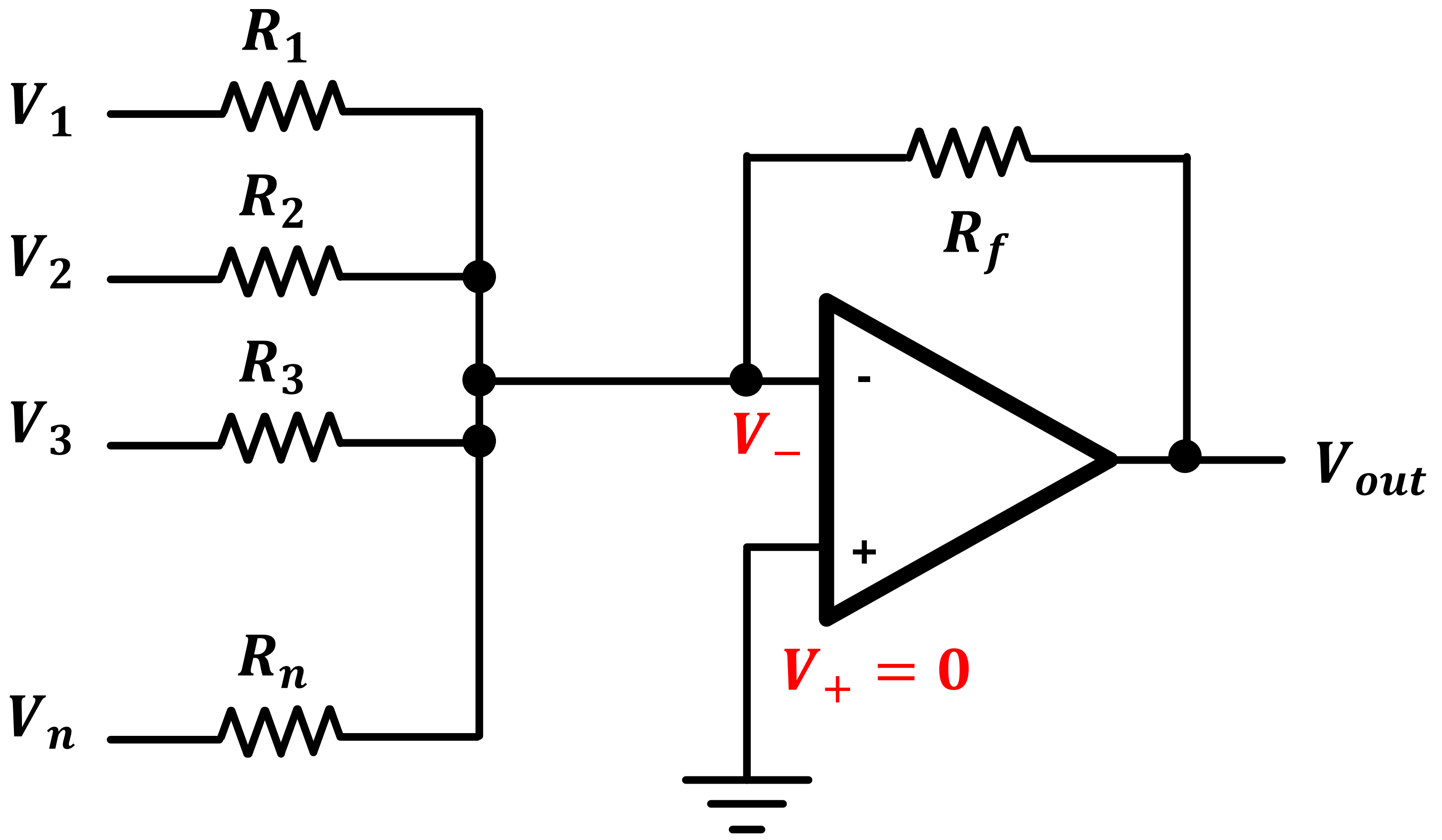


- $V_1 - V_- = i_1 R_1$
- $V_2 - V_- = i_2 R_2$
- $V_{out} - V_- = i_f R_f$
- $-A_{OL} V_- = V_{out}$

- $i_1 + i_2 = -i_f$
- $\frac{V_1 - V_-}{R_1} + \frac{V_2 - V_-}{R_2} = \frac{V_- - V_{out}}{R_f}$
- $\frac{V_1 + \frac{V_{out}}{A_{OL}}}{R_1} + \frac{V_2 + \frac{V_{out}}{A_{OL}}}{R_2} = \frac{-\frac{V_{out}}{A_{OL}} - V_{out}}{R_f}$
- $\frac{V_1 + 0}{R_1} + \frac{V_2 + 0}{R_2} = \frac{0 - V_{out}}{R_f}$

$$V_{out} = \frac{-R_f}{R_1} V_1 + \frac{-R_f}{R_2} V_2$$

Summing Amplifier



$$V_{out} = -R_f\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \cdots + \frac{V_n}{R_n}\right)$$

Remember superposition principle – we discussed this when we were solving for the R-2R Ladder circuit

Each input branch is effectively an individual and independent Inverting Amplifier circuit

Output voltage is essentially the sum of each input branch

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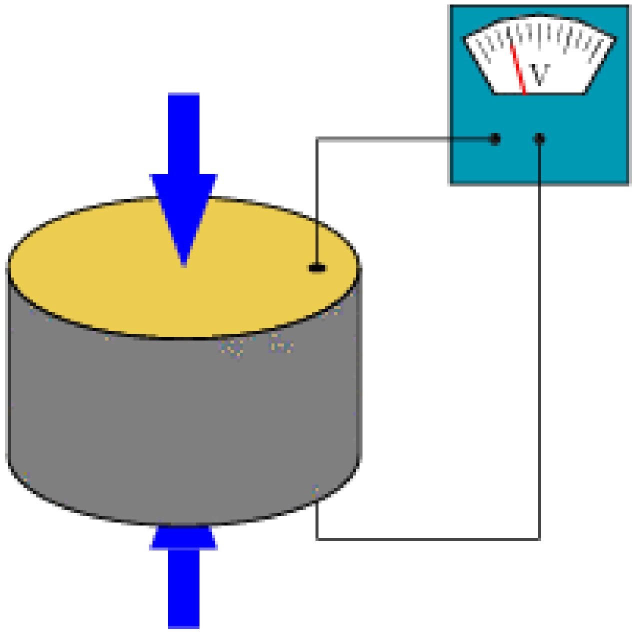
Piezoelectric effect of Quartz



Piezoelectricity is the electric charge that gets accumulated in some materials upon application of **mechanical stress**

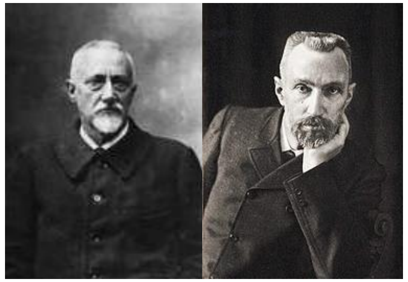
Words derived from *Piezein* (meaning 'to squeeze') and *Elektron* (electricity)

$$\boldsymbol{Q \propto F}$$

This relation is of extreme importance!

This allows us to measure force in terms of electricity

Let us see how we do this



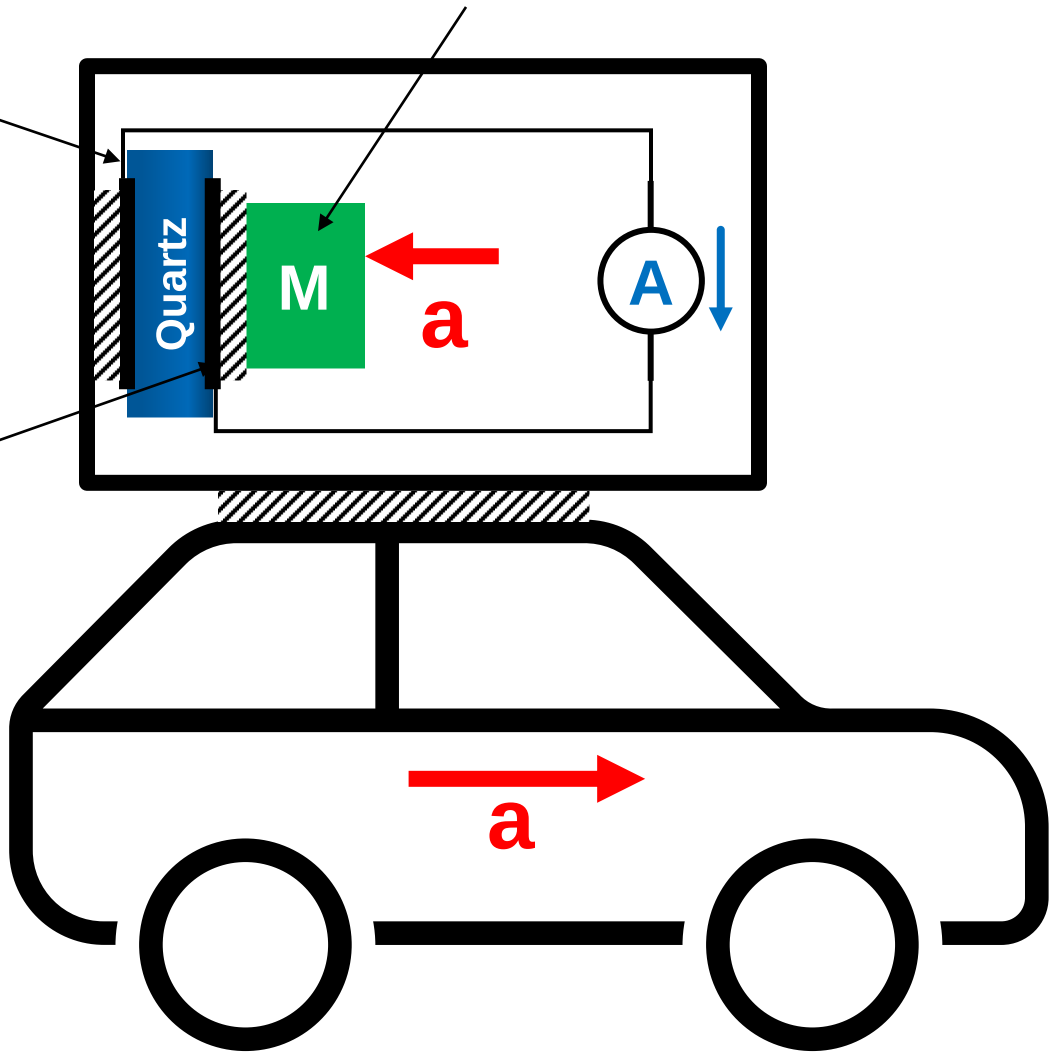
Jacques & Pierre Curie, French physicist brothers discovered piezoelectricity in 1880. Pierre is the same guy who won the Nobel prize with wife Marie Sklodowska-Curie for radiation

Piezoelectric effect of Quartz

Quartz crystal (or any other piezo device) rigidly attached on one face

Other face of quartz rigidly attached to the suspended mass

A very well known mass suspended freely



Let us find a relation between acceleration and current

$$Q \propto F$$

$$Q = k_1 F$$

$$Q = k_1 M a$$

Differentiating both sides with respect to time

$$\frac{dQ}{dt} = k_1 M \frac{da}{dt}$$

We know current is rate of movement of charge

$$i = k_1 M \frac{da}{dt}$$

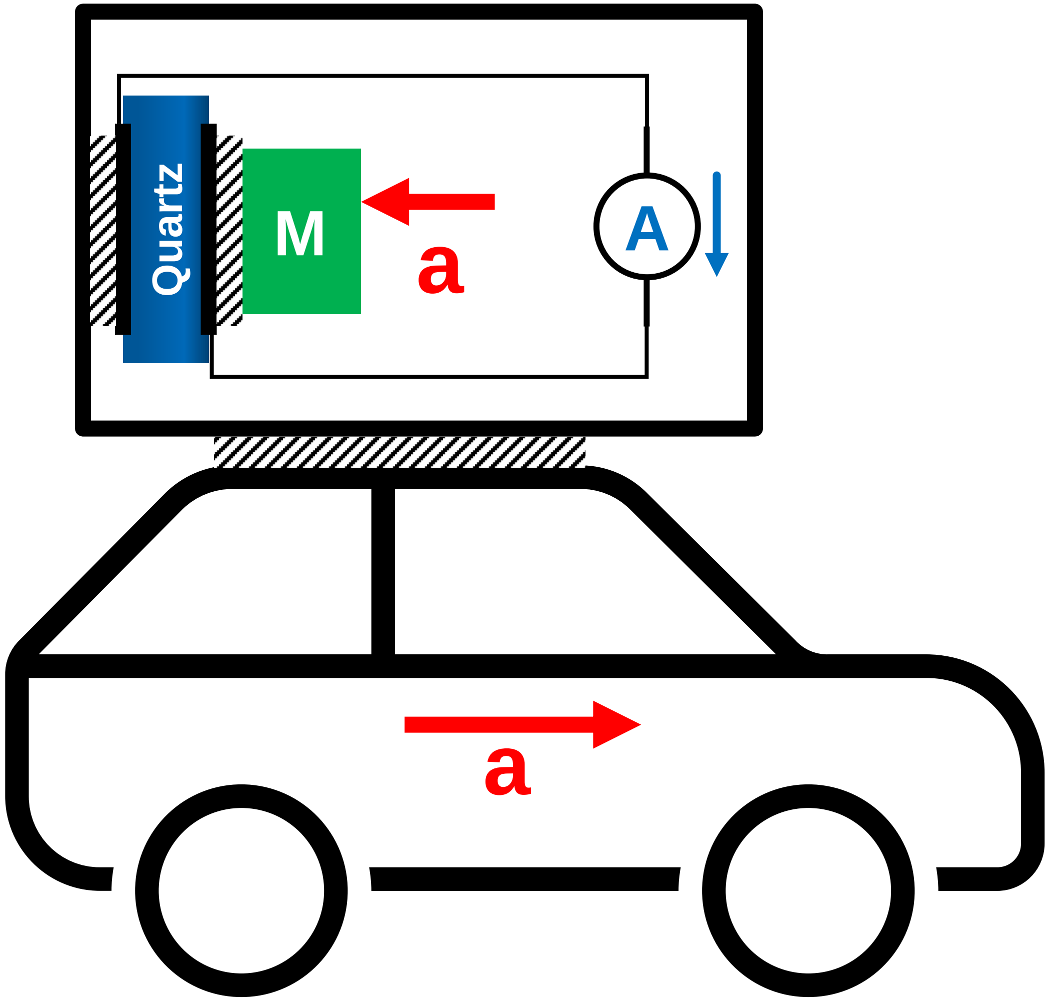
Piezoelectric effect of Quartz

How do we get acceleration signal?

We need to **integrate** the **current signal**

One way is to convert this to a **digital signal** and process it in a computer – but this is too much effort!

Let us try an analog way!



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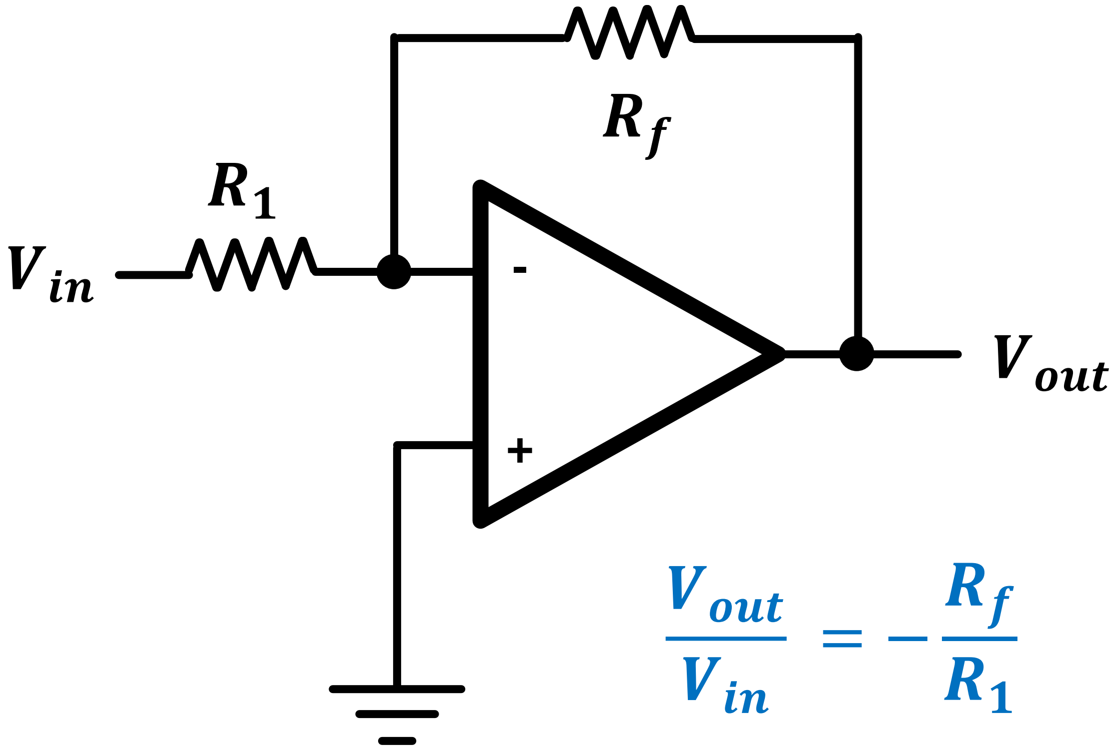
Integrating Amplifier

Recall the Inverting Amplifier



$$\frac{V_{out}}{V_{in}} = -\frac{R_f}{R_1}$$

What if we use an energy storing element (like capacitor/inductor) in place of the resistive element in feedback path?

We know that we need to integrate the current signal – this is done by a capacitor

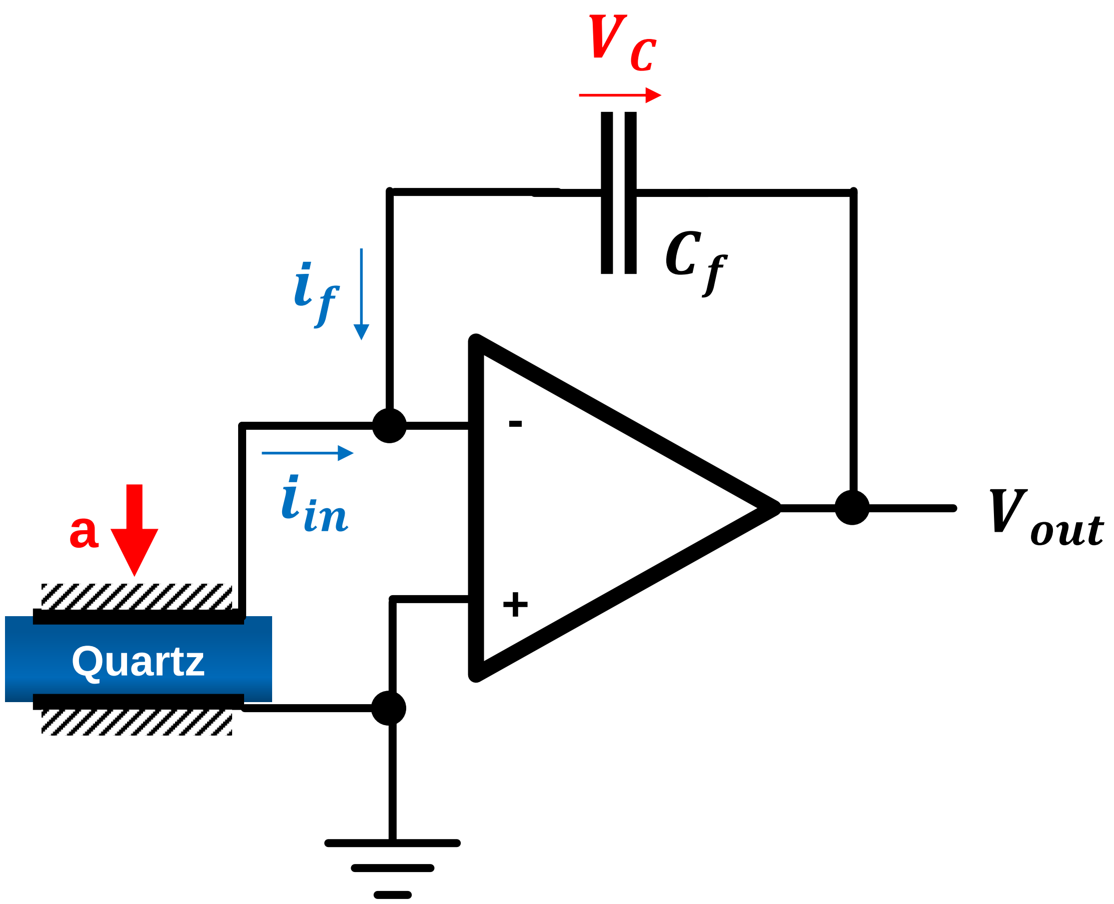
Recalling the Capacitor equation:

$$Q = CV$$

$$\frac{dQ}{dt} = i = C\frac{dV}{dt}$$

Let us replace the $\boldsymbol{V_{in}}$ and $\boldsymbol{R_1}$ at the inverting input with the piezoelectric element, and $\boldsymbol{R_f}$ with a capacitor:

Integrating Amplifier



Let us solve the circuit again like we did for inverting amplifier:

$$V_{out} = A_{OL}(V_+ - V_-)$$

$$V_{out} = A_{OL}(0 - V_-)$$

$$V_{out} = -A_{OL}V_-$$

But we can calculate V_- from the current:

$$V_- = V_{out} - V_C$$

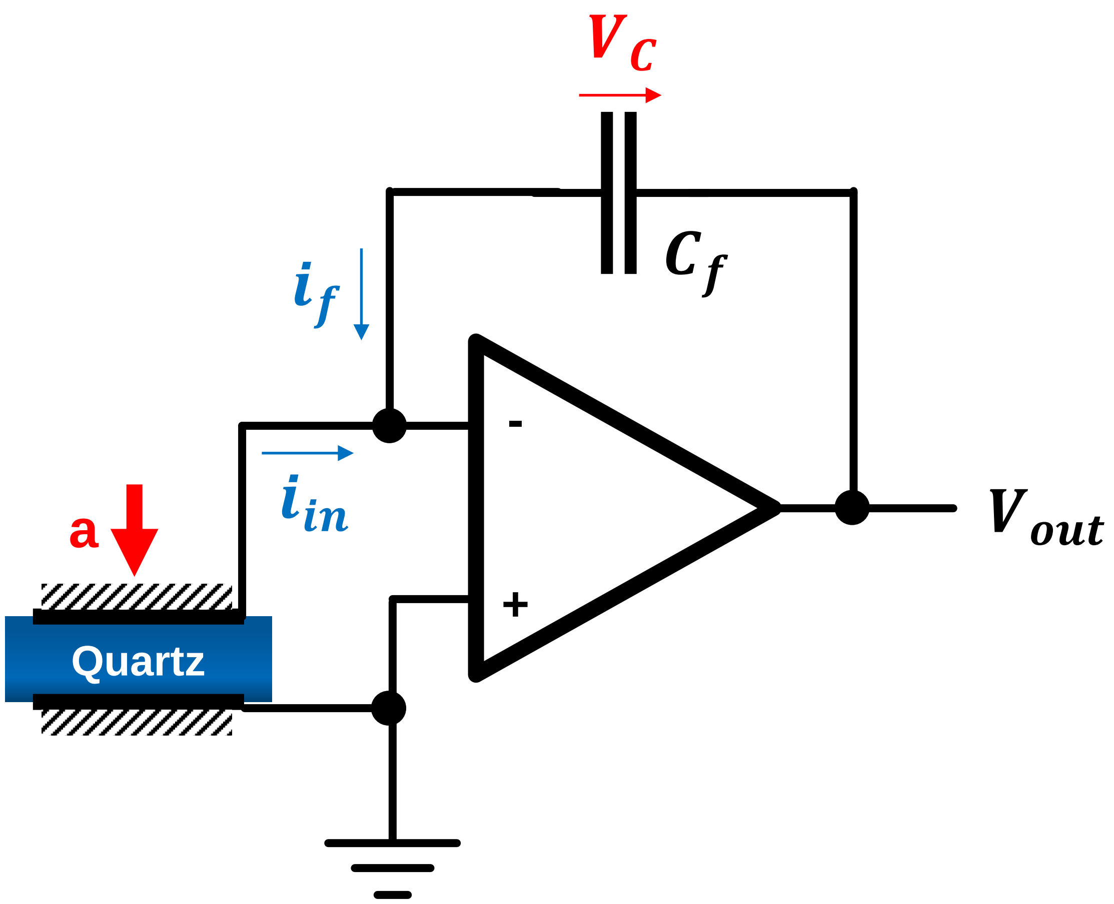
As input resistance of op amp is ∞:

$$i_f = -i_{in} = -k_1 M \frac{da}{dt}$$

From the capacitor equation:

$$\boldsymbol{i_f = C_f \frac{dV_C}{dt} = -k_1 M \frac{da}{dt}}$$

Integrating Amplifier



- $V_{out} = -A_{OL}V_-$
- $V_- = V_{out} - V_C$

$$i_f = C_f\frac{dV_C}{dt} = -k_1M\frac{da}{dt}$$

Integrating both sides w/r/t time:

$$C_fV_C = -k_1Ma$$

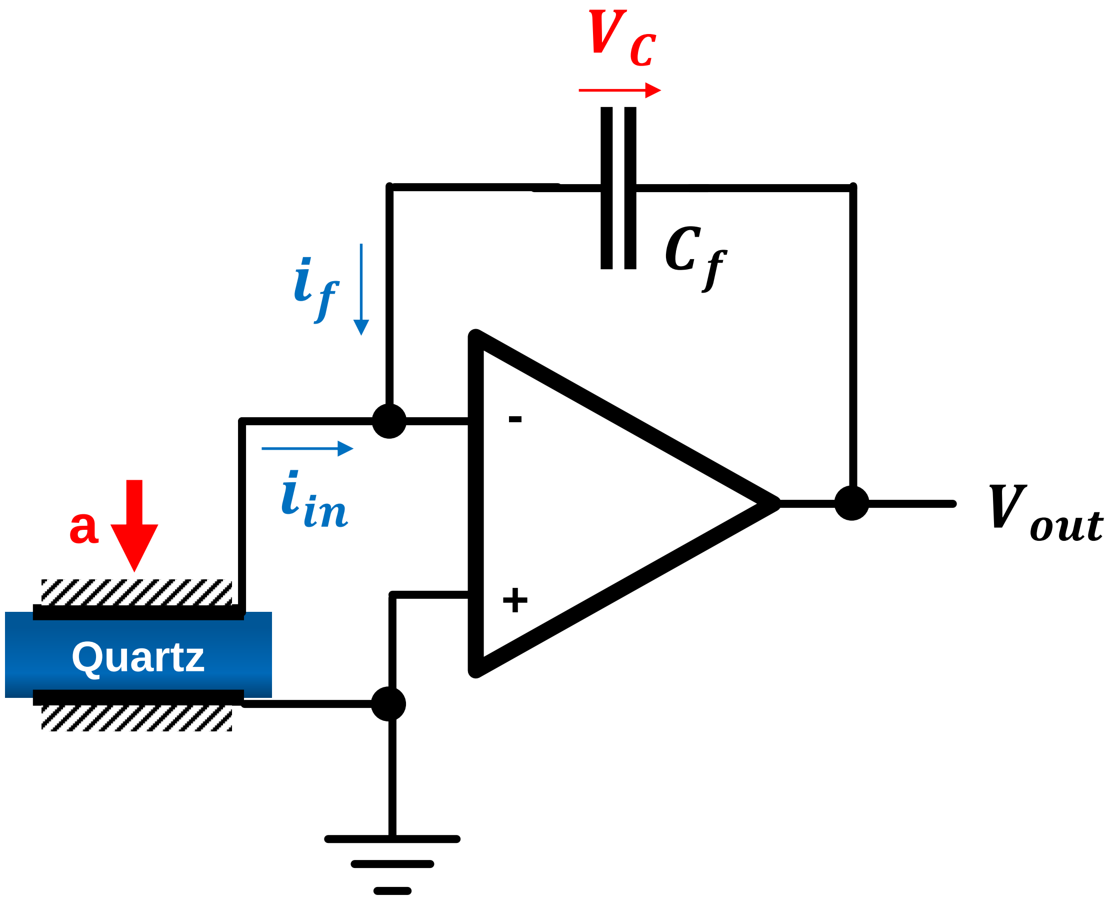
$$\boldsymbol{V_C = -\frac{k_1M}{C_f}a}$$

Applying this relation to resolve for output

$$V_{out} = -A_{OL}(V_{out} - V_C)$$

$$\boldsymbol{V_C = -V_{out}\frac{(1 + A_{OL})}{A_{OL}}}$$

Integrating Amplifier



$$V_C = -V_{out}\frac{(1 + A_{OL})}{A_{OL}}$$

As $A_{OL} \gg 1$:

$$V_C = -V_{out}\frac{(A_{OL})}{A_{OL}}$$

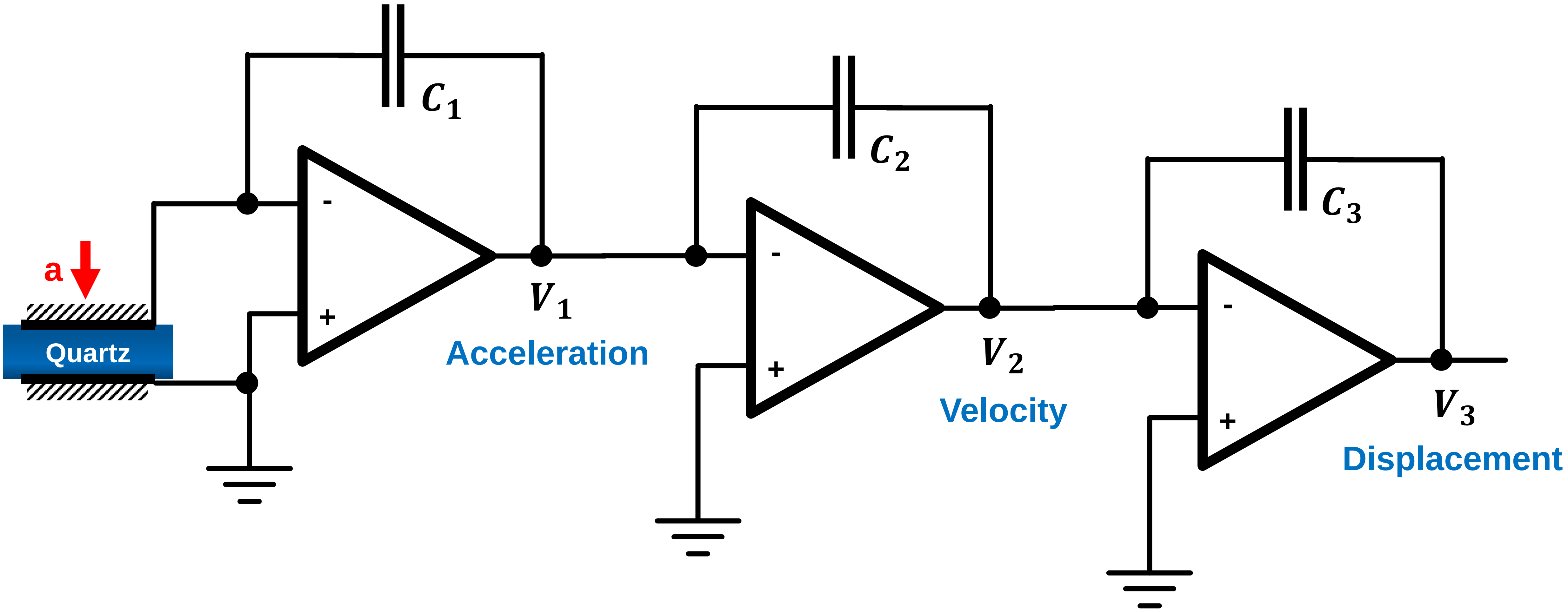
$$V_C = -V_{out}$$

$$-\frac{k_1 M}{C_f}a = -V_{out}$$

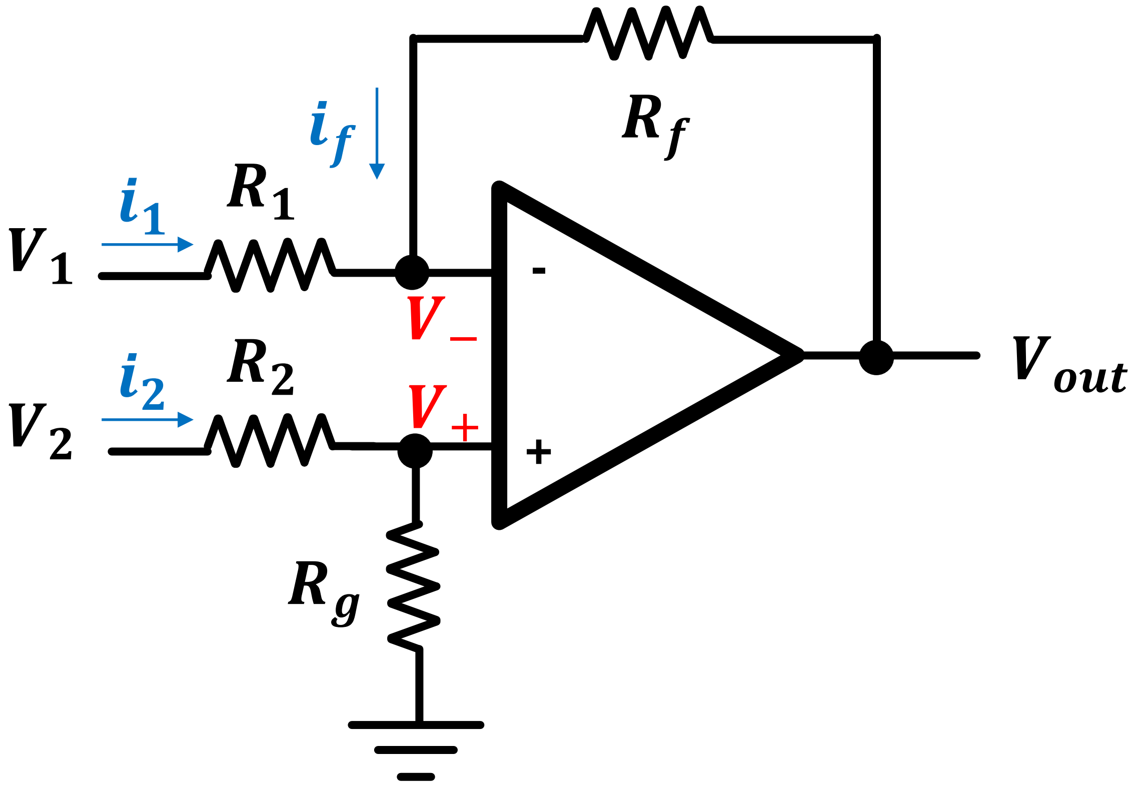
$$\boldsymbol{V_{out} = \frac{k_1 M}{C_f}\, a}$$

Integrating Amplifier

We can stack multiple integrators to get velocity and displacement



Differencing Amplifier

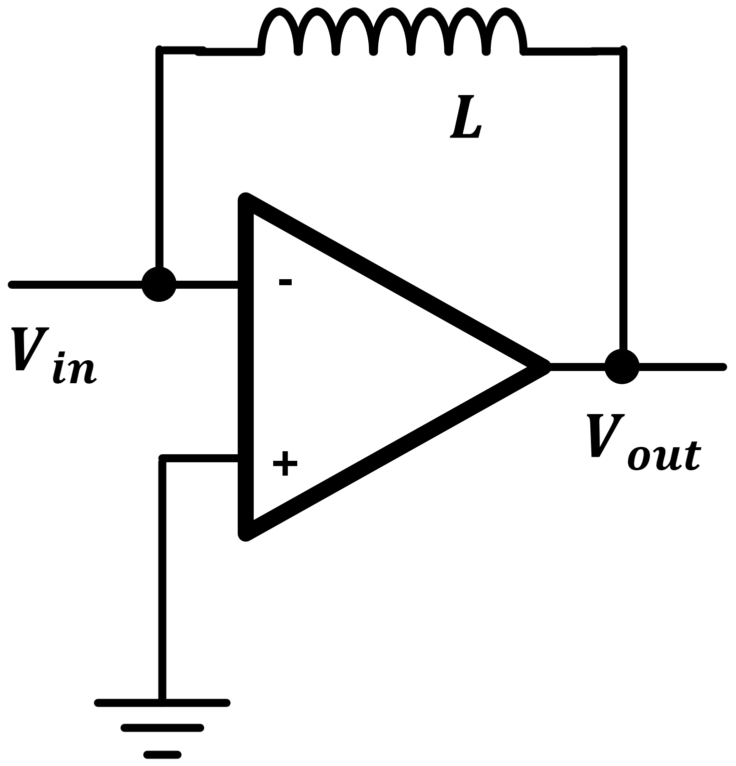


- $V_1 - V_- = i_1 R_1$
- $V_{out} - V_- = i_f R_f$
- $V_+ = \frac{R_g}{R_2+R_g} V_2$
- $A_{OL}(V_+ - V_-) = V_{out}$

- $i_1 = -i_f$
  - $\frac{V_1 - V_-}{R_1} = -\frac{V_- - V_{out}}{R_f}$
  - $V_- = \frac{V_1 R_f + V_{out} R_1}{R_f + R_1}$
- $A_{OL}(V_+ - V_-) = V_{out}$
  - $A_{OL}\left(\frac{R_g}{R_2+R_g} V_2 - \frac{V_1 R_f + V_{out} R_1}{R_f + R_1}\right) = V_{out}$

$$V_{out}\left(\frac{R_1}{R_1 + R_f}\right) = V_2\left(\frac{R_g}{R_2 + R_g}\right) - V_1\left(\frac{R_f}{R_1 + R_f}\right)$$

Differencing Amplifier



You have learnt how to do integration operation using a capacitor in the feedback circuit

What would happen if you replace the capacitor with an inductor?

Recall that capacitor and inductor do exactly opposite things

Does this mean you can do derivative operation using op amp in this configuration?

Try solving this circuit on your own

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Attendance

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