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MMME2046 Dynamics and Control

Machine Dynamics Revision

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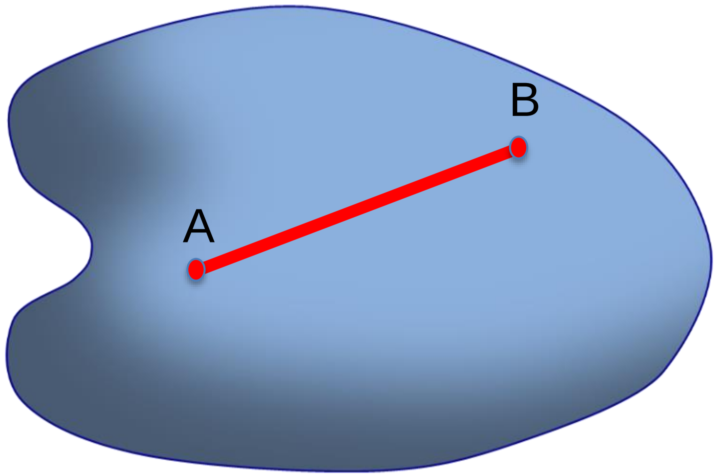
Lecture objectives

- Revise kinematics and dynamics of rigid bodies

- Solve several exam style problems

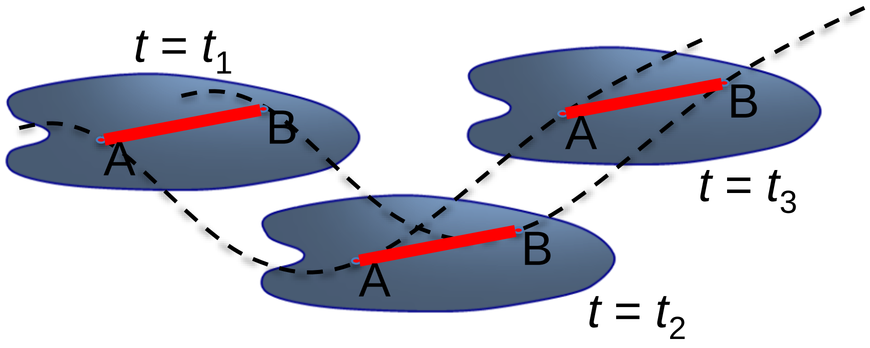
Rigid Body definition



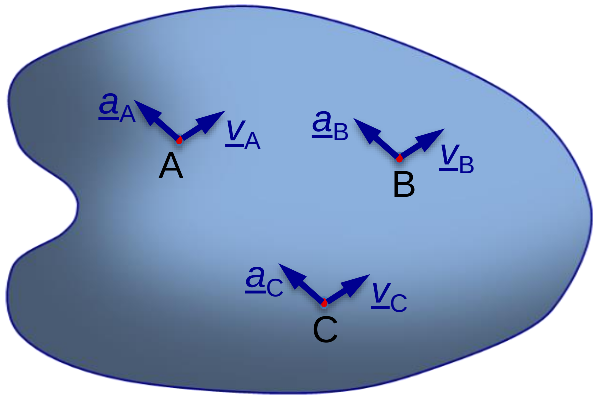
- System of particles
- Distances between particles remain unchanged
- Deformations are neglected

Particle − Rigid body − System of rigid bodies

Rigid Body motion: Pure translation



- Line segments maintain orientation
- Points move on "parallel" trajectories

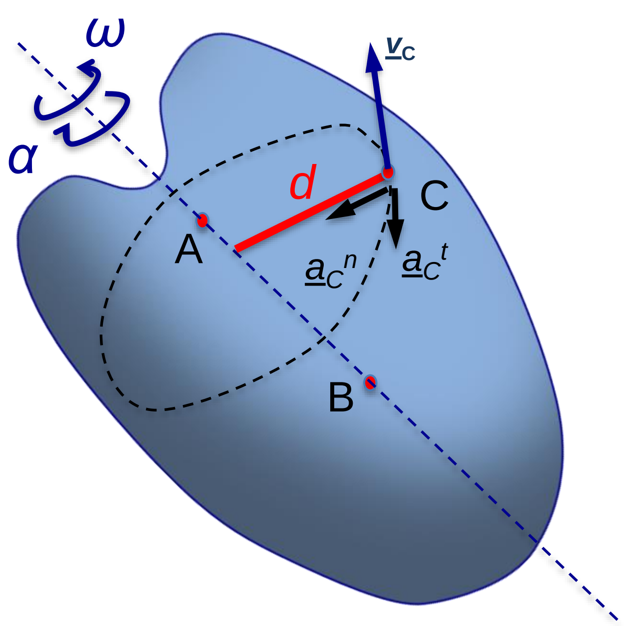


At any instant of time:

$\underline{v}_A = \underline{v}_B = \underline{v}_C = \ldots$

$\underline{a}_A = \underline{a}_B = \underline{a}_C = \ldots$

Rigid Body motion: Rotation about fixed axis



Kinematics of rigid body governed by:

$\theta(t)$ angle of rotation

$\dot{\theta}(t) = \omega(t)$ angular velocity

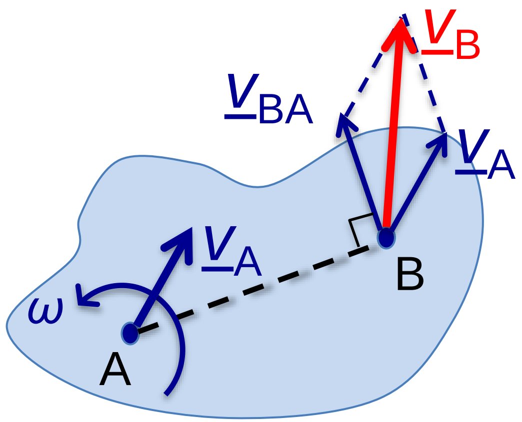
$\ddot{\theta}(t) = \alpha(t)$ angular acceleration

Each point performs circular motion.
E.g. for point C:

$v_c = \omega d$ velocity magnitude

$$\left.\begin{array}{l} a_C^n = \omega^2 d \\ a_C^t = \alpha d \end{array}\right]$$ acceleration components

Velocity relations in planar motion



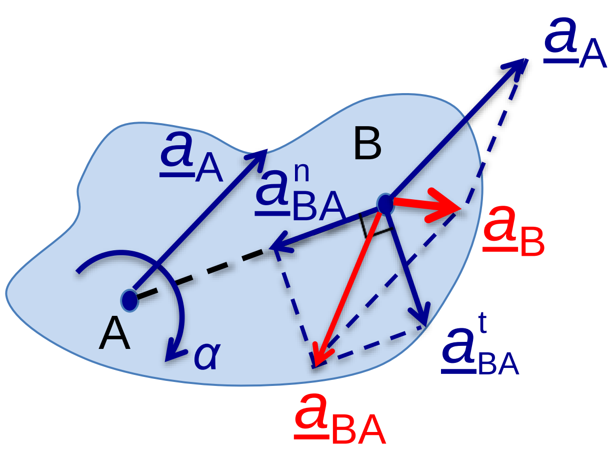
Given: velocity at A & angular velocity

Known: $\underline{v}_B = \underline{v}_A + \underline{v}_{BA}$ (1)

Relative motion at B is **circular** around A:

1) magnitude: $v_{BA} = \omega \, AB$
2) direction: perpendicular to AB
3) sense: governed by the angular velocity

Acceleration relations in planar motion



Given: acceleration at A, angular velocity & angular acceleration

Known: $\underline{a}_B = \underline{a}_A + \underline{a}_{BA} = \underline{a}_A + \underline{a}_{BA}{}^n + \underline{a}_{BA}{}^t$ (2)

Relative motion at B is **circular** around A:

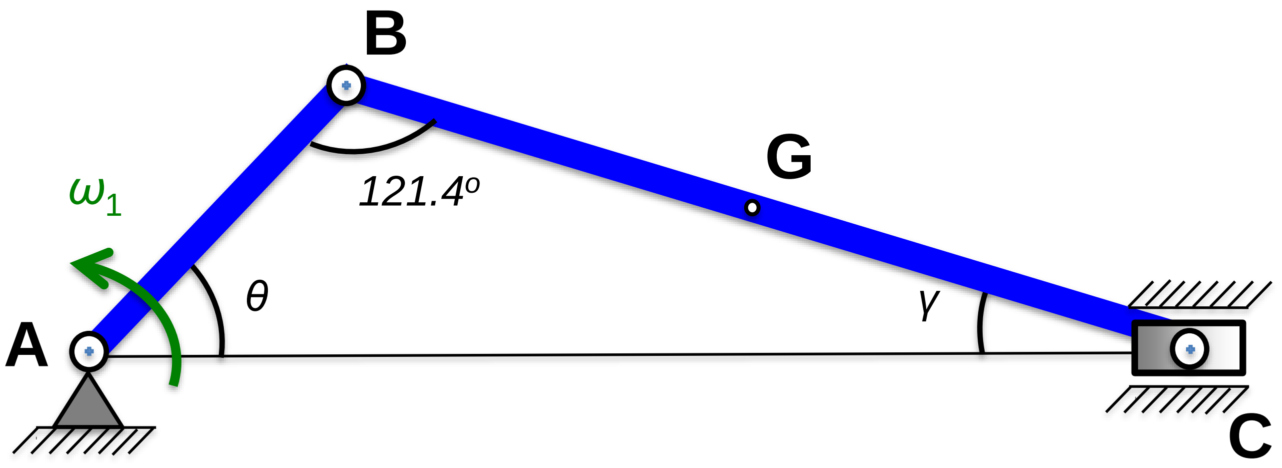
1) magnitudes: $a_{BA}^n = \omega^2\, AB \qquad a_{BA}^t = \alpha\, AB$

2) directions & senses:

- Normal component always has direction towards the reference point.
- Tangential component is perpendicular to AB with direction defined by α.

Example II.3: Slider-Crank Mechanism

ω_1 = 100 rad/s = const.



BC = 240 mm

θ = 45°

AB = 80 mm

BG = 120 mm

Geometry:

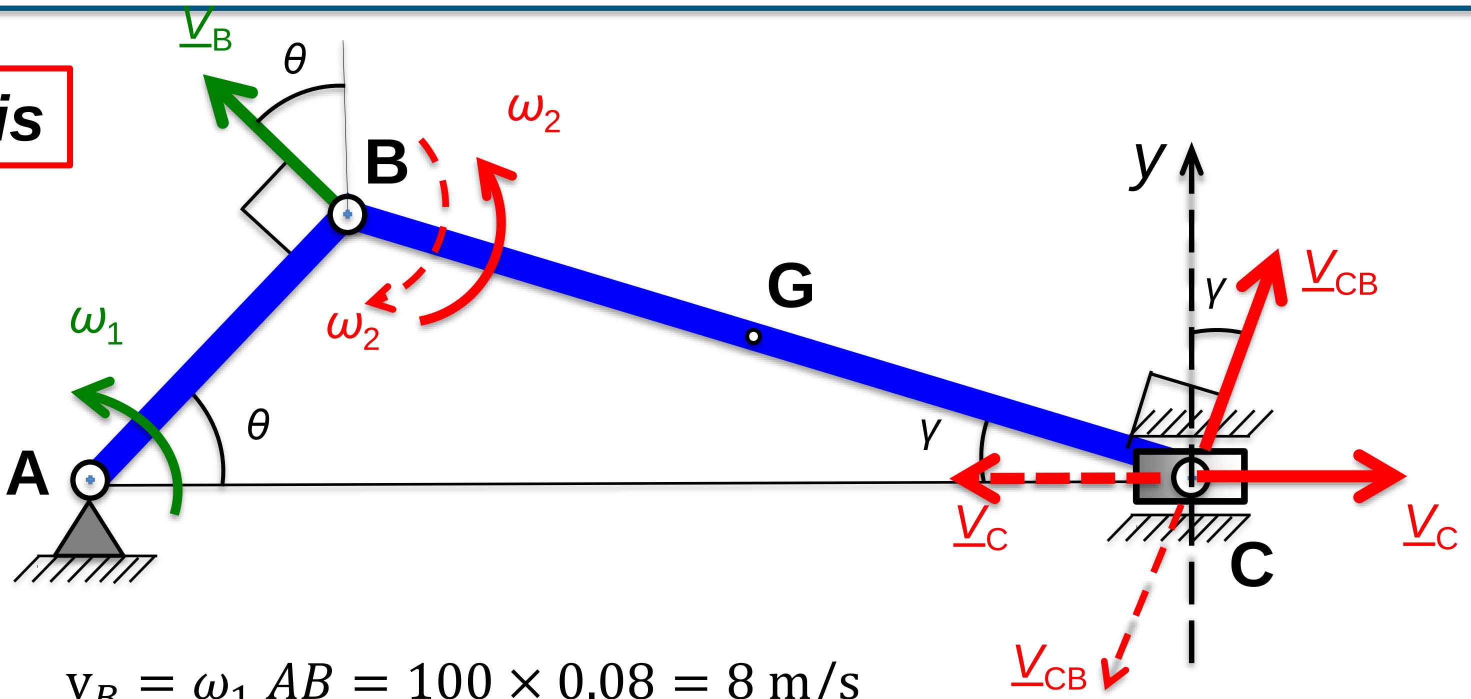
$$\frac{\sin\gamma}{AB} = \frac{\sin\theta}{BC}, \text{or } \gamma = 13.63°$$

$$AC^2 = AB^2 + BC^2 - 2AB.BC\cos 121.4^{\circ}$$

$$AC = 0.2897m$$

Example II.3: Slider-Crank Mechanism

Velocity Analysis



AB = 80 mm

BC = 240 mm

θ = 45°

γ = 13.63°

$$\mathrm{v}_B = \omega_1\, AB = 100 \times 0.08 = 8\ \mathrm{m/s}$$

$$\underline{\mathrm{v}}_C = \underline{\mathrm{v}}_B + \underline{\mathrm{v}}_{CB}$$

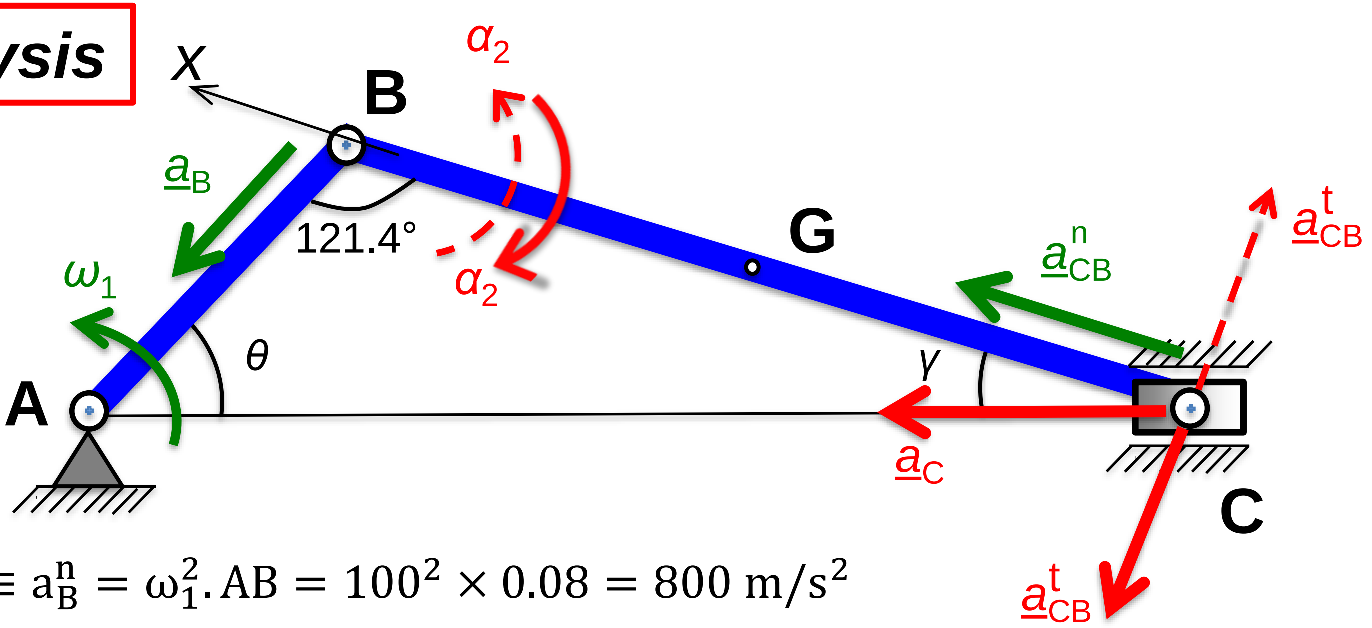
$$\mathrm{v}_{CB} = \omega_2\, BC = 0.24\omega_2$$

$$\uparrow^+: \quad 0 = \mathrm{v}_B \cos\theta + \mathrm{v}_{CB}\cos\gamma \quad \Longrightarrow \quad \omega_2 = -\frac{\mathrm{v}_B\cos\theta}{BC\cos\gamma} = -\frac{8\cos 45^\circ}{0.24\cos 13.63^\circ} = -24.25\ \mathrm{rad/s}$$

$$\rightarrow^+: \quad \mathrm{v}_C = -\mathrm{v}_B\cos\theta + \mathrm{v}_{CB}\sin\gamma \quad \Longrightarrow \quad \mathrm{v}_c = -7.028\ \mathrm{m/s}$$

Example II.3: Slider-Crank Mechanism

Acceleration Analysis



$$a_B \equiv a_B^n = \omega_1^2 . AB = 100^2 \times 0.08 = 800 \text{ m/s}^2$$

$$\mathbf{\underline{a}}_C = \mathbf{\underline{a}}_B + \mathbf{\underline{a}}_{CB}^n + \mathbf{\underline{a}}_{CB}^t$$

$$a_{CB}^n = \omega_2^2 . BC = 24.25^2 \times 0.24 = 141.1 \text{ m/s}^2$$
$$a_{CB}^t = \alpha_2 . BC = 0.24\alpha_2,$$

$\nwarrow^+$: $\quad a_C \cos 13.63^o = 800 \cos 58.63^o + 141.1 + 0 \ \rightarrow a_C = 573.7 \text{ m/s}^2$

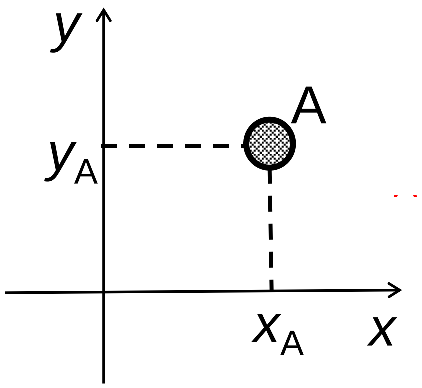
$\uparrow^+$: $\quad 0 = -800 \cos 45^o + 141.1 \sin 13.63^o - 0.24\, \alpha_2 \cos 13.63^o \ \rightarrow \ \alpha_2 = -2283 \text{ rad/s}^2.$

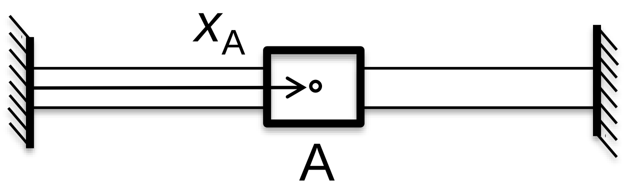
Degrees of Freedom

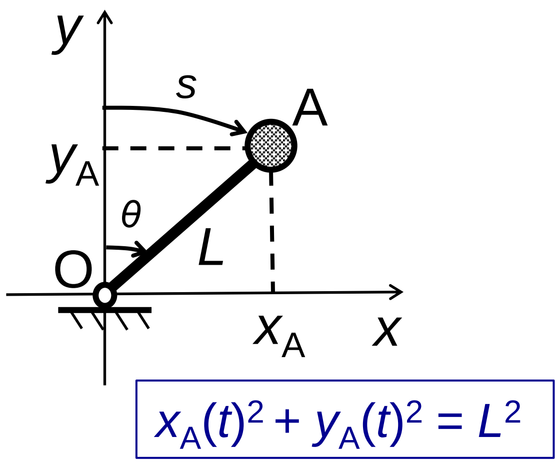
The **degrees of freedom** of a mechanical system in motion are the independent coordinates needed to uniquely specify the position of the system.

The **number of degrees of freedom** is the smallest number of different coordinates in a mechanical system that must be fixed in order to prevent the system from moving.

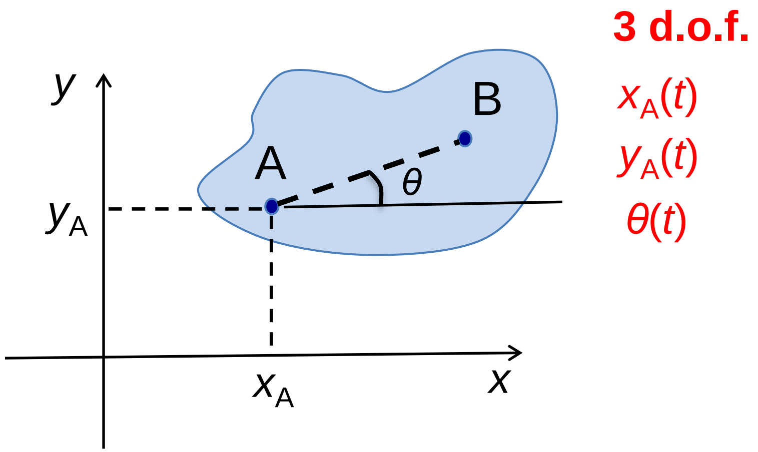
Quiz:

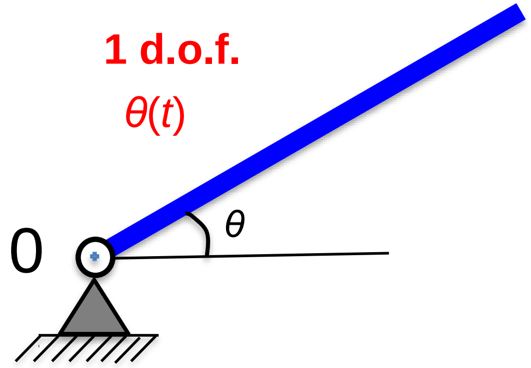


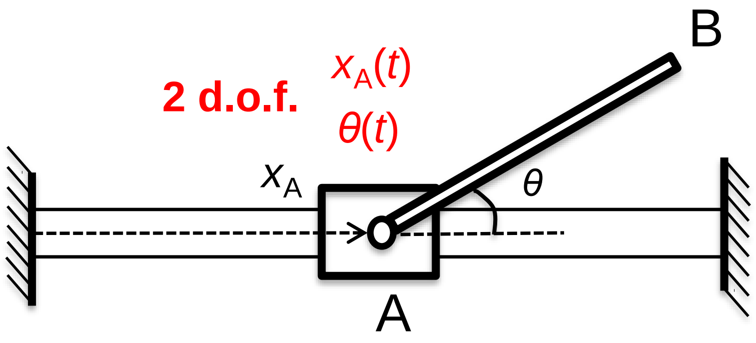


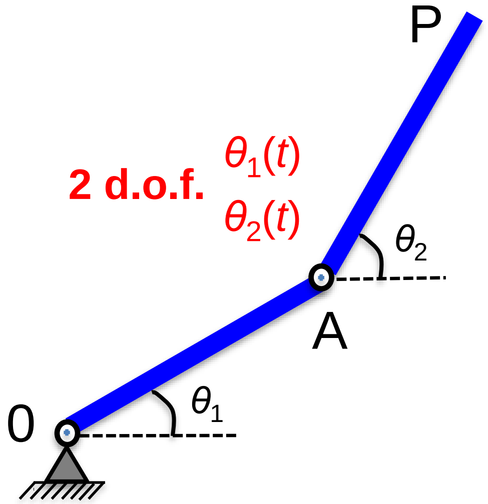


Degrees of Freedom

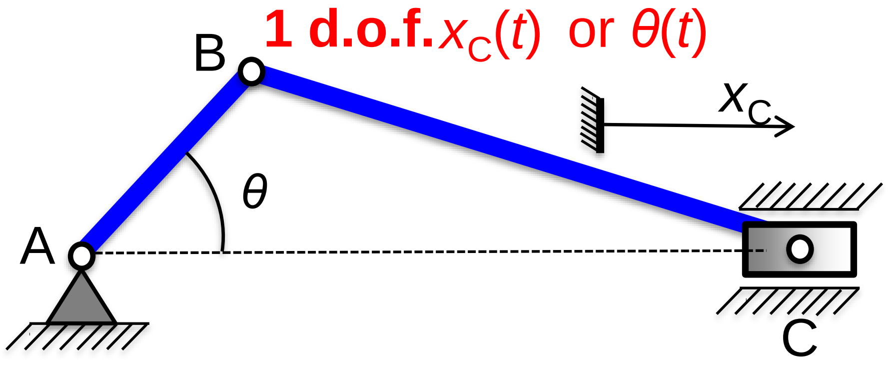






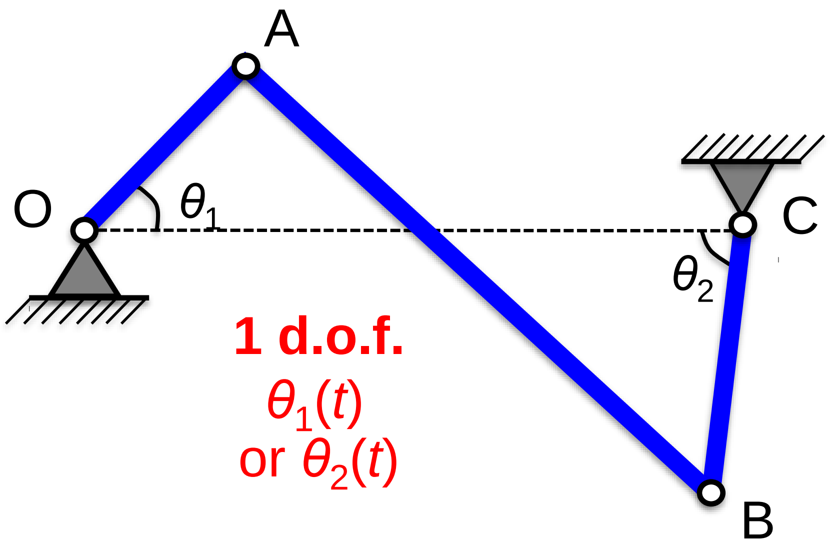


Degrees of Freedom



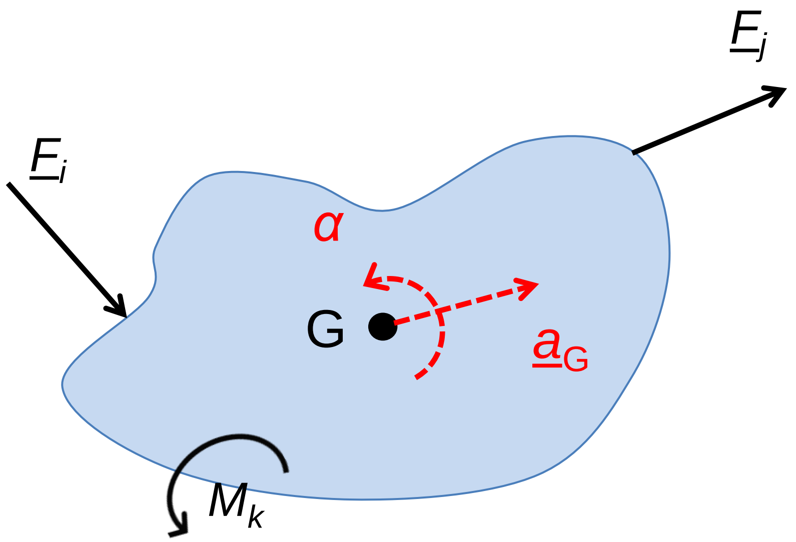
Unconstrained Particle: 2 E.o.M.

Unconstrained R.B.: 3 E.o.M.



Constrained Systems: some E.o.M. used for calculating reaction forces.

Fundamental Laws of Rigid Body Motion



$$\underline{\boldsymbol{F}} = m\underline{\mathbf{a}}_G \qquad (1)$$

$\underline{\boldsymbol{F}}$: resultant of the external forces
$\underline{\boldsymbol{a}}_G$: acceleration of mass centre

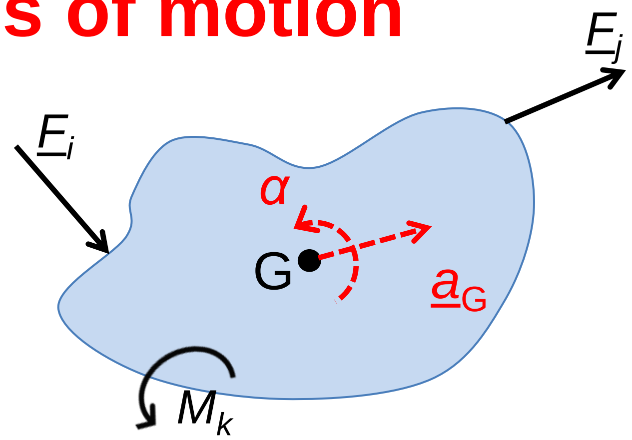
MMME1028 result:

$$M_G = J_G\alpha \qquad (2)$$

M_G : resultant of the applied moments about the axis of rotation
J_G : mass moment of inertia about the axis of rotation
α : angular acceleration of the rigid body

Fundamental Laws of Rigid Body Motion

Equations of motion

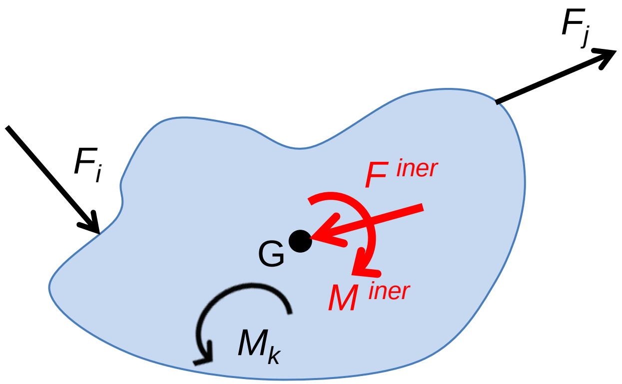


$$\rightarrow^{+}: \ \Sigma F_x = m_G a_{G,x}$$

$$\uparrow^{+}: \ \Sigma F_y = m_G a_{G,y}$$

$$\curvearrowleft^{+}: \ \Sigma M_G = J_G \alpha$$

D'Alembert's principle



$$\rightarrow^{+}: \ \Sigma F_x - F_x^{inertia} = \Sigma F_x - m_G a_{G,x} = 0$$

$$\uparrow^{+}: \ \Sigma F_y - F_y^{inertia} = \Sigma F_y - m_G a_{G,y} = 0$$

$$\curvearrowleft^{+}: \ \Sigma M_G - M^{inertia} = \Sigma M_G - J_G \alpha = 0$$

Exam question 2021/22

FIGURE Q1 shows a rigid bar AB, of length L, that slides down an incline. At the instant shown end A is sliding along a horizontal plane and end B is sliding along an inclined plane (β = 60°).

(a) How many degrees of freedom does the system have at the instant shown? [1]

(b) Find the angle α at the moment when ends A and B have equal speed. [4]

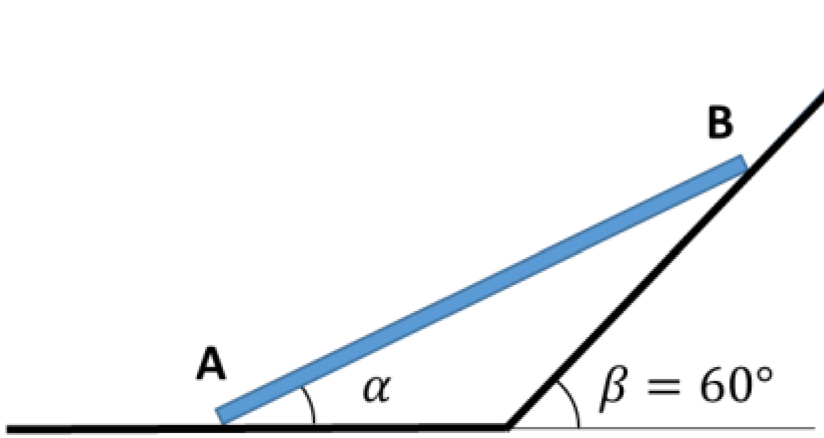
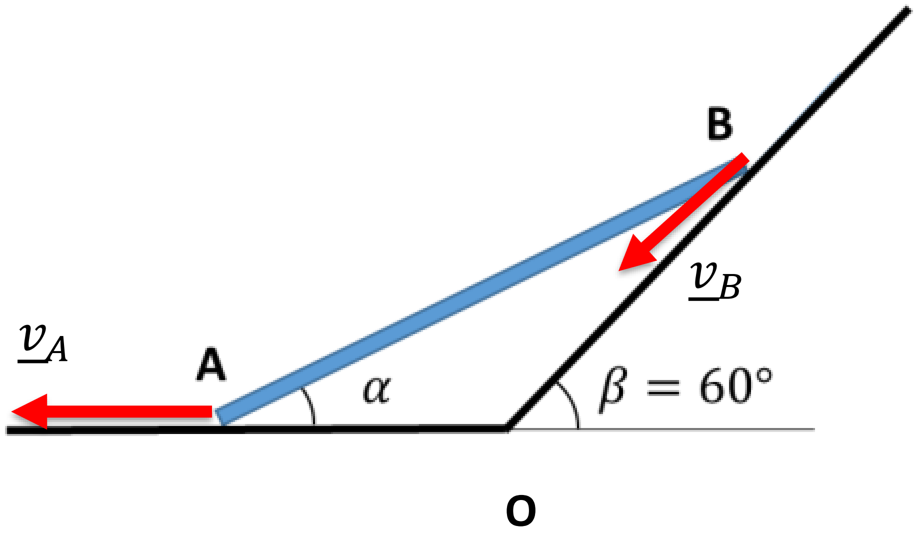


FIGURE Q1

1 DOF

Exam question 2021/22



AB: $v_A \cos(\alpha) = v_B \cos(\angle ABO)$

When $v_A = v_B$:

$$\cos(\alpha) = \cos(\angle ABO)$$

So: $\alpha = \angle ABO$

Finally, from the triangle:

$$\alpha + \angle ABO + 120° = 180°$$
$$2\alpha + 120° = 180°$$
$$\alpha = 30°$$

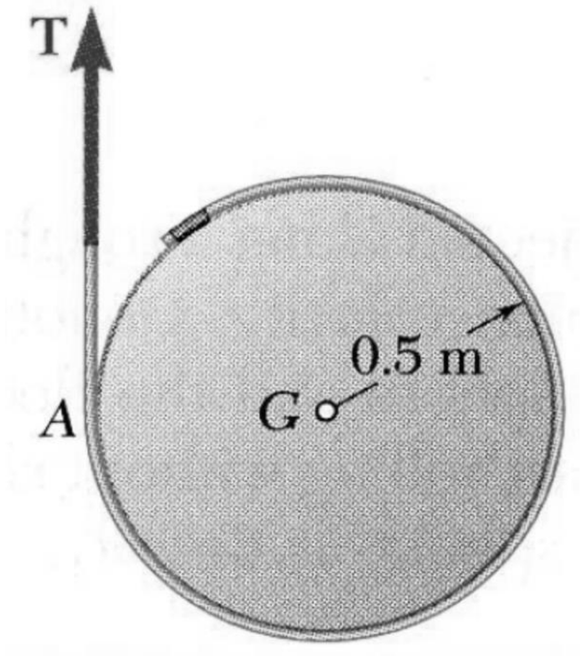
Rigid body dynamics problem

A cord is wrapped around a homogeneous disk of mass m=15 kg. The cord is pulled upwards with a force T=180 N as shown below.

Using d'Alembert's principle, determine:

(a) The **translational acceleration** of the centre of the disk,

(b) The **angular acceleration** of the disk,



Rigid body dynamics problem

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A cord is wrapped around a homogeneous disk of mass **m=15kg**. The cord is pulled upwards with a force **T=180N**.

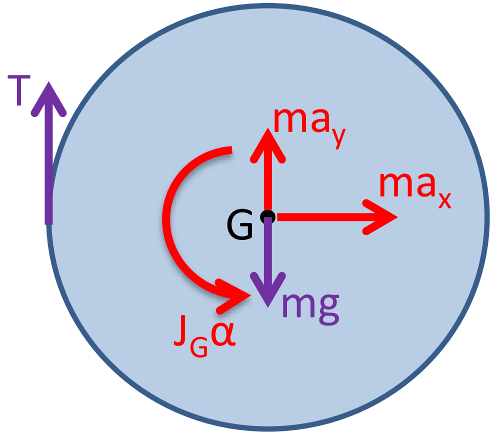
r=0.5m

Using d'Alembert's principle, determine:
(a) The translational acceleration of the centre of the disk,
(b) The angular acceleration of the disk,

$$J_G = \frac{1}{2}mr^2 = 1.875\ kgm^2$$

FBD for the disk:

T = 180 N

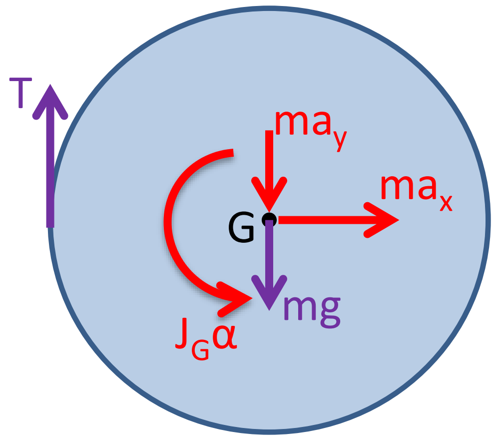


Inertial ma_y downwards, thus **actual a_y upwards**

Inertial $J_G\alpha$ counterclock, thus **actual α clock**

Rigid body dynamics problem

EOMs for the disk:



r=0.5m

$$J_G = \frac{1}{2}mr^2 = 1.875\ kgm^2$$

T = 180 N

Inertial ma_y downwards, thus **actual a_y upwards**

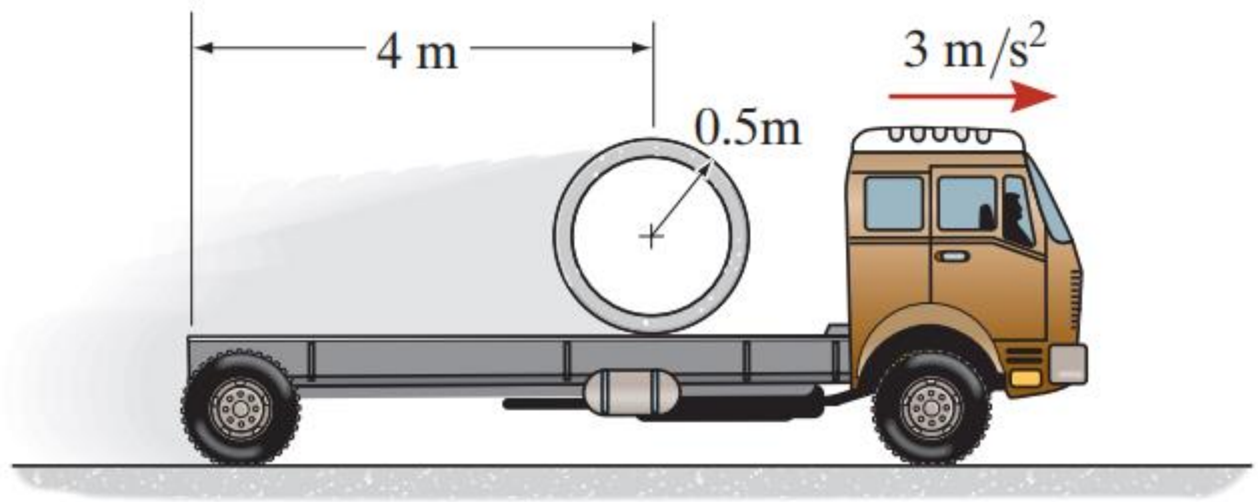
Inertial $J_G\alpha$ counterclock, thus **actual α clock**

$\overset{\leftarrow}{+}\Sigma F_x=0$: $\quad ma_x = 0 \rightarrow a_x = 0$

$\uparrow + \Sigma F_y=0$: $\quad T - ma_y - mg = 0 \rightarrow a_y = 2.19 m/s^2$

$\circlearrowleft + \Sigma M_G=0$: $\quad J_G\alpha - Tr = 0 \rightarrow \alpha = 48 rad/s^2$

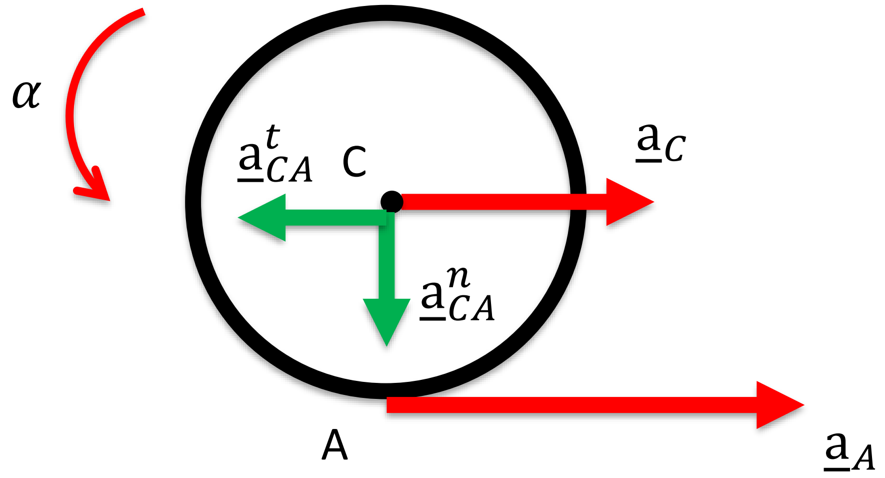
Rigid body dynamics problem

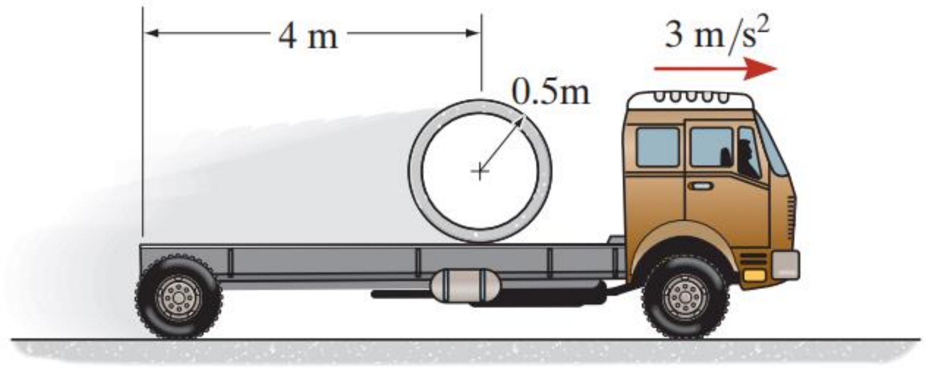


The 500-kg concrete culvert has a mean radius of 0.5 m. If the truck has an acceleration of 3 m/s^2, determine the culvert's angular acceleration. Assume that the culvert does not slip on the truck bed, and neglect its thickness.

Rigid body dynamics problem

Kinematics





A – point of contact between the culvert and the truck.

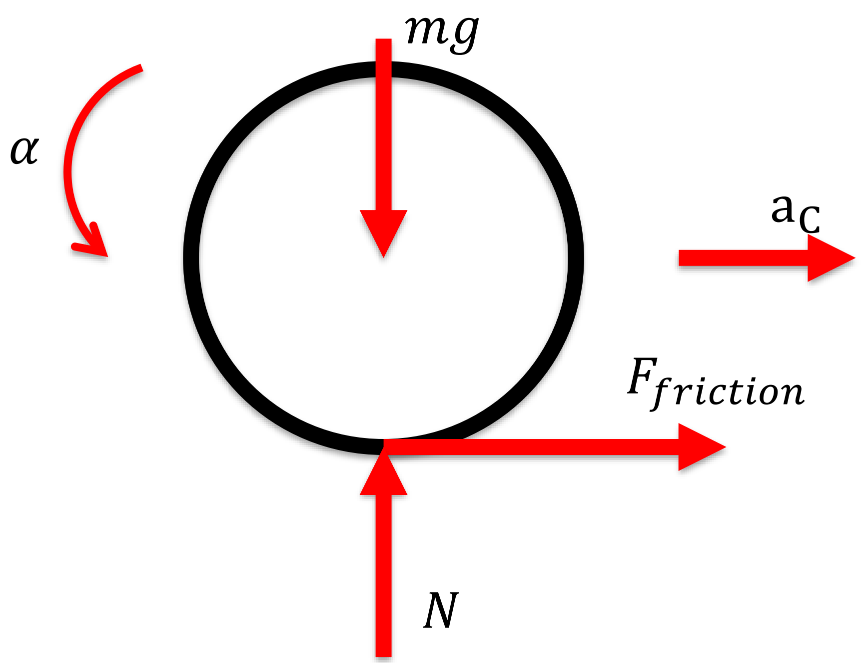
$a_A = a_{truck} = 3\ m/s^2$

$$\underline{a}_C = \underline{a}_A + \underline{a}_{CA}^{n} + \underline{a}_{CA}^{t}$$

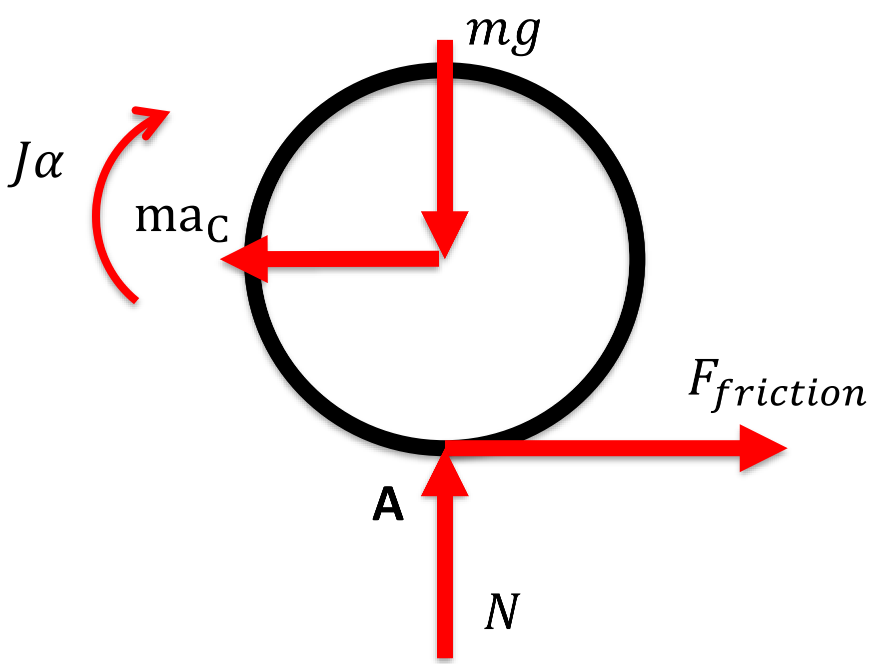
$$\rightarrow +:\ \ a_C = a_A - a_{CA}^{t} = a_A - \alpha r_{culv}$$

Rigid body dynamics problem

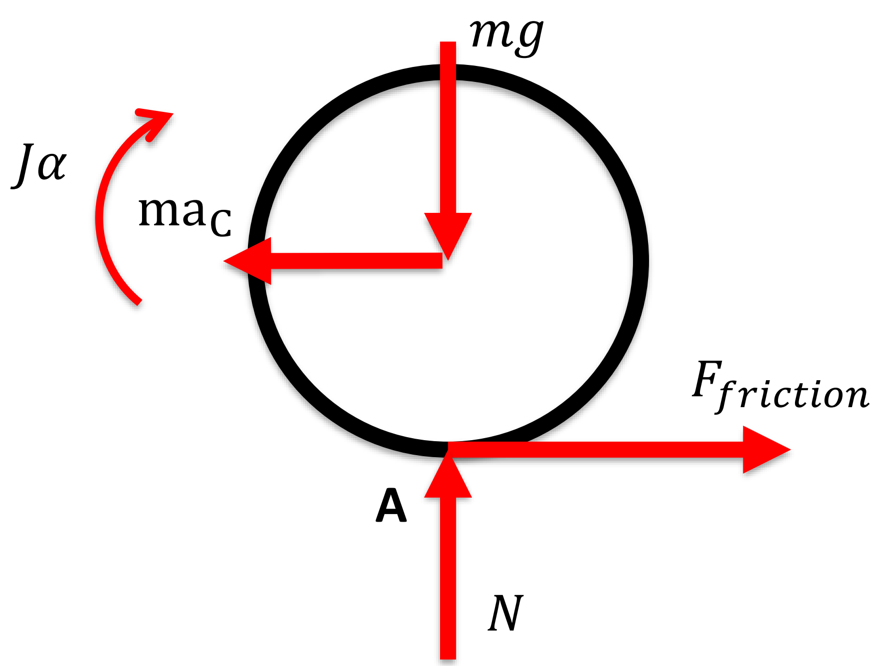
FBD for the culvert:



D'Alambert principle
(introduces inertia forces and moments):



Rigid body dynamics problem



$$J = mr_{culv}^2 = 500 \times (0.5)^2 = 125\ kg \times m^2$$

Moments about A:

$$\sum M = 0$$

$$J\alpha - (\mathrm{ma_C})r_{culv} = 0$$

Kinematics: $\mathrm{a}_C = \mathrm{a}_A - \alpha r_{culv}$

$$J\alpha - m(\mathrm{a_A} - \alpha r_{culv})r_{culv} = 0$$

$$\alpha(J + mr_{culv}^2) - \mathrm{ma_A} r_{culv} = 0$$

$$\alpha = \frac{\mathrm{ma_A} r_{culv}}{J + mr_{culv}^2} = \frac{500 \times 3 \times 0.5}{250} = 3\ rad/s^2$$

$$\mathrm{a}_C = a_A - \alpha r_{culv} = 3 - 3 \times 0.5 = 1.5\ m/s^2$$