DYNAMICS AND CONTROL

CONTROL CONSOLIDATION 2

EXAM PREPARATION – 20/05/2021

- General advice:
  - Don’t panic – systematic saves the day
  - You won’t be asked to take anything 3rd order or higher to the time domain.
  - Don’t forget the Routh-Hurwitz criteria for system stability.
- And now: 2017-18 Exam paper (I didn’t set the 2019 questions, and the 2020 paper was pass/fail)

4. The control diagram for a flap actuation system of a large airliner is shown in Figure Q4, where $\Phi(s)$ is the desired position of the flap, and $\theta(s)$ is the current position.

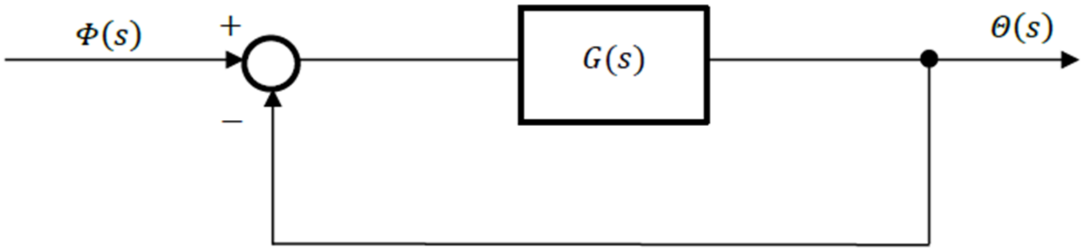

Figure Q4

i. Determine the overall Transfer Function $\frac{\Theta(s)}{\Phi(s)}$ for the system.

[3]

ii. If the transfer function $G(s) = \frac{K}{s}$, where K is a constant, determine an expression for the resulting displacement as a function of time, $\theta(t)$, in response to a unit step input of amplitude α.

[3]

iii. Calculate the steady state output of the system for $\alpha = 0.2$ radians and $K = 5$.

[2]

iv. Calculate the time taken to reach 95% of the steady state value.

[2]

7. Figure Q7a shows an active suspension system for an off-road vehicle. The vertical position of the wheel, x_{out}, can be modelled as a single-degree-of-freedom Mass/Spring/Damper system subjected to an actuator force, *f*(*t*), in the vertical direction.

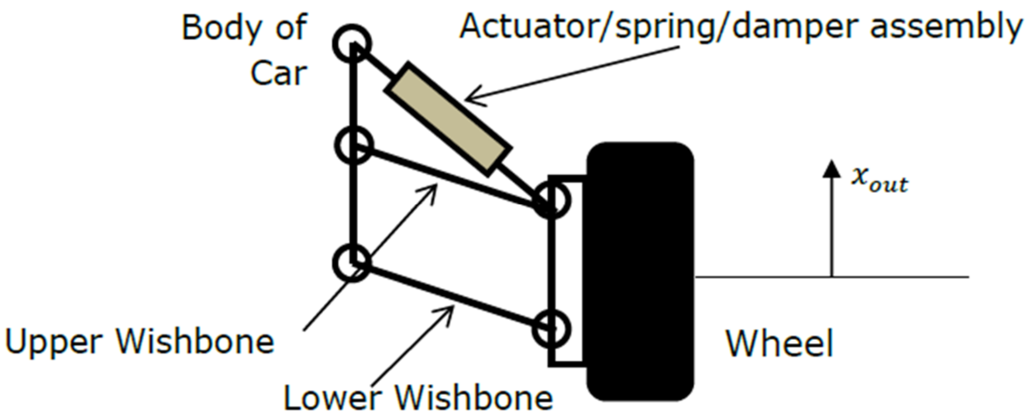

Figure Q7a

The resulting transfer function for the wheel and suspension assembly plant model is given by:

$$H(s) = \frac{X_{out}(s)}{F(s)} = \frac{1}{Ms^2 + Cs + K}$$

i. If M= 500 kg, C=200 N/m/s, K=10 N/m, calculate the damping ratio for the suspension system. Is the system underdamped, critically damped, or overdamped? How do you know that the system will not be unstable?

[6]

Solution:

$$\frac{1}{500s^2 + 200s + 10} = \frac{0.002}{s^2 + 0.4s + 0.02} = 0.1\left(\frac{\omega^2}{s^2 + 2\gamma\omega s + \omega^2}\right)$$

Then A=0.1 $\omega = \sqrt{0.02}$ and

$$\gamma = \frac{0.4}{2\sqrt{0.02}} = 1.414$$

and the system is overdamped. The system will be stable because there is no change of sign in the characteristic function.

The wheel suspension assembly is incorporated into an active feedback control system as shown in Figure Q7b, where $X_{in}(s)$ is the desired position set by the driver, $G(s)$ is the transfer function of the controller, $H(s)$ is the transfer function of the suspension system and $X_{out}(s)$ is the resulting wheel position in the Laplace domain.

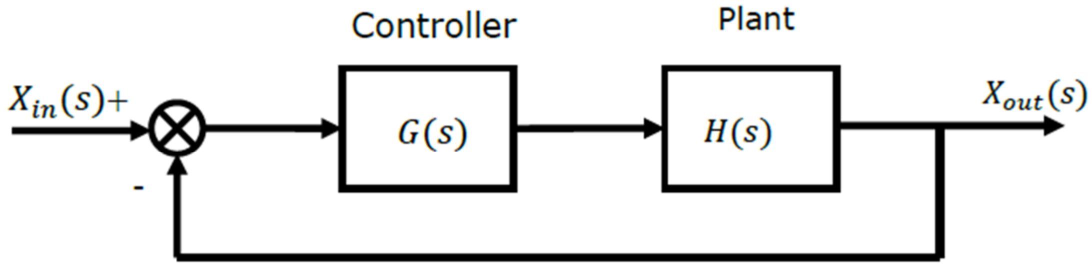

Figure Q7b

If the controller, $G(s)$, has the transfer function:

$$G(s) = D\left(\frac{s+1}{s}\right)$$

ii. Determine the closed loop transfer function for $\frac{X_{out}(s)}{X_{in}(s)}$.

[7]

iii. Using the values of *M*, *C* and *K* in part i), determine the range of values for *D* when the system is stable.

[12]

END

$$G(s) = D\left(\frac{s+1}{s}\right)$$

$$H(s) = \frac{1}{Ms^2 + Cs + K}$$

Solution:
Combined transfer function

$$G(s)H(s) = \frac{D(s+1)}{s(Ms^2 + Cs + K)}$$

Feedback transfer function is therefore:

$$J(s) = \frac{G(s)H(s)}{1 + G(s)H(s)} = \frac{D(s+1)}{D(s+1) + s(Ms^2 + Cs + K)}$$

Note: Students may go further to

$$J(s) = \frac{D(s+1)}{Ms^3 + Cs^2 + (K+D)s + D}$$

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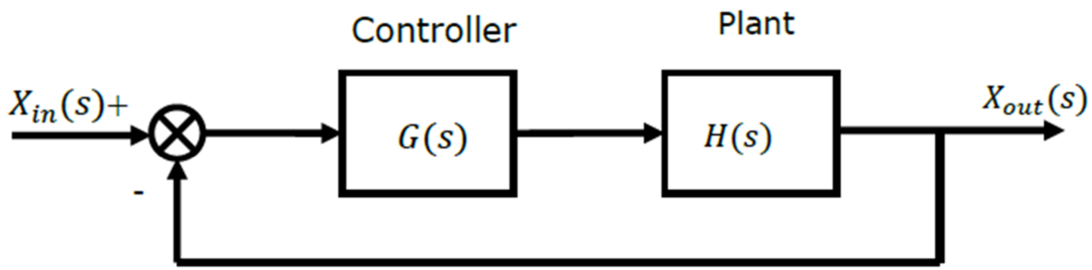

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[12]

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Solution:

Bottom line of J(s) (characteristic function) is:

$$Ms^3 + Cs^2 + (K + D)s + D = 500s^3 + 200s^2s + (10 + D)s + D$$

Routh table is given by:

s^3	500	10+D	0	0
s^2	200	D	0	0
s	$\frac{(200(10+D) - 500D)}{200}$	0		
s^0	D	0		

The system will be unstable for values of D such that

$$\frac{(200(10 + D) - 500D)}{200} < 0$$

$$(10 + D) - 2.5D < 0$$

$$10 < 1.5D$$

And hence the system will be unstable for values of D greater than $6\frac{2}{3}$

THE END?

Any questions?