Asymmetrical Bending

Lecture 2 – Stresses in an Asymmetrically Loaded Beam

Department of Mechanical, Materials & Manufacturing Engineering

MMME2053 – Mechanics of Solids

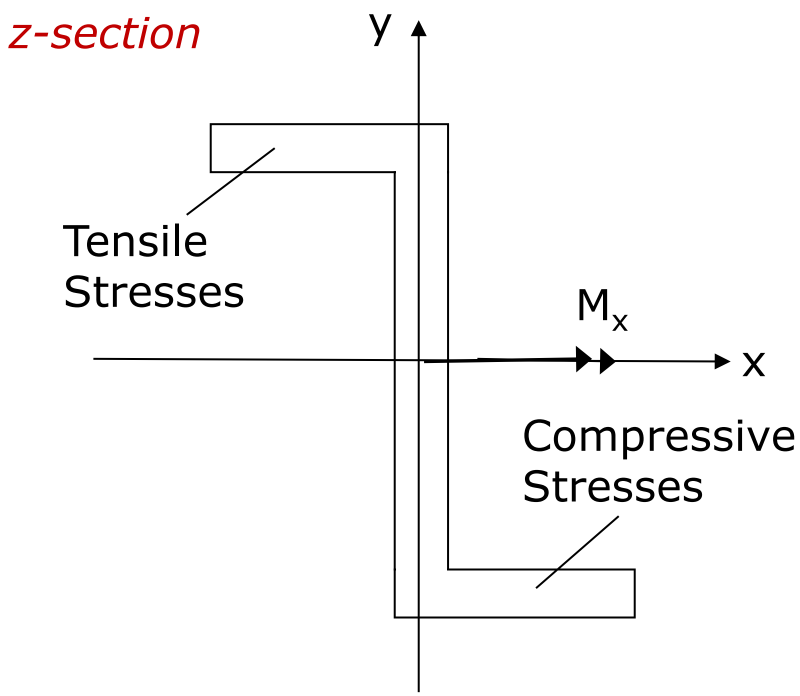
University of Nottingham
UK | CHINA | MALAYSIA

Asymmetrical Bending

Learning Outcomes

1. Know that an asymmetric cross-section, in addition to its 2nd moments of area about the x and y axes, I_x and I_y, possesses a geometric quantity called the Product Moment of Area, I_{xy}, with respect to these axes (knowledge);
2. Be able to calculate the 2nd moments of area and the product moment of area about a convenient set of axes (application);
3. Know that an asymmetric section will have a set of axes, at some orientation, for which the product moment of area is zero, and that these axes are called the Principal Axes (knowledge);
4. Know that the 2nd moments of area about the principal axes are called the principal 2nd moments of area (knowledge);
5. Be able to determine the 2nd moments of area and the product moment of area about any oriented set of axes, including the principal axes, using a Mohr's circle construction (application);
6. Understand that it is convenient to analyse the bending of a beam with an asymmetric section by resolving bending moments onto the principal axes of the section (knowledge);
7. Be able to follow a basic procedure for analysing the bending of a beam with an asymmetric cross-section (application).

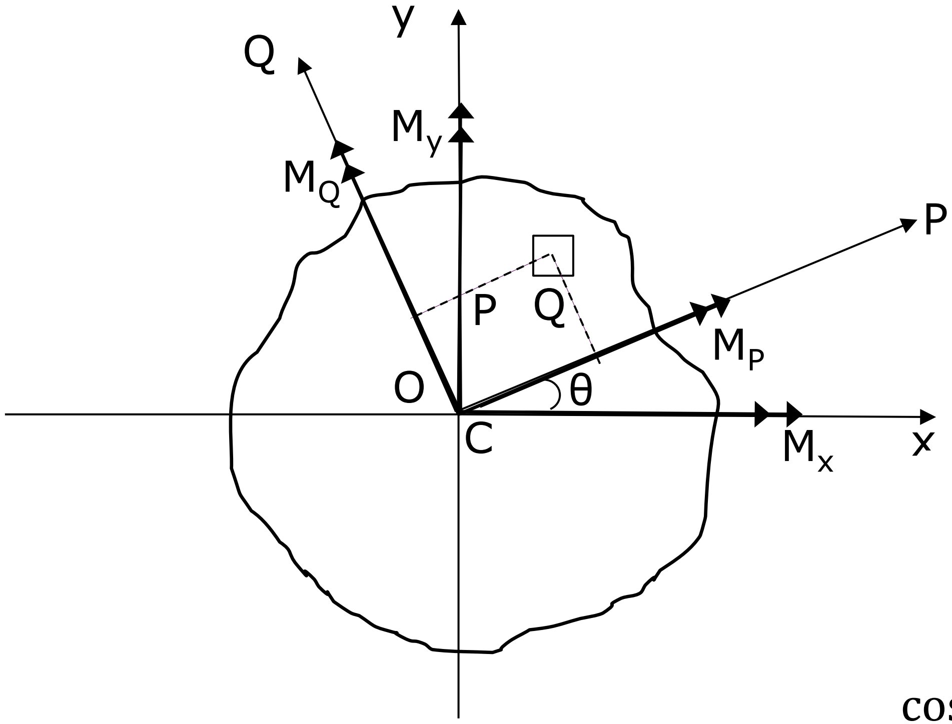
Bending of a Beam with an Asymmetric Section



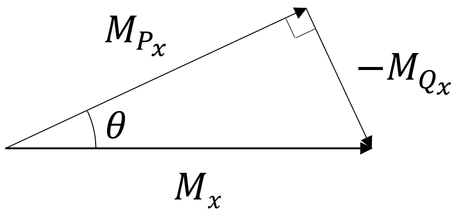
If a bending moment, M_x, is applied about the x-axis only, then the stresses in the flanges will cause bending to take place about both the x and y axes. This is a consequence of I_{xy} not being zero.

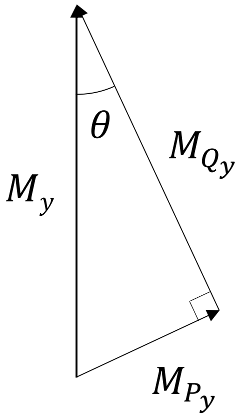
To avoid this moment coupling effect, it is usually convenient to solve bending problems by considering bending about the Principal Axes of a section for which the Product Moment of Area is zero.

Resolving on to Principal Axes



If bending moments, M_x and M_y, are applied about the x and y axes respectively, these van be resolved onto the principal, P-Q, axes as follows:



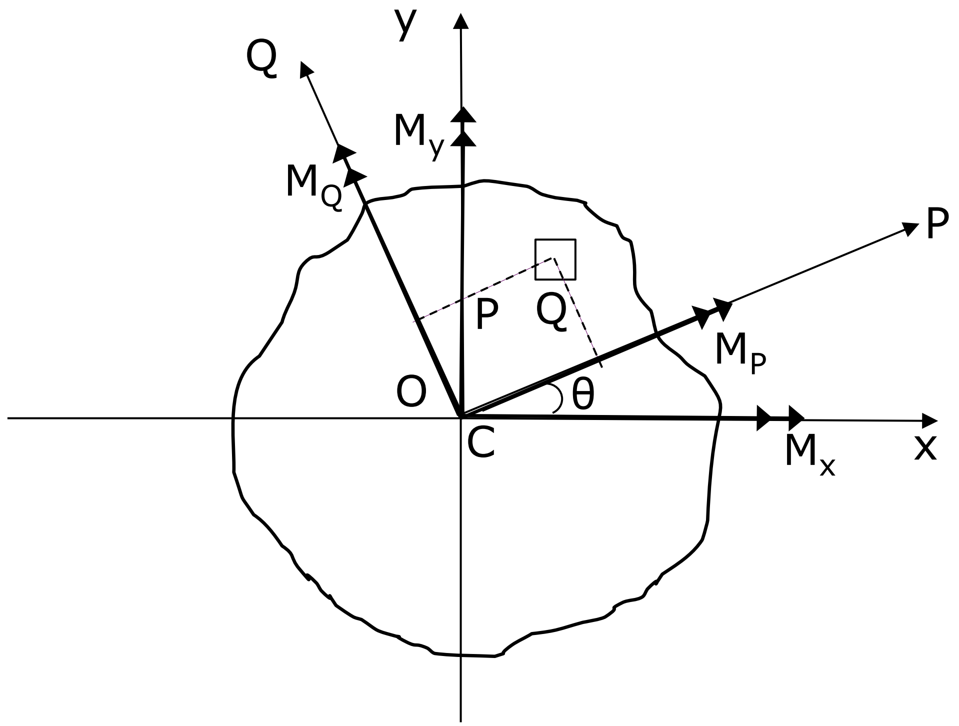


$$\cos\theta = \frac{M_{P_x}}{M_x} \qquad \sin\theta = \frac{-M_{Q_x}}{M_x} \qquad \sin\theta = \frac{M_{P_y}}{M_y} \qquad \cos\theta = \frac{M_{Q_y}}{M_y}$$

$$\therefore M_{P_x} = M_x\cos\theta \qquad \therefore M_{Q_x} = -M_x\sin\theta \qquad \therefore M_{P_y} = M_y\sin\theta \qquad \therefore M_{Q_y} = M_y\cos\theta$$

$$M_P = M_{P_x} + M_{P_y} = M_x\cos\theta + M_y\sin\theta \qquad M_Q = M_{Q_x} + M_{Q_y} = -M_x\sin\theta + M_y\cos\theta$$

Resolving on to Principal Axes

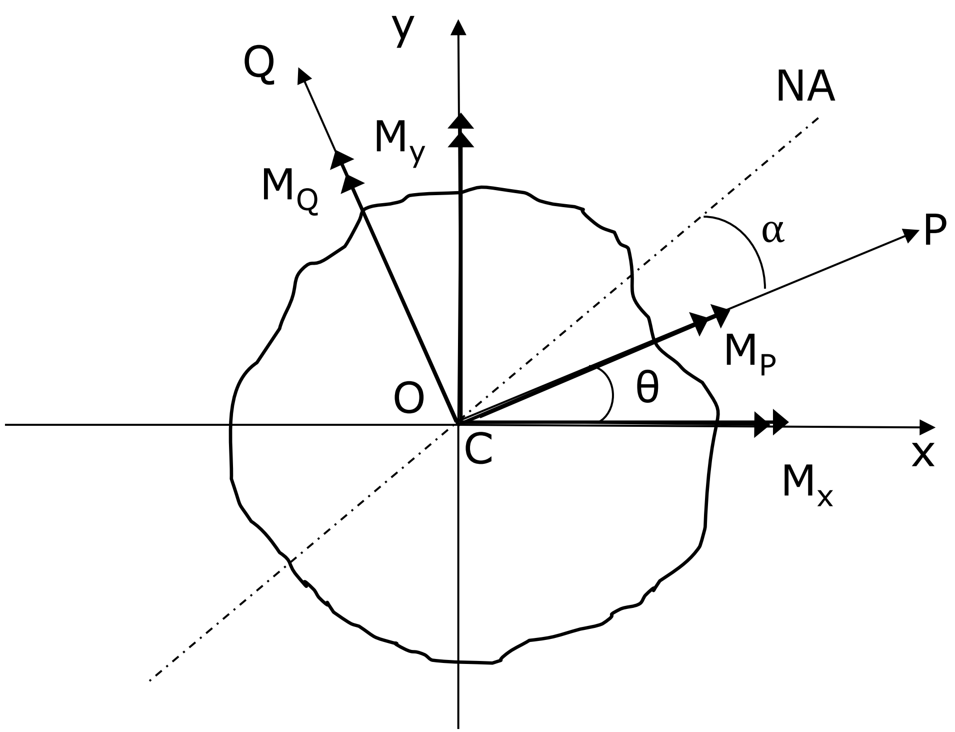


Bending stress at position P, Q can be calculated using the beam bending equation and summing the components from M_P and M_Q:

$$\sigma = \frac{M_P Q}{I_P} - \frac{M_Q P}{I_Q}$$

Note the –ve sign arising because a positive stress results in a –ve moment about the y-axis

Position/Orientation of the Neutral Axis



At the neutral axis there is no stress. Therefore:

$$\sigma = \frac{M_P Q}{I_P} - \frac{M_Q P}{I_Q} = 0$$

$$\therefore \frac{M_P Q}{I_P} = \frac{M_Q P}{I_Q}$$

$$\therefore \frac{Q}{P} = \frac{M_Q I_P}{M_P I_Q}$$

$$\therefore \alpha = \tan^{-1}\left(\frac{Q}{P}\right) = \tan^{-1}\left(\frac{M_Q I_P}{M_P I_Q}\right)$$

The first equation on this slide can therefore be used to determine the magnitude of the stress at any position (P, Q) and the last equation can be used to determine the orientation of the neutral axis and hence the position of the maximum stress which is at the extreme distance from the neutral axis.

Asymmetrical Bending

Learning Outcomes

1. Know that an asymmetric cross-section, in addition to its 2nd moments of area about the x and y axes, I_x and I_y, possesses a geometric quantity called the Product Moment of Area, I_{xy}, with respect to these axes (knowledge);
2. Be able to calculate the 2nd moments of area and the product moment of area about a convenient set of axes (application);
3. Know that an asymmetric section will have a set of axes, at some orientation, for which the product moment of area is zero, and that these axes are called the Principal Axes (knowledge);
4. Know that the 2nd moments of area about the principal axes are called the principal 2nd moments of area (knowledge);
5. Be able to determine the 2nd moments of area and the product moment of area about any oriented set of axes, including the principal axes, using a Mohr's circle construction (application);
6. Understand that it is convenient to analyse the bending of a beam with an asymmetric section by resolving bending moments onto the principal axes of the section (knowledge);
7. Be able to follow a basic procedure for analysing the bending of a beam with an asymmetric cross-section (application).

Procedure for Solving for Bending Stress and Neutral Axis Position in Asymmetrical Bending Problems

1. Determine the Principal Axes of the section, P and Q, about which $I_{xy} = 0$.

2. Consider bending about the principal axes, i.e. resolve bending moments onto these axes.

3. Knowing M_P, M_Q, I_P and I_Q, determine the general expression for the bending stress at position (P, Q) as follows,

$$\sigma = \frac{M_P Q}{I_P} - \frac{M_Q P}{I_Q} = 0$$

4. Determine the angle of the neutral axis with respect to the P-axis as follows,

$$\therefore \alpha = \tan^{-1}\left(\frac{Q}{P}\right) \tan^{-1}\left(\frac{M_Q I_P}{M_P I_Q}\right)$$

5. Evaluate the bending stress at any position in the section including the extreme positions from the neutral axis which give the maximum bending stresses.

See worked example 2.

Asymmetrical Bending

Learning Outcomes

1. Know that an asymmetric cross-section, in addition to its 2nd moments of area about the x and y axes, I_x and I_y, possesses a geometric quantity called the Product Moment of Area, I_{xy}, with respect to these axes (knowledge);
2. Be able to calculate the 2nd moments of area and the product moment of area about a convenient set of axes (application);
3. Know that an asymmetric section will have a set of axes, at some orientation, for which the product moment of area is zero, and that these axes are called the Principal Axes (knowledge);
4. Know that the 2nd moments of area about the principal axes are called the principal 2nd moments of area (knowledge);
5. Be able to determine the 2nd moments of area and the product moment of area about any oriented set of axes, including the principal axes, using a Mohr's circle construction (application);
6. Understand that it is convenient to analyse the bending of a beam with an asymmetric section by resolving bending moments onto the principal axes of the section (knowledge);
7. Be able to follow a basic procedure for analysing the bending of a beam with an asymmetric cross-section (application).