MMME2046 Dynamics and Control: Lecture 5

Planar Dynamics of Rigid Bodies

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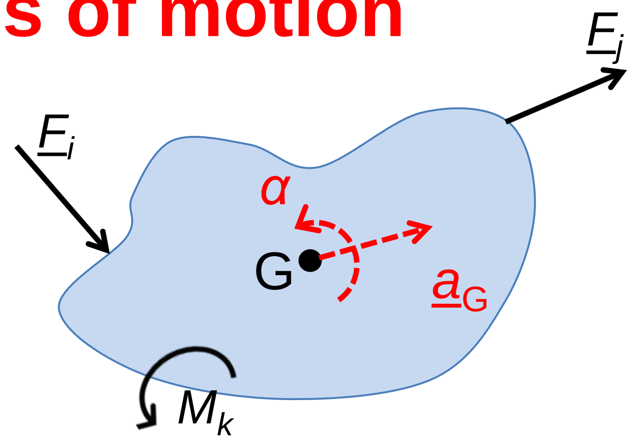
Handouts Chapter III

Lecture objectives

- Dynamics of a linkage mechanism
- Introduction to the coursework

Fundamental Laws of Rigid Body Motion

Equations of motion

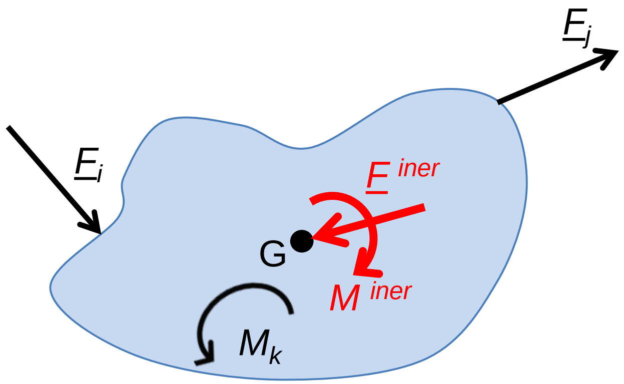


$$\rightarrow^{+}: \ \Sigma F_x = m_G a_{G,x}$$

$$\uparrow^{+}: \ \Sigma F_y = m_G a_{G,y}$$

$$\curvearrowleft^{+}: \ \Sigma M_G = J_G \alpha$$

D'Alembert's principle



$$\rightarrow^{+}: \ \Sigma F_x - F_x^{inertia} = \Sigma F_x - m_G a_{G,x} = 0$$

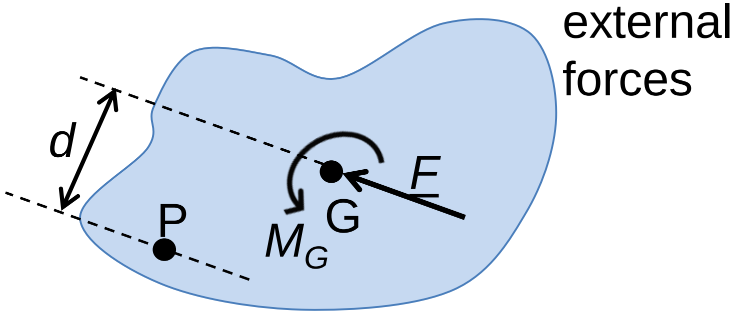
$$\uparrow^{+}: \ \Sigma F_y - F_y^{inertia} = \Sigma F_y - m_G a_{G,y} = 0$$

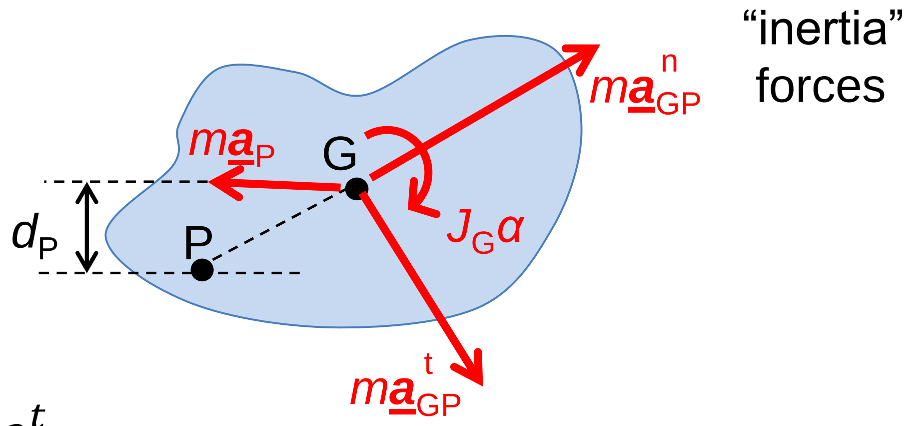
$$\curvearrowleft^{+}: \ \Sigma M_G - M^{inertia} = \Sigma M_G - J_G \alpha = 0$$

Fundamental Laws of Rigid Body Motion

D'Alembert's principle

Given: arbitrary point P with known $\underline{\boldsymbol{a}}_P$



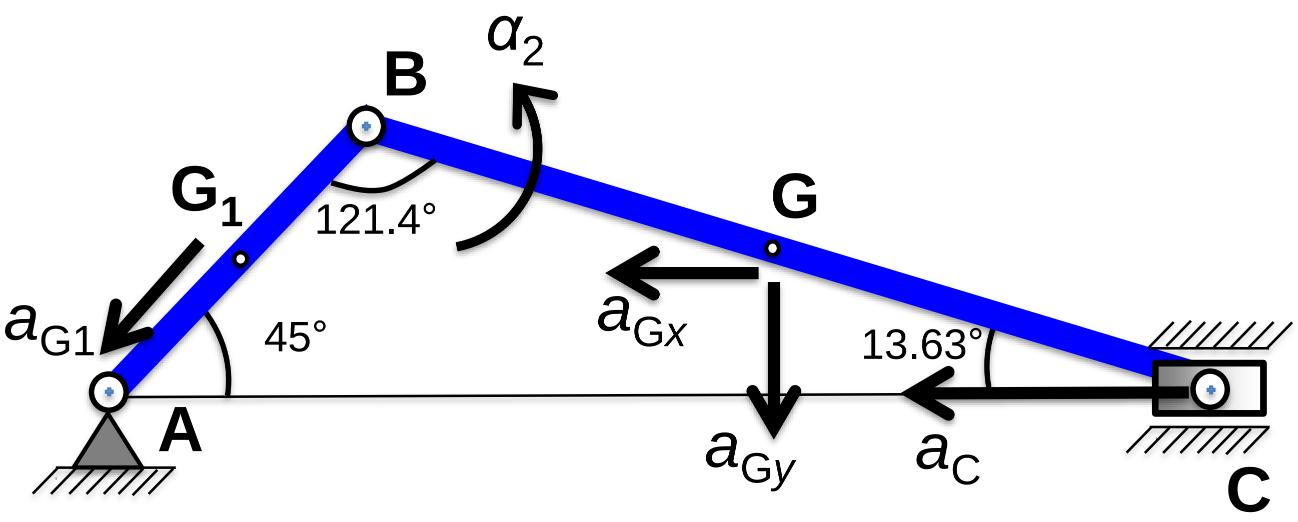


$$\underline{a}_G = \underline{a}_P + \underline{a}_{GP}^n + \underline{a}_{GP}^t$$

$$\circlearrowleft^{+}: \quad M'_P = M_G + Fd - J_G\alpha - mPG^2\alpha + ma_P d_P = 0$$

$$M_P + ma_P d_P = J_P\alpha \qquad (3)$$

Worked Example III.6: Slider-Crank mechanism



ω_1 = 100 rad/s = const.

BC = 240 mm

$\theta = 45^O$

AB = 80 mm

BG = 120 mm

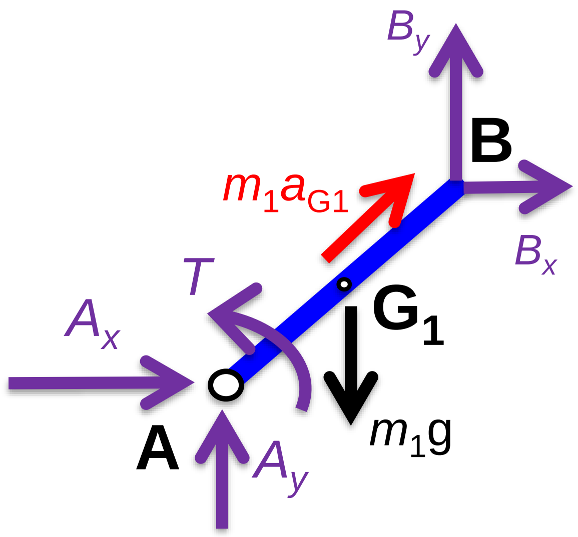
From Worked Example II.4 in Planar Kinematics of Rigid Bodies:

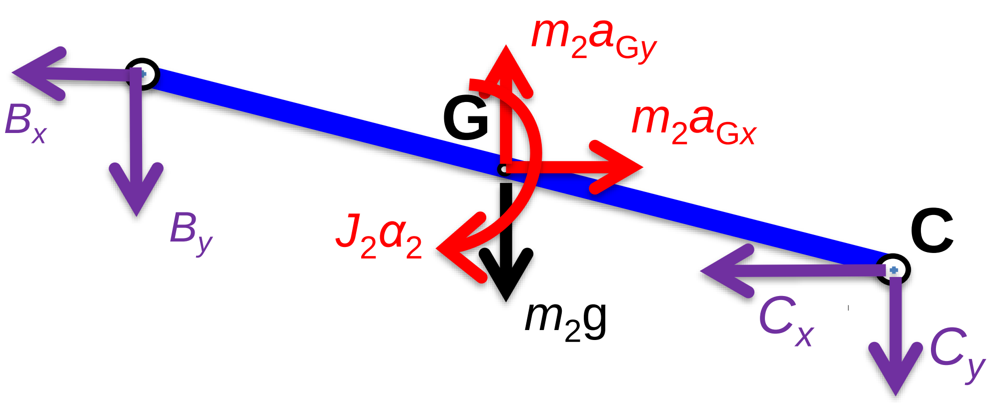
$$\mathrm{a}_{G1} = 400\ m/s^2\,; \quad \mathrm{a}_{Gx} = 579.7\,m/s^2\,; \quad \mathrm{a}_{Gy} = 282.8\,m/s^2\,; \quad \mathrm{a}_c = 573.7\,m/s^2\,; \quad \alpha = 2283\,rad/s^2$$

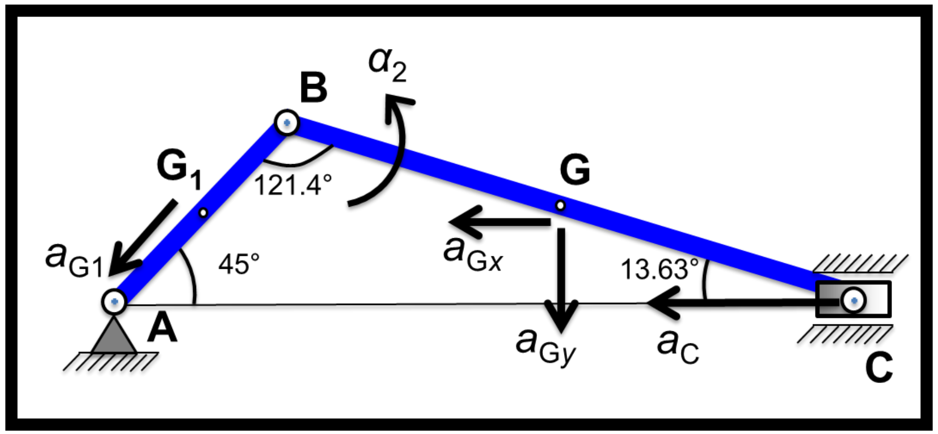
Assume frictionless cylinder and:

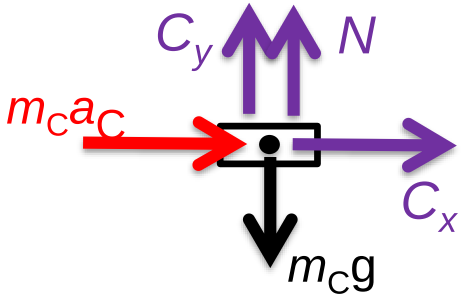
$$m_1 = 0.02\ \text{kg}, \quad m_2 = 0.06\ \text{kg} \quad \text{and} \quad m_C = 0.2\ \text{kg}$$

Worked Example III.6: Slider-Crank mechanism





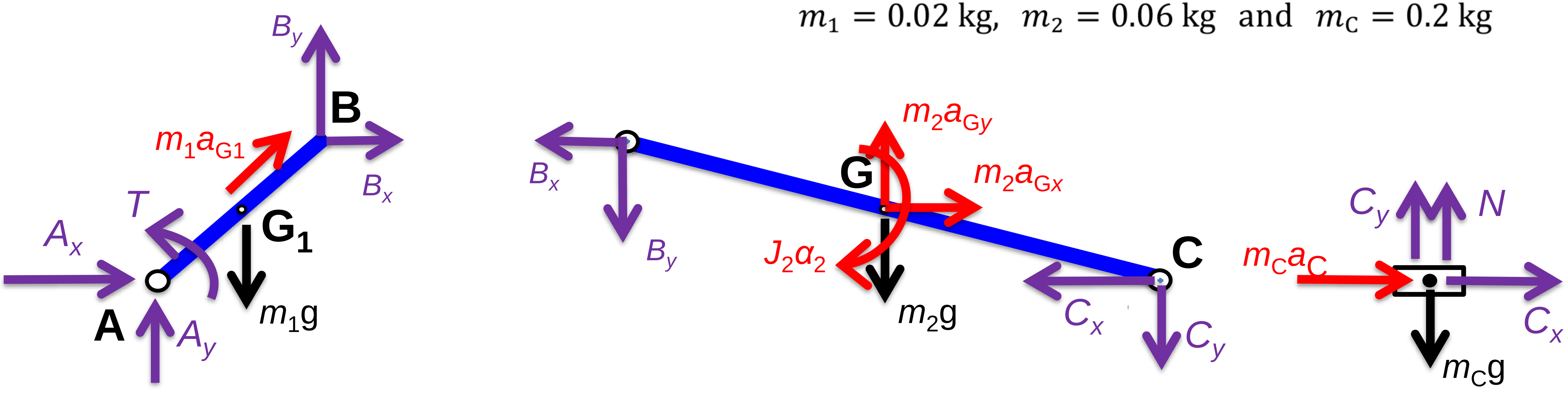




Worked Example III.6: Slider-Crank mechanism

$a_{G1} = 400\ m/s^2$; $a_{Gx} = 579.7\, m/s^2$; $a_{Gy} = 282.8\, m/s^2$; $a_c = 573.7\, m/s^2$; $\alpha = 2283\, rad/s^2$

$m_1 = 0.02$ kg, $m_2 = 0.06$ kg and $m_C = 0.2$ kg



$m_1 g = 0.02 \times 9.8 = 0.196$ N

$m_1 a_{G1} = 0.02 \times 400 = 8\ N$

$m_2 g = 0.06 \times 9.8 = 0.588\ N$

$m_2 a_{Gx} = 0.06 \times 569.7 = 34.18\ N$

$m_2 a_{Gy} = 0.06 \times 282.8 = 16.97\ N$

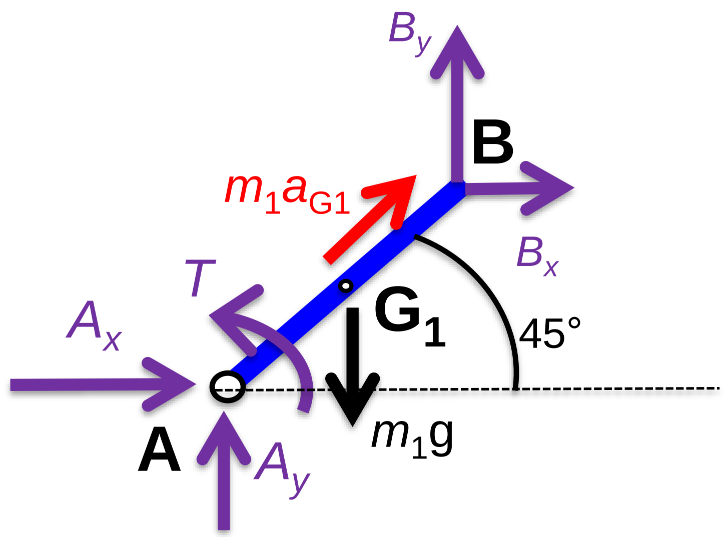
$m_c g = 0.2 \times 9.8 = 1.96\ N$

$m_c a_C = 0.2 \times 573.7 = 114.7\ N$

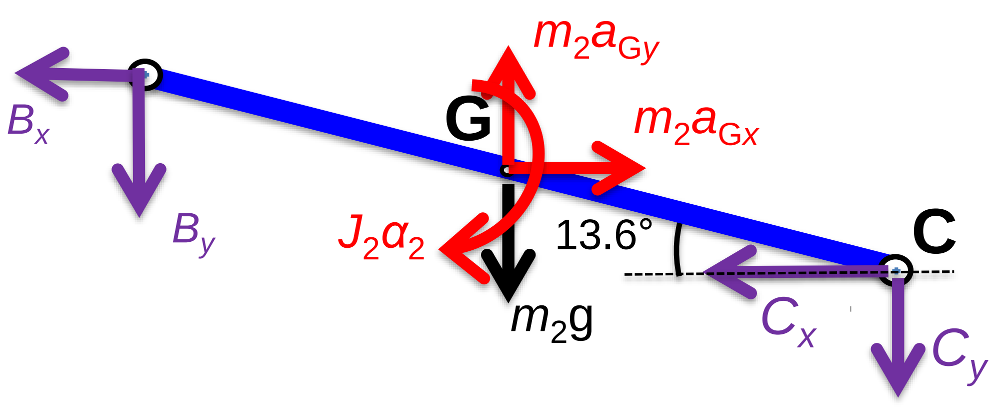
$$J_2\alpha_2 = \left(\frac{1}{12} m_2 BC^2\right)\alpha_2 = \frac{1}{12} 0.06 \times 0.24^2 \times 2283 = 0.6575\ Nm$$

Worked Example III.6: Slider-Crank mechanism

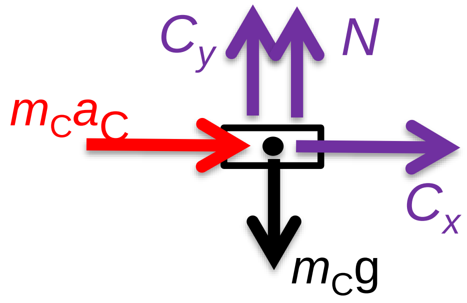
Total 8 E.O.M for 8 unknowns



Rigid body: 3 E.O.M



Rigid body: 3 E.O.M



Particle: 2 E.O.M

Worked Example III.6: Slider-Crank mechanism

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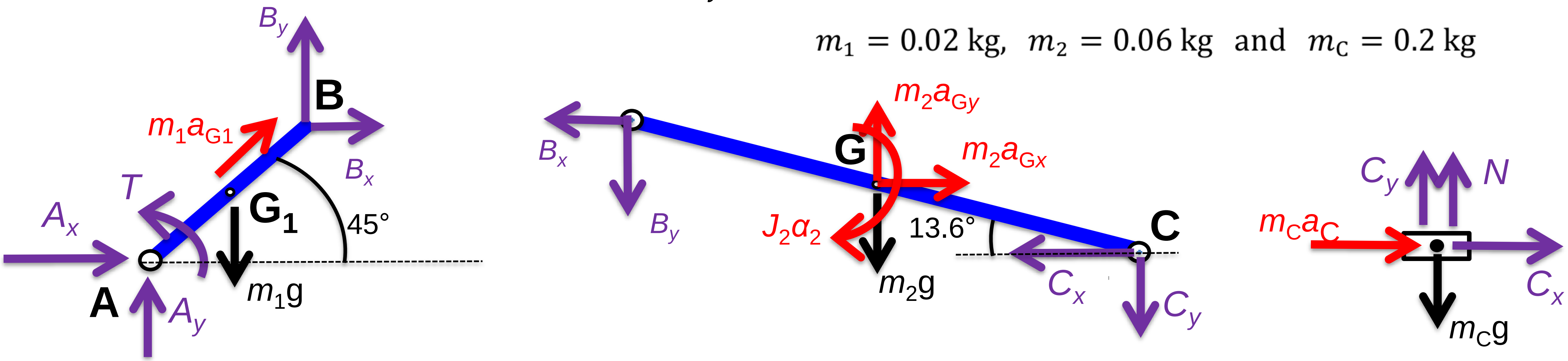
$m_1 = 0.02$ kg, $m_2 = 0.06$ kg and $m_C = 0.2$ kg



Rigid body: 3 E.O.M

Rigid body: 3 E.O.M

Particle: 2 E.O.M

$\rightarrow^+ \Sigma F_x^C = 0$: $C_x + m_C a_c = 0 \quad \rightarrow \quad C_x = -m_c a_c = -114.7$ N

$\circlearrowleft^+ \Sigma M_B^{BC} = 0$:

$$-C_y BC\cos 13.6° - C_x BC\sin 13.6° + m_2 a_{Gx} BG\sin 13.6° + m_2 a_{Gy} BG\cos 13.6° - m_2 g BG\cos 13.6° - J_2\alpha_2 = 0 \quad \rightarrow$$

$$C_y = 37.34 \text{ N}$$

$\uparrow^+ \Sigma F_y^C = 0$: $C_y + N - m_C g = 0 \quad \rightarrow \quad N = m_c g - C_y = -35.38$ N

Worked Example III.6: Slider-Crank mechanism

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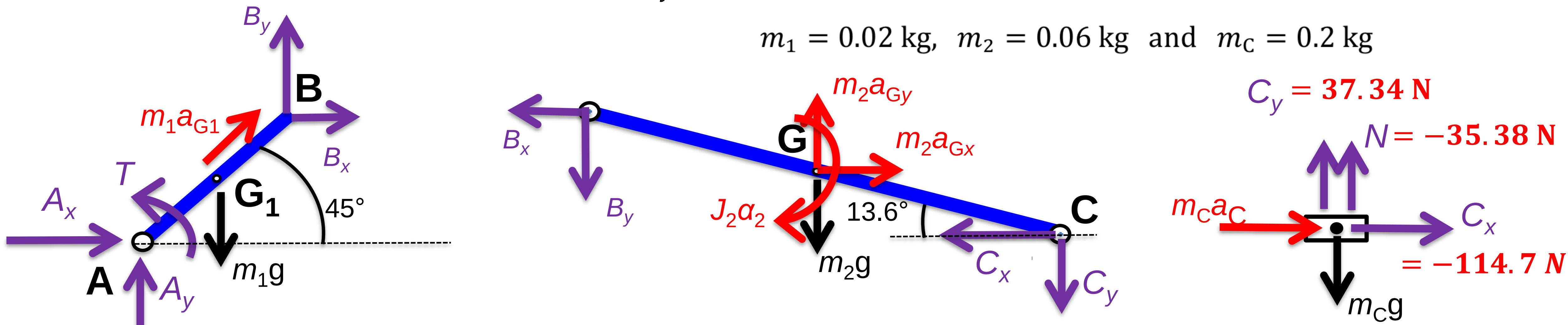

Rigid body: 3 E.O.M

Rigid body: 2 E.O.M

Particle: 0 E.O.M

$\rightarrow^{+}\ \Sigma F_x^{(BC)} = 0$: $-C_x + m_2 a_{Gx} - B_x = 0 \quad \rightarrow \quad B_x = -C_x + m_2 a_{Gx} = 149.48\text{N}$

$\uparrow^{+}\ \Sigma F_y^{(BC)} = 0$: $-C_y - m_2 g + m_2 a_{Gy} - B_y = 0 \quad \rightarrow \quad B_y = -20.96$ N

Worked Example III.6: Slider-Crank mechanism

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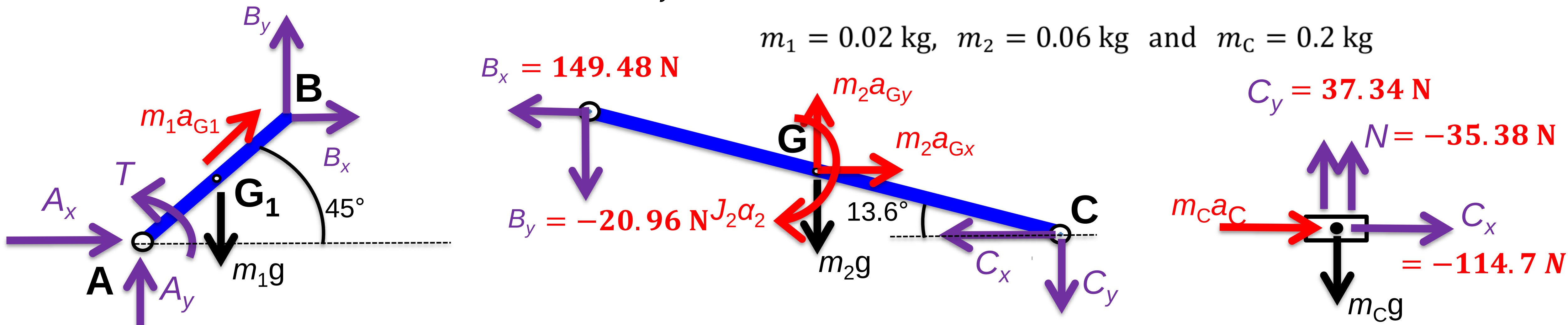


Rigid body: 3 E.O.M

Rigid body: 0 E.O.M

Particle: 0 E.O.M

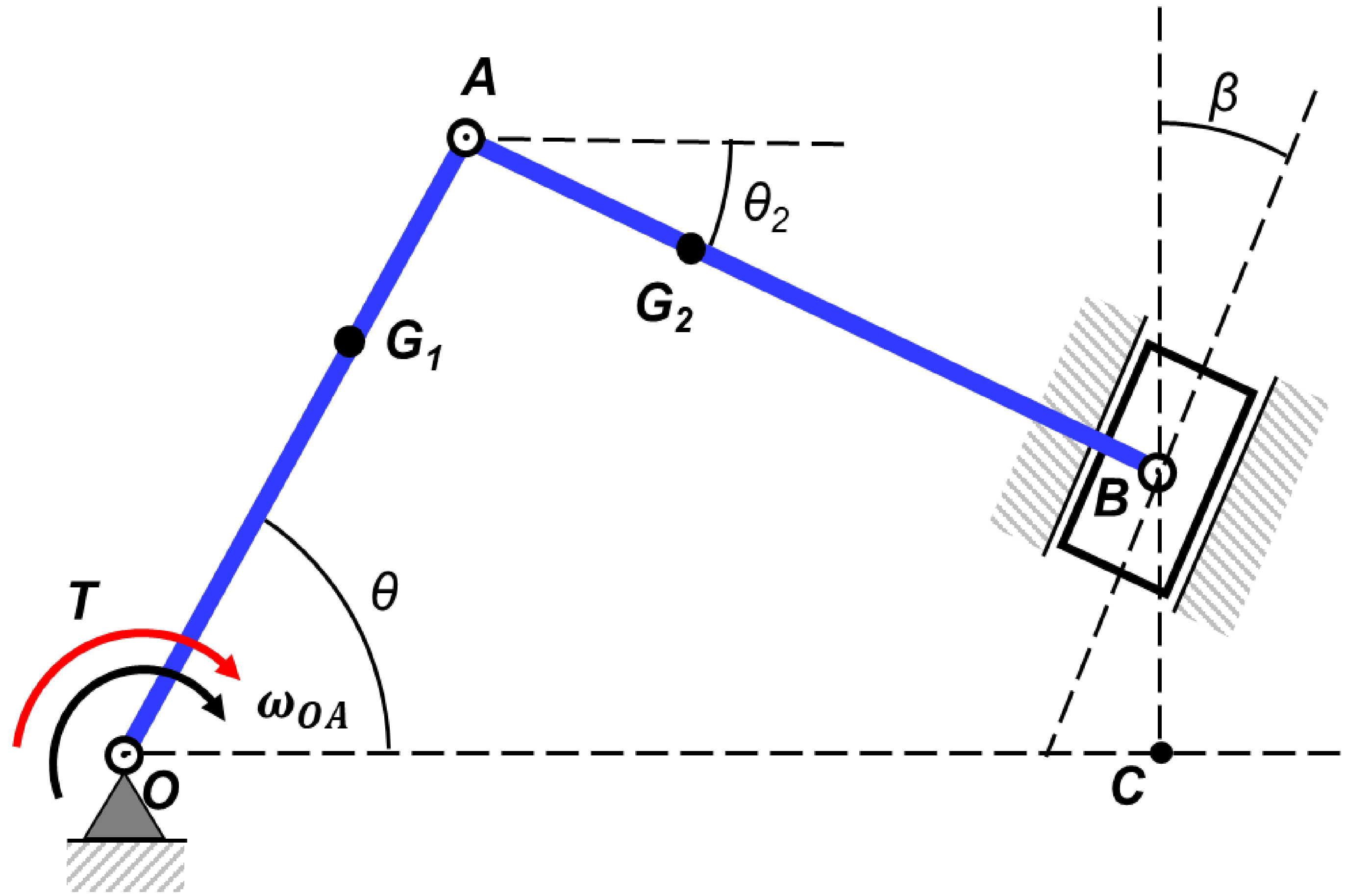
$\rightarrow^+ \ \Sigma F_x^{(AB)} = 0: \quad A_x + m_1 a_1 \cos 45° + B_x = 0 \quad \rightarrow \quad A_x = -155.16 \text{ N}$

$\uparrow^+ \ \Sigma F_y^{(AB)} = 0: \quad A_y + m_1 a_1 \sin 45° + B_y - m_1 g = 0 \quad \rightarrow \quad A_y = 15.50 \text{ N}$

$\curvearrowleft^+ \ \Sigma M_A^{AB} = 0:$

$T - m_1 g A G_1 \cos 45° - B_x AB \sin 45° + B_y AB \cos 45° = 0 \quad \rightarrow \quad T = 9.65 \text{ Nm}$

Machine Dynamics Case Study



Machine Dynamics Case Study

- Use your individual values
- Follow the submission process guidelines