Mechanics of Solids Formula Sheet

Yield Criterion

Principal Stresses:

$$\sigma_1 > \sigma_2 > \sigma_3$$

Tresca:

$$\sigma_y = \sigma_1 - \sigma_3$$

von Mises:

$$2{\sigma_y}^2 = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$$

Tension/Compression (Elastic)

$$\sigma = \frac{F}{A} \qquad \varepsilon = \frac{\Delta L}{L} \qquad \sigma = E\varepsilon$$

Generalised Hooke's Law:

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu(\sigma_y + \sigma_z)\right) \qquad \varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu(\sigma_x + \sigma_z)\right) \qquad \varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu(\sigma_x + \sigma_y)\right)$$

Bending (Elastic)

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R} \qquad EI\frac{d^2y}{dx^2} = M$$

$$M = \int_A y\sigma dA$$

Torsion (Elastic)

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L} \qquad \tau = G\gamma$$

$$T = \int_A r\tau dA = 2\pi\int_0^R \tau r^2 dA$$

Second Moment of Area

General form:

$$I_{xx} = \int_A y^2 dA$$

$$I_{yy} = \int_A x^2 dA$$

Rectangle:

$$I = \frac{bd^3}{12}$$

Circle:

$$I = \frac{\pi D^4}{64}$$

Product Moment of Area:

$$I_{xy} = \int_A xydA$$

Parallel Axis Theorem:

$$I_{x'x'} = I_{xx} + Ab^2$$

$$I_{y'y'} = I_{yy} + Aa^2$$

$$I_{x'y'} = I_{xy} + Aab$$

Polar Second Moment of Area

General form:

$$J = I_{xx} + I_{yy}$$

$$J = \int_A r^2 dA$$

Circle:

$$J = \frac{\pi D^4}{32}$$

Shear Stresses

Transverse Shear Stress:

$$\tau = \frac{S}{Iz}\int_A ydA = \frac{S}{Iz}A\bar{y}$$

Horizontal Shear Stress:

$$\tau = \frac{S}{Iz}\int_A ydA = \frac{S}{Iz}A\bar{y}$$

Shear Force:

$$S = \int_A \tau dA$$

Buckling

$$P_C = \frac{\pi^2 EI}{{L_{eff}}^2}$$

End Condition	Effective Length, L_{eff}
Pinned – Pinned	L
Pinned – Pinned (with initial curvature)	L
Fixed – Free	$2L$
Fixed – Pinned	$0.7L$
Fixed – Fixed	$L/2$

Thermal Stress & Strain

General:

$$\varepsilon_{THERMAL} = \alpha T$$

$$\gamma = \frac{\tau}{G}$$

Beams:

$$\varepsilon_x = \bar{\varepsilon} + \frac{y}{R} = \frac{\sigma_x}{E} + \alpha T$$

$$\sigma_x = E\left\{\bar{\varepsilon} + \frac{y}{R} - \alpha T\right\}$$

$$P = \int_A \sigma_x dA = E\bar{\varepsilon}A - E\alpha \int_A T dA$$

$$M = \int_A y\sigma_x dA = \frac{EI}{R} - E\alpha \int_A TydA$$

Thin Discs:

$$\sigma_r + r\frac{d\sigma_r}{dr} - \sigma_\theta = 0$$

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0$$

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d(u_r)}{dr}\right] = (1+\nu)\alpha\frac{d\big(T(r)\big)}{dr}$$

$$\varepsilon_\theta = \frac{u}{r}$$

$$\varepsilon_r = \frac{du}{dr}$$

$$\sigma_r = \frac{E}{1-\nu^2}\left[\varepsilon_r + \nu\varepsilon_r - (1+\nu)\alpha\big(T(r)\big)\right]$$

$$\sigma_\theta = \frac{E}{1-\nu^2}\left[\varepsilon_\theta + \nu\varepsilon_r - (1+\nu)\alpha\big(T(r)\big)\right]$$

$$u = (1+\nu)\frac{\alpha}{r}\int_0^r \big(T(r)\big)rdr + C_1 r + \frac{C_2}{r}$$

Cylinders:

$$\sigma_r = 0$$

$$\varepsilon_\theta = \frac{1}{E}(\sigma_\theta - \nu\sigma_z) + \alpha T$$

$$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu\sigma_\theta) + \alpha T$$

Compatibility:

$$\varepsilon_\theta = \varepsilon_z = 0$$

$$\sigma_\theta = \sigma_z = \frac{E\alpha(\Delta T)}{(1-\nu)}\left(\frac{y}{t}\right)$$

Strain Energy

Tension/Compression:

$$U = \int_0^L \frac{P^2}{2AE} dx$$

Bending:

$$U = \int_0^L \frac{M^2}{2EI} dx$$

Torsion:

$$U = \int_0^L \frac{T^2}{2GJ} dx$$

Castigliano's Theorem:

$$u = \frac{\partial U}{\partial P}$$

Cylinders

Thin-Walled:

$$\sigma_r = \tau_{rz} = \tau_{r\theta} = 0$$

$$\sigma_\theta = \frac{PR_M}{t}$$

$$\sigma_z = \frac{PR_M}{2t}$$

Thick-Walled:

$$\sigma_r = A - \frac{B}{r^2}$$

$$\sigma_\theta = A + \frac{B}{r^2}$$

$$\sigma_z = \frac{{R_i}^2 p_i - {R_o}^2 p_o}{\left({R_o}^2 - {R_i}^2\right)}$$

$$\varepsilon_\theta = \frac{u}{r}$$

$$\varepsilon_z = \frac{\Delta L}{L}$$

Thick-walled rotating discs

$$\sigma_r = A - \frac{B}{r^2} - \frac{3+\nu}{8}\rho\omega^2 r^2$$

$$\sigma_\theta = A + \frac{B}{r^2} - \frac{1+3\nu}{8}\rho\omega^2 r^2$$

Fracture

$$K_I = Y\sigma\sqrt{\pi a}$$

$$\frac{da}{dN} = C(\Delta K)^m$$

Finite Element Analysis

$$\{F\} = [K]\{u\}$$

Principle of Virtual Work:

$$\delta W_{EXT} + \delta W_{INT} = 0$$

Element Stiffness Matrices for Direct Stiffness Method:

1D Spring Element:

$$[K^e] = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$$

2D truss element (in the global co-ordinate system):

$$[K^e]_g = \left(\frac{AE}{L}\right)\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta & -\cos^2\theta & -\cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & -\cos\theta\sin\theta & -\sin^2\theta \\ -\cos^2\theta & -\cos\theta\sin\theta & \cos^2\theta & \cos\theta\sin\theta \\ -\cos\theta\sin\theta & -\sin^2\theta & \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$

where $k = \frac{AE}{L}$