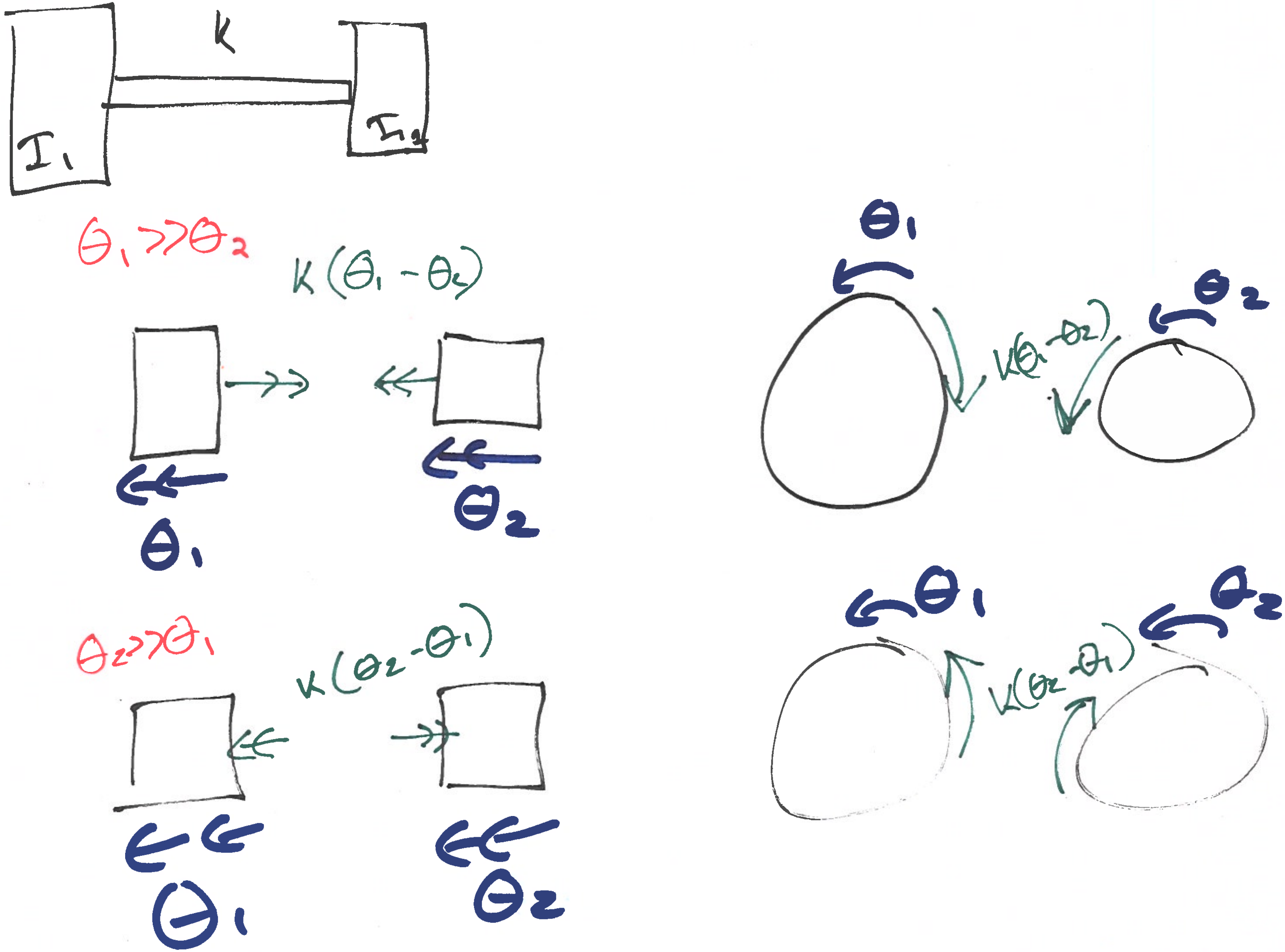
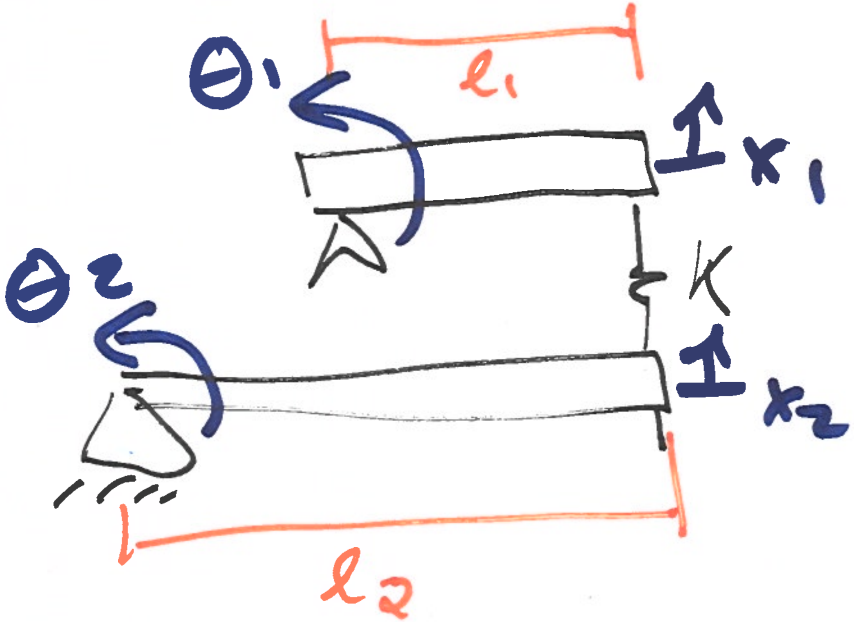


k
I_1
I_2
$\theta_1 >> \theta_2$
$k(\theta_1 - \theta_2)$
θ_1
θ_2
$\theta_2 >> \theta_1$
$k(\theta_2 - \theta_1)$
θ_1
θ_2
θ_1
θ_2
$k(\theta_1 - \theta_2)$
θ_1
θ_2
$k(\theta_2 - \theta_1)$



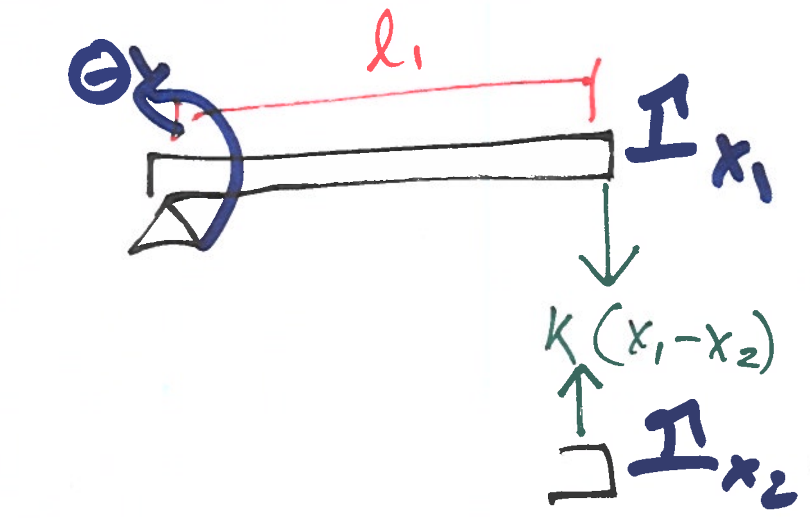
$$[Z]\{\Theta\} = \{0\}$$

$$\{\Theta\} = \begin{Bmatrix} \Theta_1 \\ \Theta_2 \end{Bmatrix} \qquad \{0\} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$x_1 = l_1 \Theta_1 \qquad x_2 = l_2 \Theta_2$$

$$\dot{x}_1 = l_1 \dot{\Theta}_1 \qquad x_2 = l_2 \dot{\Theta}_2$$

$$\ddot{x}_1 = l_1 \ddot{\Theta}_1 \qquad x_2 = l_2 \ddot{\Theta}_2$$

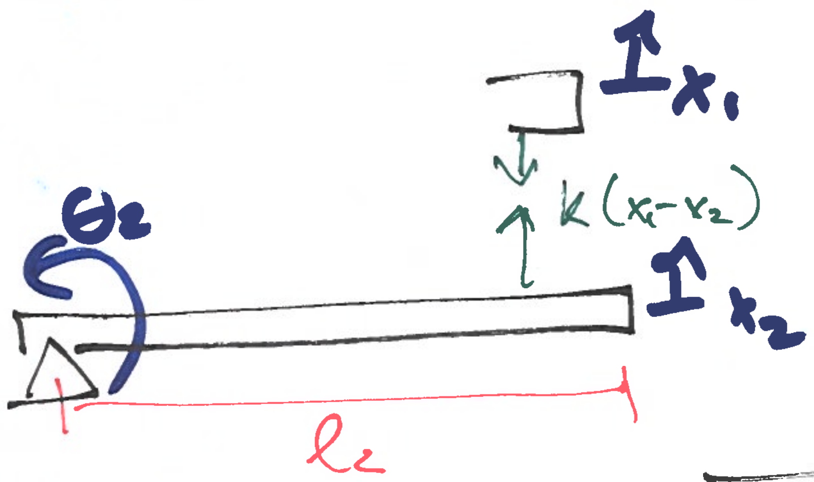


$x_1 >> x_2$

ΣOM_{θ_1} $\quad \Sigma T = I_1 \ddot{\theta}_1$

$$-l_1 k(x_1 - x_2) = I_1 \ddot{\theta}_1$$

$$\boxed{I_1\ddot{\theta}_1 + l_1^2 k \theta_1 - l_1 l_2 k \theta_2 = 0} \quad (1)$$



EOM_{θ_2} $\quad \Sigma T = I_2 \ddot{\theta}_2$

$$+ l_2 k (x_1 - x_2) = I_2 \ddot{\theta}_2$$

$$\boxed{I_2 \ddot{\theta}_2 + l_2^2 k \theta_2 - l_1 l_2 k \theta_1 = 0} \quad (2)$$

$$\boxed{[M]}\{\ddot{\Theta}(t)\} + \boxed{[k]}\{\Theta(t)\} = \{0\}$$

$$\cancel{\begin{bmatrix} I_1 & 0 \\ & \end{bmatrix} \begin{Bmatrix} \ddot{\Theta}_1(t) \\ \ddot{\Theta}_2(t) \end{Bmatrix}}$$

$$\boxed{\begin{bmatrix} I_1 & 0 \\ 0 & I_2 \end{bmatrix}} \begin{Bmatrix} \ddot{\Theta}_1(t) \\ \ddot{\Theta}_2(t) \end{Bmatrix} + \boxed{\begin{bmatrix} l_1^2 k & -l_1 l_2 k \\ -l_1 l_2 k & l_2^2 k \end{bmatrix}} \begin{Bmatrix} \Theta_1 \\ \Theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$[Z]\{\Theta\} = \{0\}$$

$$[Z] = [k] - \omega^2 [M]$$

$$[Z] = \begin{bmatrix} l_1^2 k - \omega^2 I_1 & -l_1 l_2 k \\ -l_1 l_2 k & l_2^2 k - \omega^2 I_2 \end{bmatrix}$$

$$I_1 \ddot{\Theta}_1 + l_1^2 k \Theta_1 - l_1 l_2 k \Theta_2 = 0$$

$$I_2 \ddot{\Theta}_2 + l_2^2 k \Theta_2 - l_1 l_2 k \Theta_1 = 0$$

$\Theta_1 = \Theta_1 \sin \omega t$ $\qquad$ $\Theta_2 = \Theta_2 \sin \omega t$

$\ddot{\Theta}_1 = -\omega^2 \Theta_1 \sin \omega t$ $\qquad$ $\ddot{\Theta}_2 = -\omega^2 \Theta_2 \sin \omega t$

$$H_1 = 1$$

$$-l_1 l_2 k\,(1) + \left(l_2^2 k - \omega^2 I_2\right) H_2 = 0$$

$$H_2 = \frac{l_1 l_2 k}{l_2^2 k - \omega^2 I_2}$$

$$\left(k l_1^2 - \omega^2 I_1\right)(1) - l_1 l_2 k\, H_2 = 0$$

$$H_2 \quad \frac{-\omega^2 I_1 + k l_1^2}{l_1 l_2 k}$$

$$\begin{Bmatrix} H_1 \\ H_2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ \dfrac{l_1 l_2 k}{l_2^2 k - \omega^2 I_2} \end{Bmatrix}$$

1b) $-\omega^2 I_1 \Theta_1 \cancel{\sin\omega t} + l_1^2 k \Theta_1 \cancel{\sin\omega t} - l_1 l_2 k \Theta_2 \cancel{\sin\omega t} = 0$

2b) $-\omega^2 I_2 \Theta_2 + l_2^2 k \Theta_2 - l_1 l_2 k \Theta_1 = 0$

$$\cancel{[Z]} = \begin{bmatrix} k l_1^2 - \omega^2 I_1 & -l_1 l_2 k \\ -l_1 l_2 k & l_2^2 k - \omega^2 I_2 \end{bmatrix} \begin{Bmatrix} \Theta_1 \\ \Theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

nat freq ω_n #

$$\det |Z| = 0$$

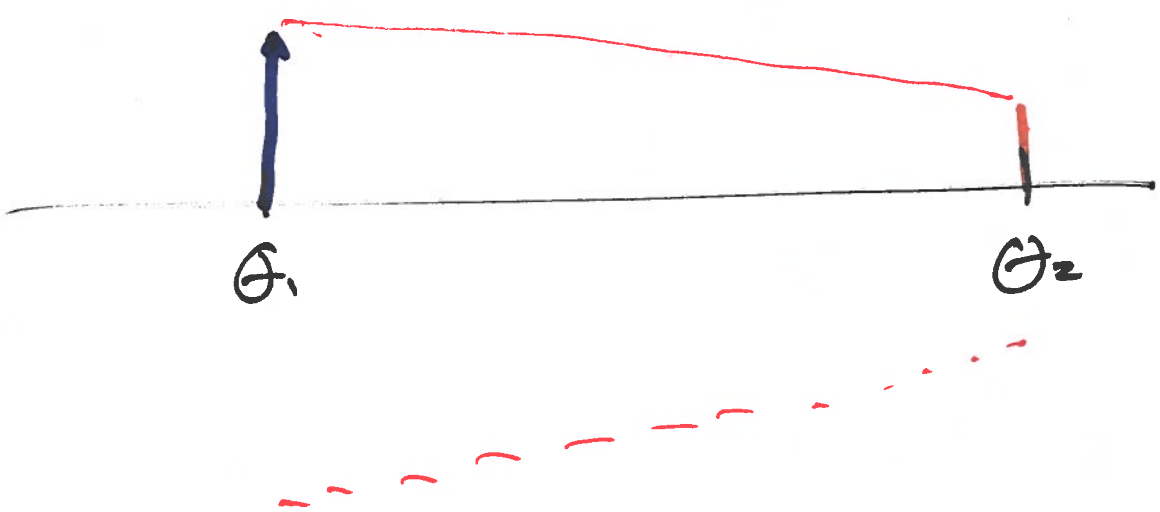
$$(k l_1^2 - \omega^2 I_1)(l_2^2 k - \omega^2 I_2) - (-l_1 l_2 k)(-l_1 l_2 k) = 0$$

$$\boxed{I_1 I_2 \omega^4 - (k l_1^2 I_2 + k l_2^2 I_1)\omega^2 = 0}$$

$$\omega_{n_1}^2 = 0 \qquad \omega_{n_2}^2 = \frac{k l_1^2 I_2 + k l_2^2 I_1}{I_1 I_2}$$

$\omega_{n_1}^{2} = 0$

$$\left\{\begin{matrix} \textcircled{H}_1 \\ \textcircled{H}_2 \end{matrix}\right\} = \left\{\begin{matrix} 1 \\ \dfrac{l_1 \cancel{l_2^{k}} \cancel{k}}{\cancel{l_2^{k}} \cancel{k}} \end{matrix}\right\} = \left\{\begin{matrix} 1 \\ \dfrac{l_1}{l_2} \end{matrix}\right\}$$



$$\omega_n^2 = \frac{k l_1^2 I_2 + k l_2^2 I_1}{I_1 I_2}$$

$$\begin{Bmatrix} \Theta_1 \\ \Theta_2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ \dfrac{l_1 l_2 k}{l_2^2 k - \left(\dfrac{k l_1^2 I_2 + k l_2^2 I_1}{I_1 I_2}\right) I_2} \end{Bmatrix}$$