DYNAMICS AND CONTROL

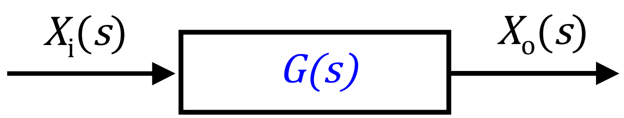
CONTROL SEMINAR 3

GENERAL INTRODUCTION – SEMINAR 3

- Block diagram manipulation
- 2nd order systems

Block Diagrams Revisited

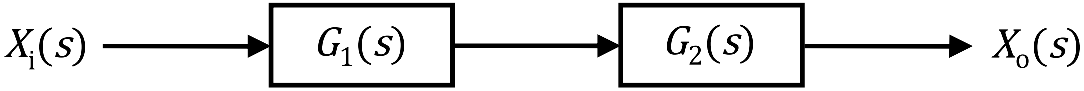
- Systems engineers represent the components of the system as a series of blocks:
- Recap:

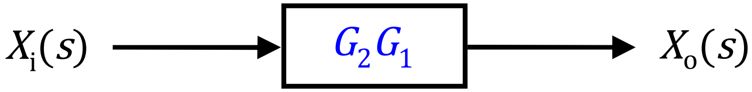


- The transfer function G(s) transforms the input X_i into the output X_0
- $X_0(s) = G(s)X_i(s)$

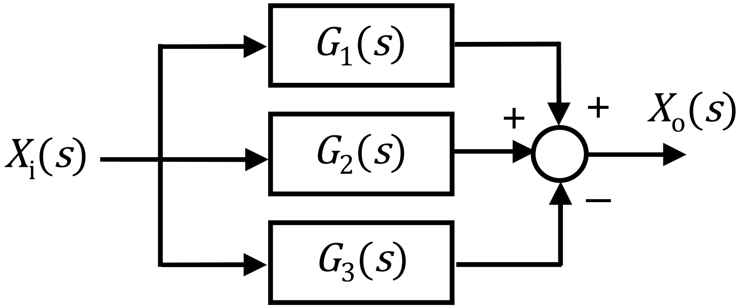
Block Diagram Manipulation: The rules

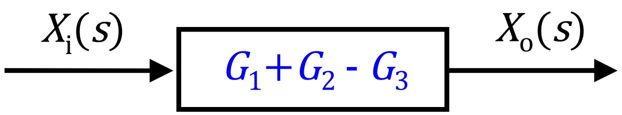
a) Elements in Series:
Multiplication



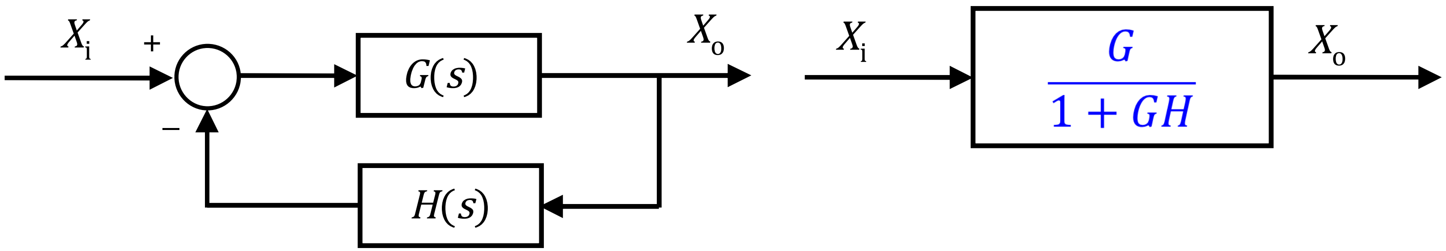


b) Elements in Parallel: Obey the
summing junction (add/subtract)

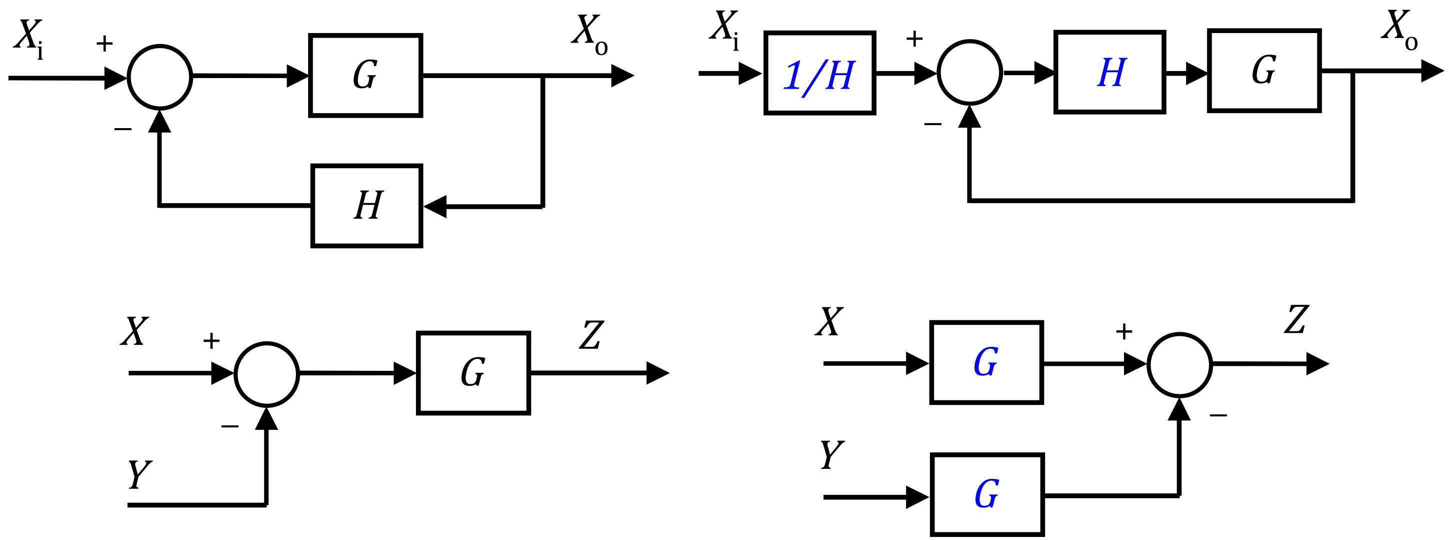




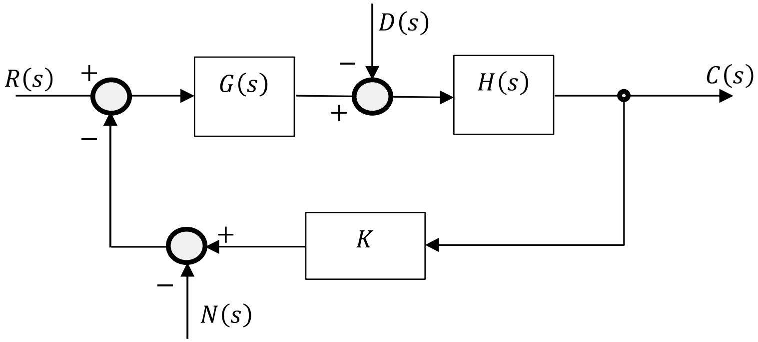
c) Feedback (Closed loop) Transfer Function



d) Changing block positions:

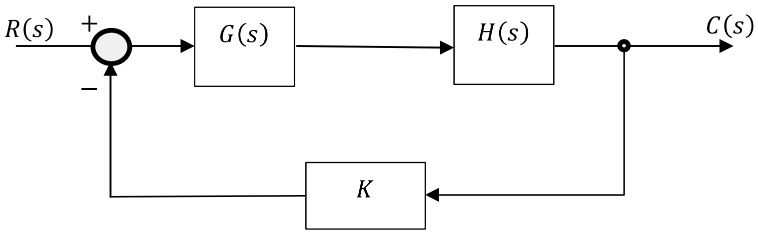


e) Dealing with disturbances



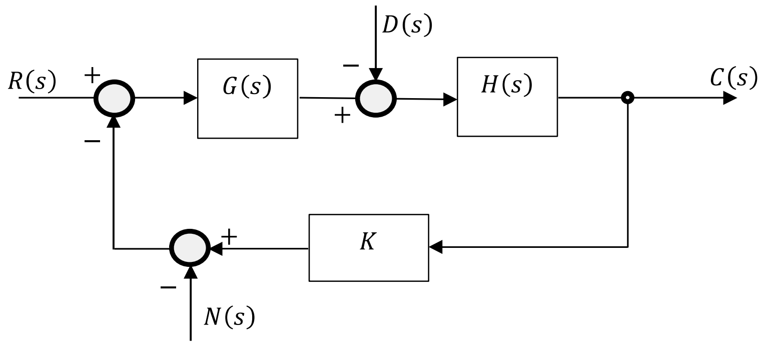
- R(s) is the input
- C(s) is the output
- D(s) is a disturbance
- N(s) is a (user controlled) compensation

- Procedure:
  - Each of the inputs has its own independent transfer function. For R(s):



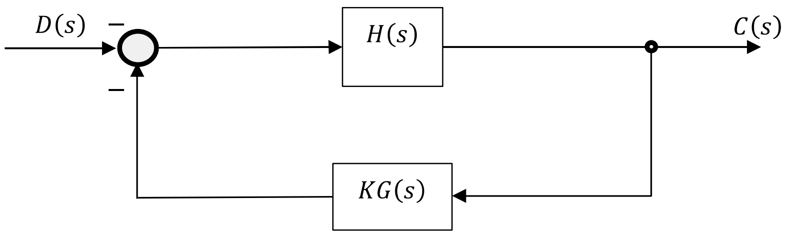
$$C(s) = \big(R(s) - KC(s)\big)G(s)H(s)$$
$$C(s)\big(1 + KG(s)H(s)\big) = R(s)G(s)H(s)$$
$$\frac{C(s)}{R(s)} = \frac{G(s)H(s)}{1 + KG(s)H(s)}$$

e) Dealing with disturbances



- R(s) is the input
- C(s) is the output
- D(s) is a disturbance
- N(s) is a (user controlled) compensation

- Procedure:
  - For D(s):

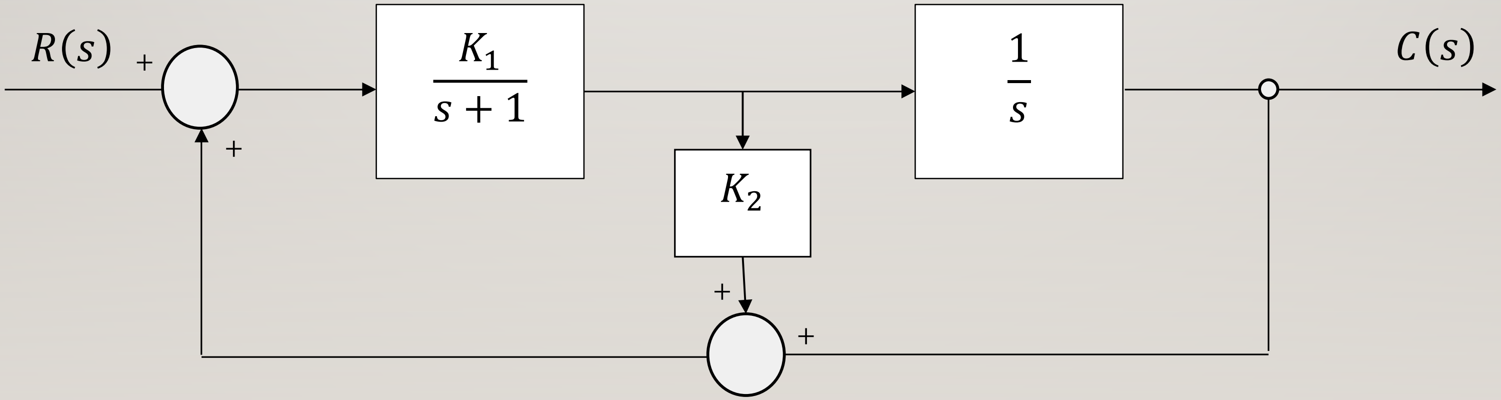


$$C(s) = -\big(D(s) + KG(s)C(s)\big)H(s)$$
$$C(s)\big(1 + KG(s)H(s)\big) = -D(s)H(s)$$
$$\frac{C(s)}{D(s)} = \frac{-H(s)}{1 + KG(s)H(s)}$$

BLOCK DIAGRAM MANIPULATION

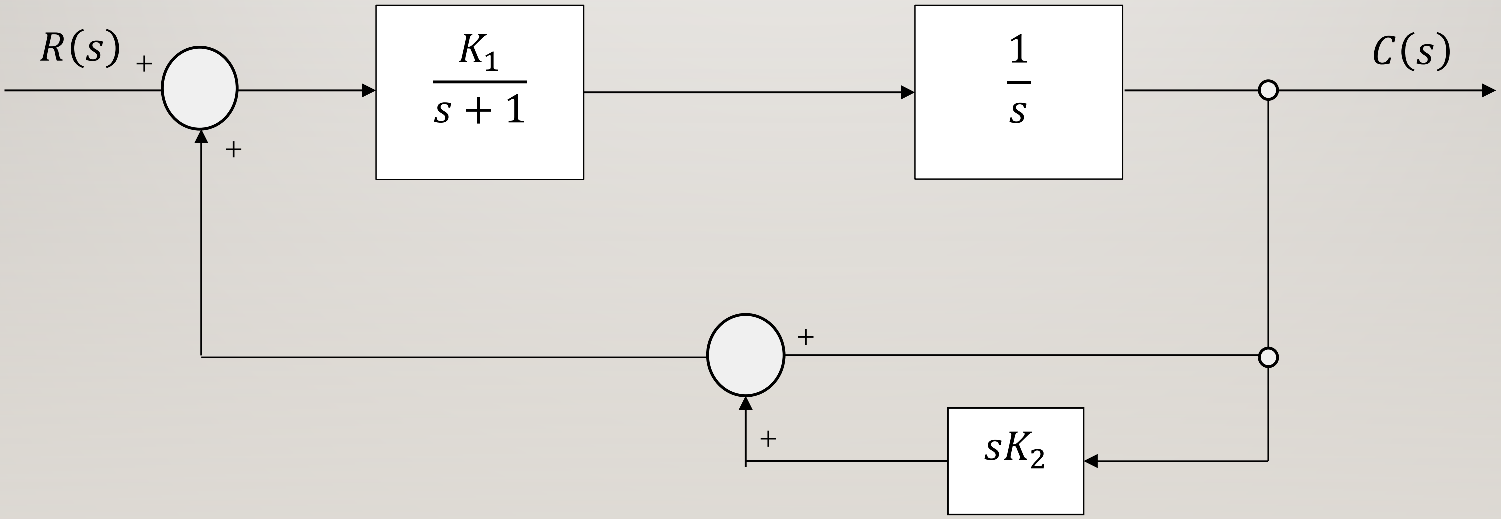
- Example 4 from Example sheet 2

Find the transfer function $\frac{C(s)}{R(s)}$ for:



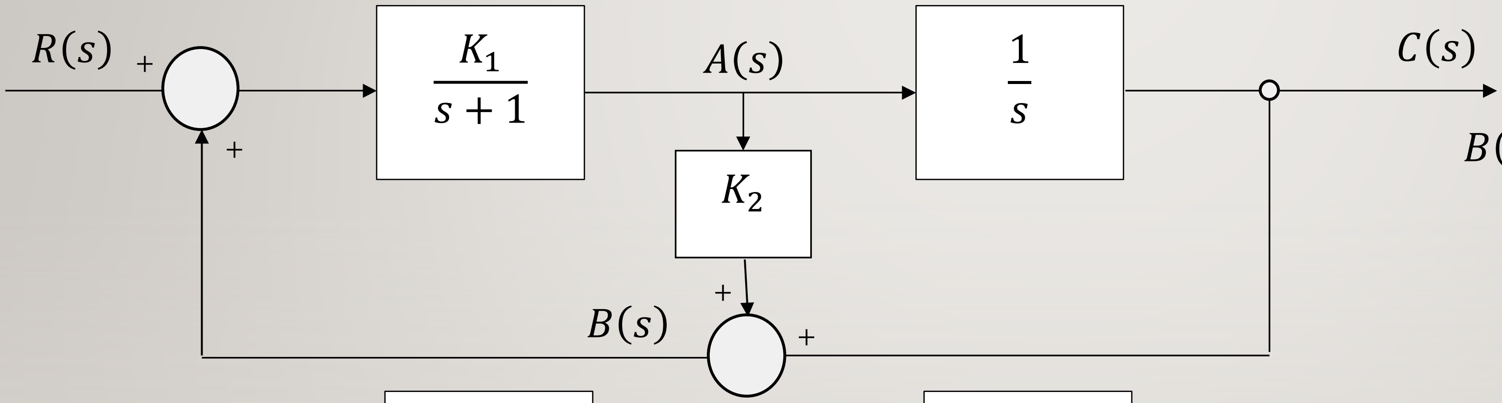
BLOCK DIAGRAM MANIPULATION

- Step 1: resolve inner feedback loop:



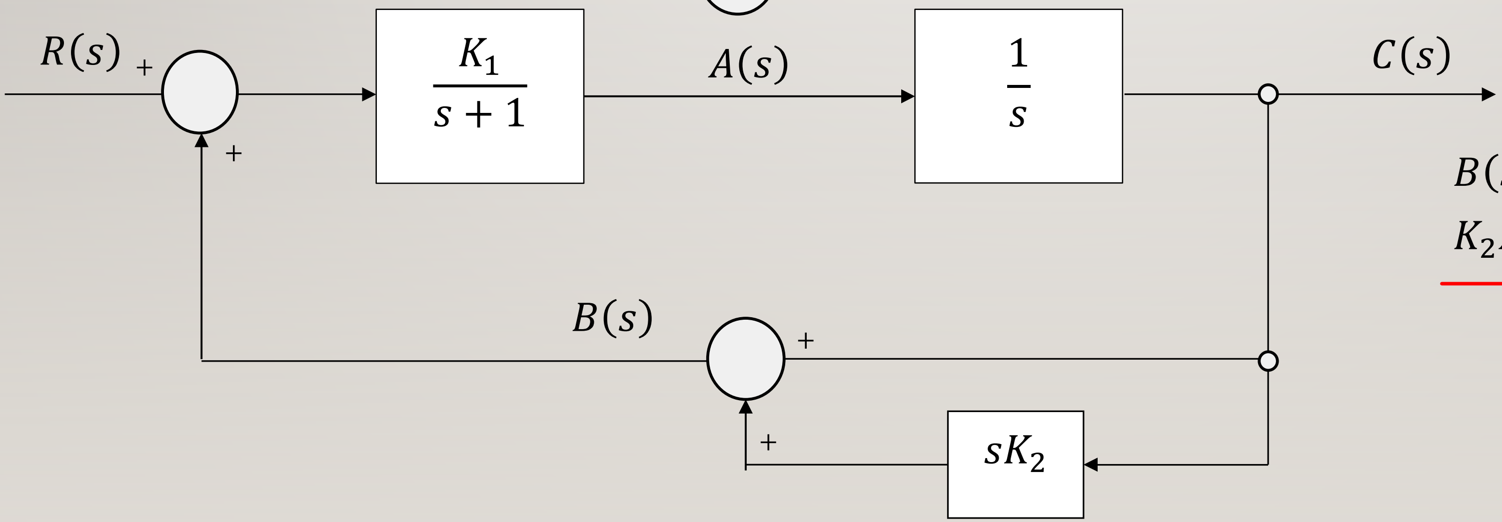
CHECK

- Step 1: resolve inner feedback loop:



$= \frac{A(s)}{s}$

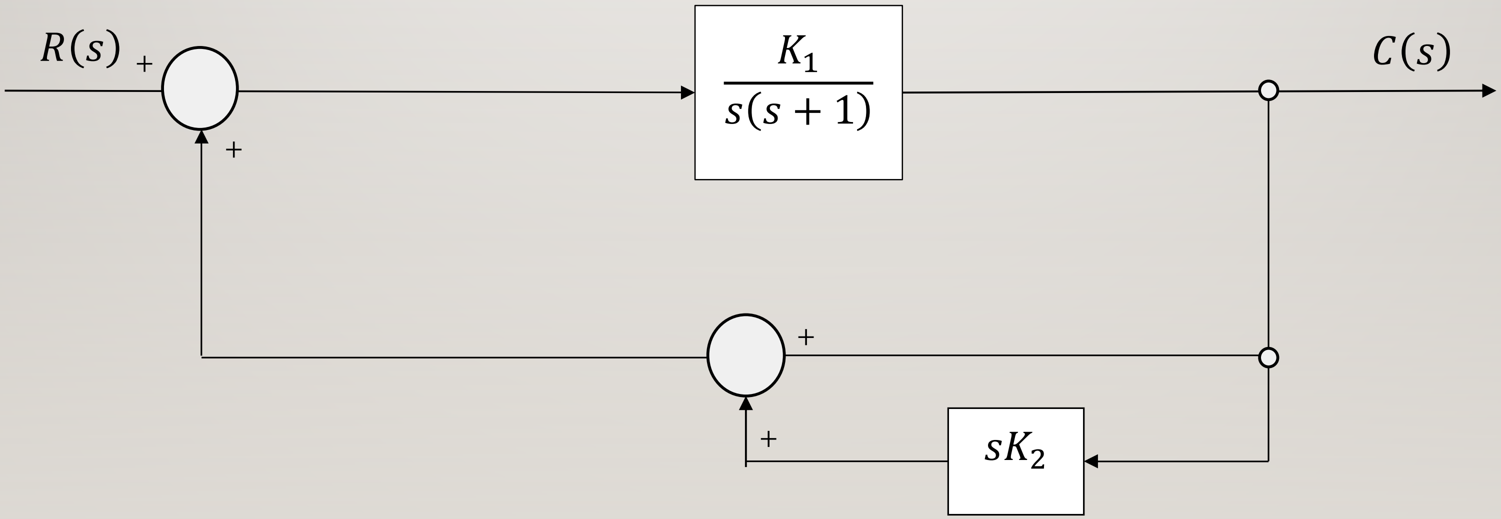
$B(s) = K_2 A(s) + C(s)$



$B(s) = A(s)\left(\frac{1}{s} \times sK_2\right) + C(s) =$
$K_2 A(s) + C(s)$

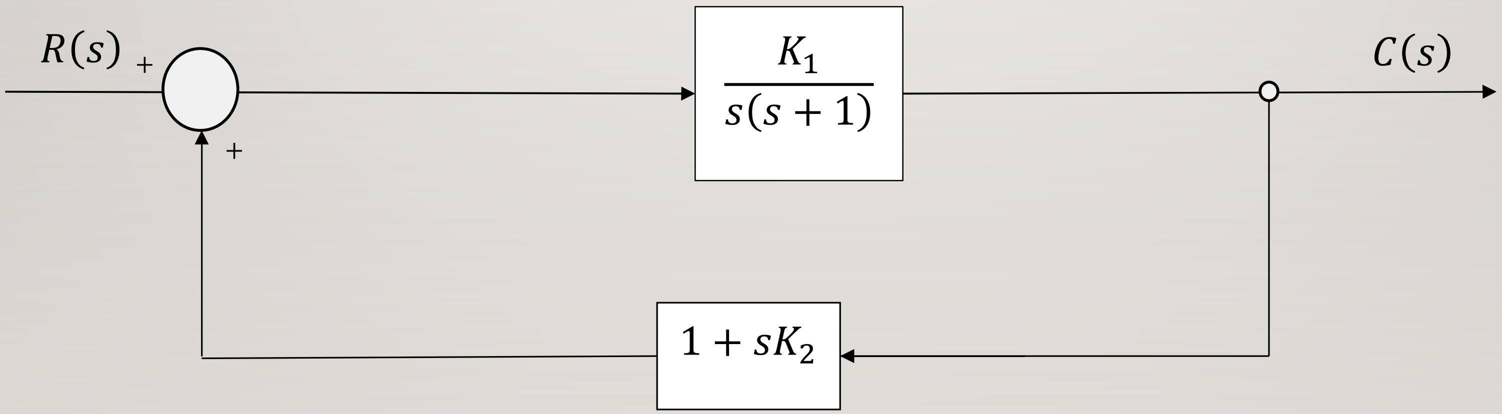
BLOCK DIAGRAM MANIPULATION

- Step 2: Combine forward transfer functions:



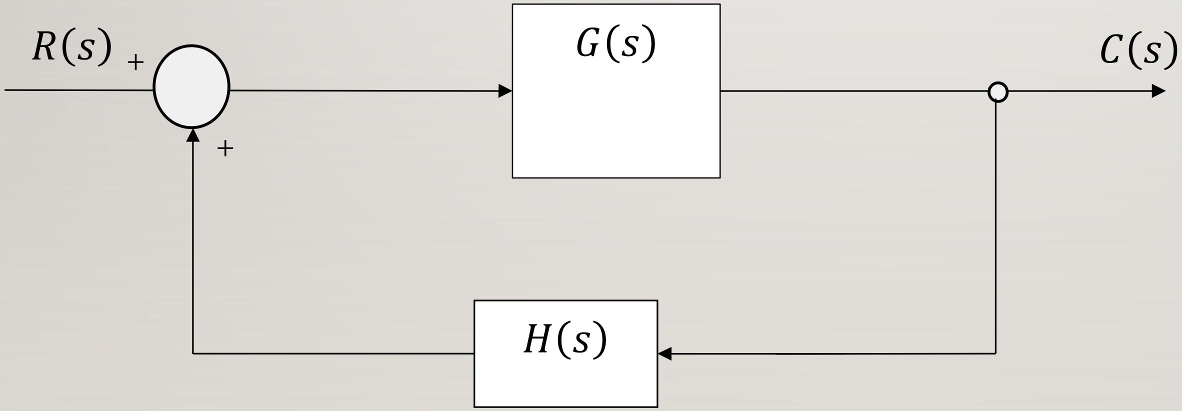
BLOCK DIAGRAM MANIPULATION

- Step 3: Combine feedback transfer functions:



BLOCK DIAGRAM MANIPULATION

- Step 4: Resolve:



$$G(s) = \frac{K_1}{s(s+1)}$$

$$H(s) = 1 + sK_2$$

$$C(s) = \big(R(s) + H(s)C(s)\big)G(s)$$

$$C(s)\big(1 - H(s)G(s)\big) = R(s)G(s)$$

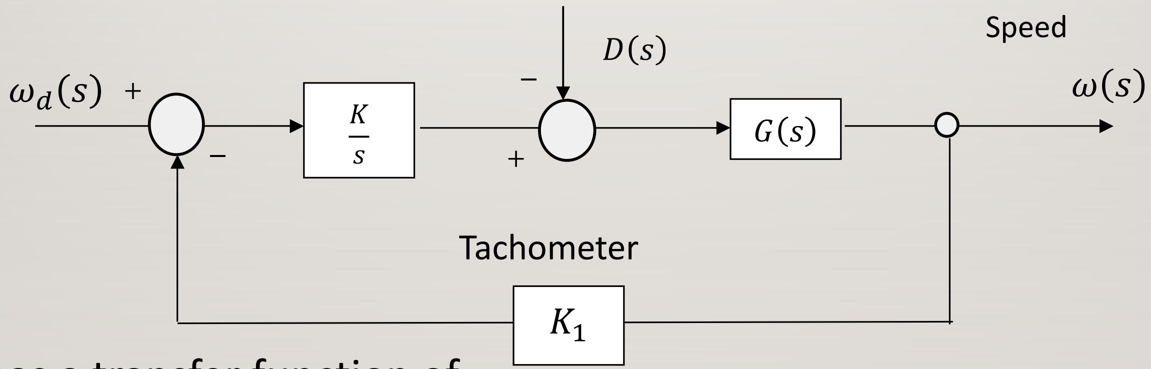
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 - H(s)G(s)}$$

$$= \frac{K_1}{s(s+1)\left(1 - \frac{K_1(1+sK_2)}{s(s+1)}\right)}$$

$$\frac{C(s)}{R(s)} = \frac{K_1}{s(s+1) - K_1(1+sK_2)}$$

EXAMPLE SHEET 2 QUESTION 8

- A control system to maintain the speed of a motor is shown in figure 8.

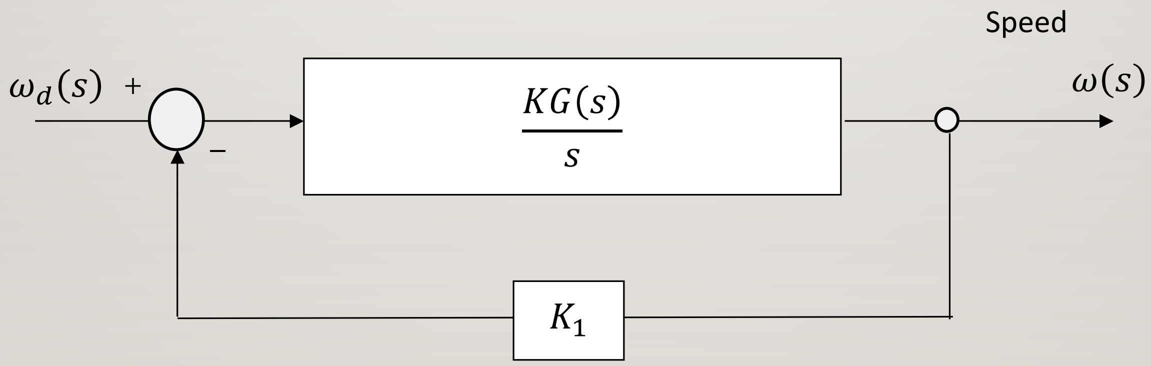


The motor has a transfer function of:

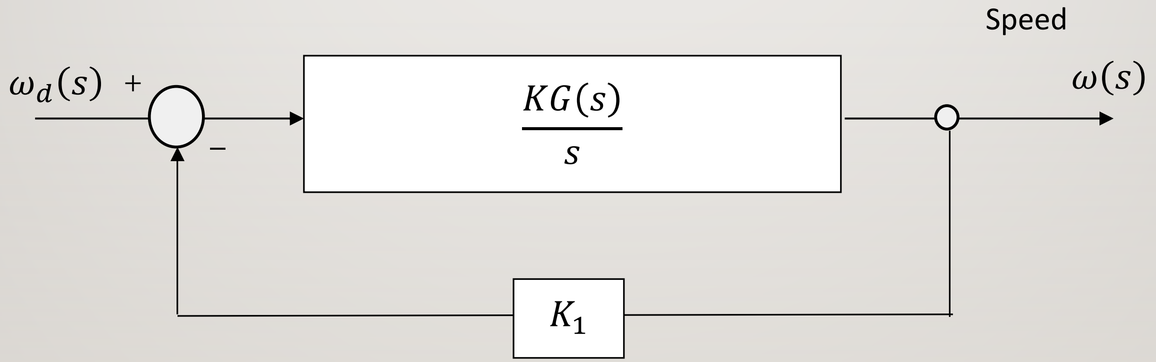
$$G(s) = \frac{1}{s+3}$$

Determine the overall transfer function of the system ω_d to ω.

STEP 1: ELIMINATE THE DISTURBANCE AND COMBINE FORWARD TRANSFER FUNCTIONS



STEP 2: RESOLVE THE FEEDBACK LOOP



$$\omega(s) = \left(\omega_d(s) - K_1\omega(s)\right)\frac{KG(s)}{s}$$

$$\left(\omega(s) + \frac{K_1KG(s)}{s}\omega(s)\right) = \omega(s)\left(1 + \frac{K_1KG(s)}{s}\right) = \frac{KG(s)}{s}\,\omega_d(s)$$

STEP 2: RESOLVE THE FEEDBACK LOOP

Continued …

$$\left(\omega(s) + \frac{K_1 KG(s)}{s}\omega(s)\right) = \omega(s)\left(1 + \frac{K_1 KG(s)}{s}\right) = \frac{KG(s)}{s}\omega_d(s)$$

We want the transfer function $\frac{\omega(s)}{\omega_d(s)}$ so:

$$\frac{\omega(s)}{\omega_d(s)} = \frac{\left(\frac{KG(s)}{s}\right)}{\left(1 + \frac{K_1 KG(s)}{s}\right)} = \frac{KG(s)}{s + K_1 KG(s)}$$

2^{nd} Order Control Systems

- 1^{st} order systems are
  - Reliable
  - Non-oscillatory
  - Slower than 2^{nd} order
- Hydraulics are slow and heavy
- First there was Claudia, now there is Melissa:
  - https://www.youtube.com/watch?v=JWyl9RnKOIQ

2^{nd} Order Control Systems

- Some examples – electro-mechanical position control
  - Power steering

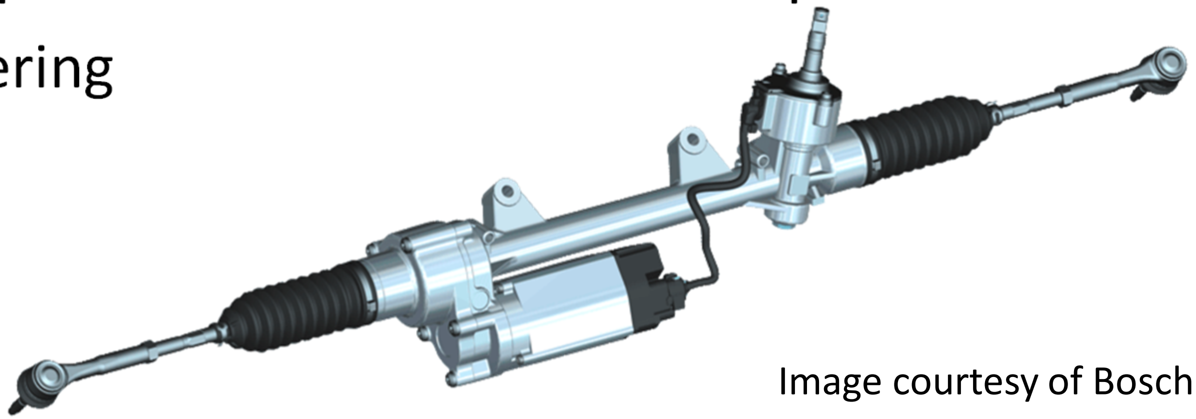


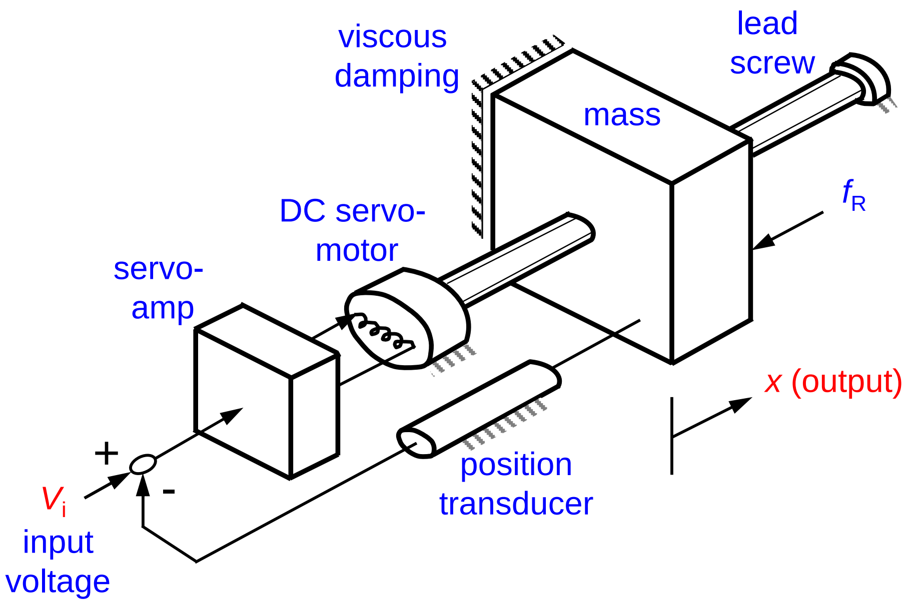
Image courtesy of Bosch AG

  - n-degree of freedom robots
  - …

2^{nd} Order Control Systems

- What is meant by "Second order systems"?
- You will be familiar with 2^{nd} order differential equations and their solutions:
  - $\frac{d^2y}{dt^2} + A\frac{dy}{dt} + B = 0$
  - 2 real roots (overdamped)
  - 1 root (critically damped)
  - 2 complex roots (underdamped)

Example: Electro-Mechanical Position Control System



2nd order system: **transfer function**

$$G(s) = \frac{\omega_n^2 X_i(s)}{x^2 + 2\gamma\omega_n s + \omega_n^2}$$

2nd order system: time domain solution

- I do not expect you to be able to solve for systems with 3rd or higher order behaviour
- Usual input is a step: $X_{in}(s) = \frac{A}{s}$
- Output: $X_{out}(s) = X_{in}(s)G(s) = \frac{A}{s}\left(\frac{1}{s^2+as+b}\right)$

2^{nd} order system: time domain solution

- Output: $X_{out}(s) = X_{in}(s)G(s) = \frac{A}{s}\left(\frac{1}{s^2+as+b}\right)$
- For this system, there are 3 possible outcomes:
  – Overdamped: $a > 2 \times \sqrt{b}$
  – Critically damped: $a = 2 \times \sqrt{b}$
  – Underdamped: $a < 2 \times \sqrt{b}$

2nd order system: time domain solution

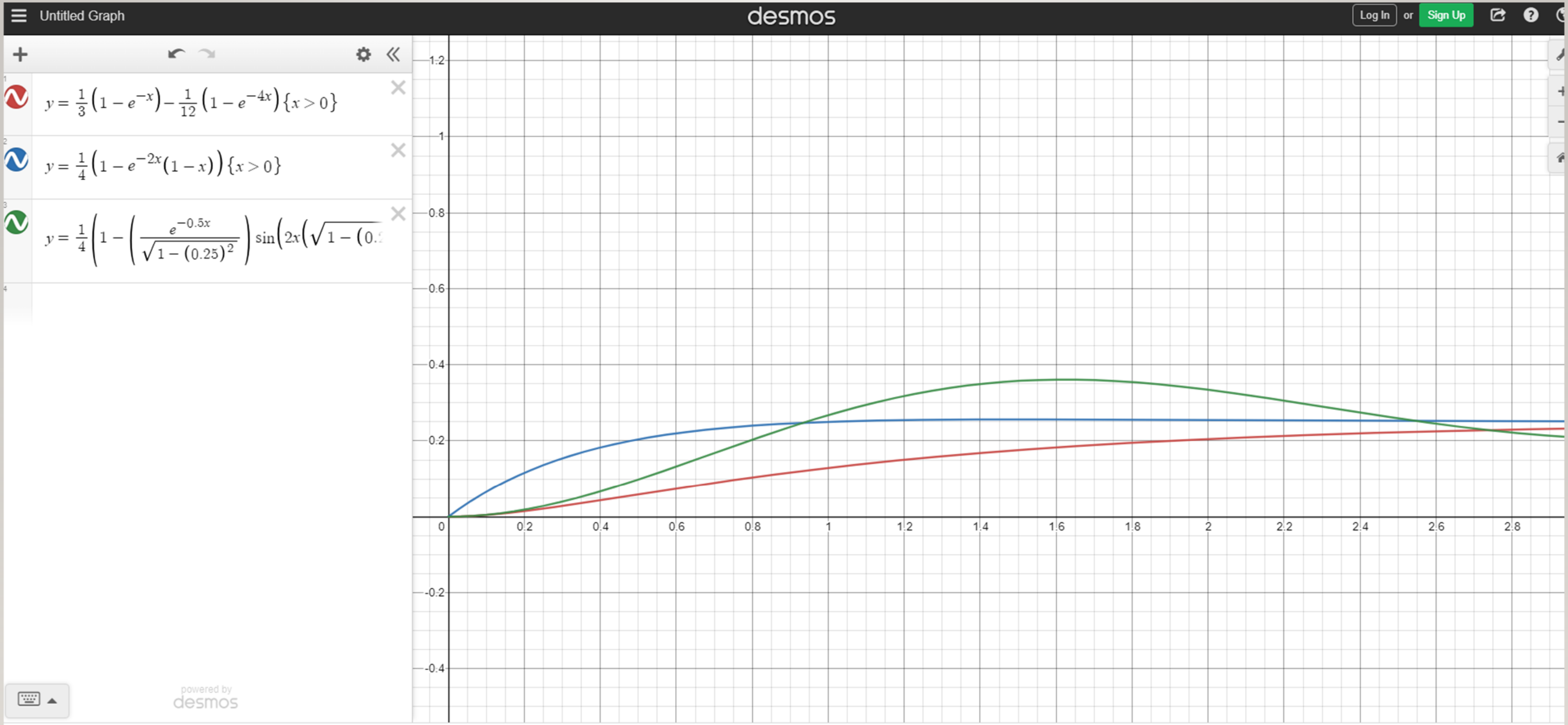
- Example: $X_{in}(s) = \frac{1}{s}$ $\qquad G(s) = \left(\frac{1}{s^2+5s+4}\right)$
- Output: $X_{out}(s) = X_{in}(s)G(s) = \frac{1}{s}\left(\frac{1}{s^2+5s+4}\right) = \frac{1}{s}\left(\frac{1}{(s+4)(s+1)}\right)$
- Overdamped: 2 real roots.
  - Use partial fractions to give:
  - $X_{out}(s) = \frac{1}{s}\left(\frac{1}{3(s+1)} - \frac{1}{3(s+4)}\right)$
  - Inverse Laplace Transforms (no. 8) give:
  - $x_{out}(t) = \frac{1}{3}(1 - e^{-t}) - \frac{1}{12}(1 - e^{-4t})$

2nd order system: time domain solution

- Example: $X_{in}(s) = \frac{1}{s}$ $\quad$ $G(s) = \left(\frac{1}{s^2+4s+4}\right)$
- Output: $X_{out}(s) = X_{in}(s)G(s) = \frac{1}{s}\left(\frac{1}{s^2+4s+4}\right) = \frac{1}{s}\left(\frac{1}{(s+2)^2}\right)$
- Critically damped: 2 duplicate real roots.
  - Use partial fractions to give:
  - $X_{out}(s) = \frac{1}{4s} - \frac{s+4}{4(s+2)^2} = \frac{1}{4s} + \frac{s}{4(s+2)^2} + \frac{1}{(s+2)^2}$
  - Inverse Laplace Transforms give:
  - $x_{out}(t) = \frac{1}{4}(1 - te^{-2t}) - \frac{1}{4}e^{-2t}(1 - 2t) = \frac{1}{4}(1 - (1 - t)e^{-2t})$

2nd order system: time domain solution

- Example: $X_{in}(s) = \frac{1}{s}$ $\quad G(s) = \left(\frac{1}{s^2+s+4}\right)$
- Output: $X_{out}(s) = X_{in}(s)G(s) = \frac{1}{s}\left(\frac{1}{s^2+s+4}\right) = \frac{1}{s}\left(\frac{1}{(s+4)(s+1)}\right)$
- Underdamped: 2 complex roots.
  - Define $\omega_n = 2$ $\quad 2\gamma\omega_n = 1$ $\quad \gamma$=0.25
  - $X_{out}(s) = \frac{1}{s}\left(\frac{1}{s^2+2\gamma\omega_n s+\omega_n^2}\right)$
  - Inverse Laplace Transforms (no. 15) give:
  - $x_{out}(t) = \frac{1}{4}\left(1 - \frac{e^{-\gamma\omega t}}{\sqrt{1-\gamma^2}}\sin\left(\omega t\sqrt{1-\gamma^2} + \phi\right)\right) = \frac{1}{4}\left(1 - \frac{e^{-0.5t}}{\sqrt{15/16}}\sin\left(2t\sqrt{15/16} + \phi\right)\right)$



Untitled Graph
desmos
Log In
or
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$y=\frac{1}{3}\left(1-e^{-x}\right)-\frac{1}{12}\left(1-e^{-4x}\right)\{x>0\}$
$y=\frac{1}{4}\left(1-e^{-2x}(1-x)\right)\{x>0\}$
$y=\frac{1}{4}\left(1-\left(\frac{e^{-0.5x}}{\sqrt{1-(0.25)^2}}\right)\sin\left(2x\left(\sqrt{1-(0.\right.\right.\right.$
1.2
1
0.8
0.6
0.4
0.2
-0.2
-0.4
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
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THE END?

Any questions?