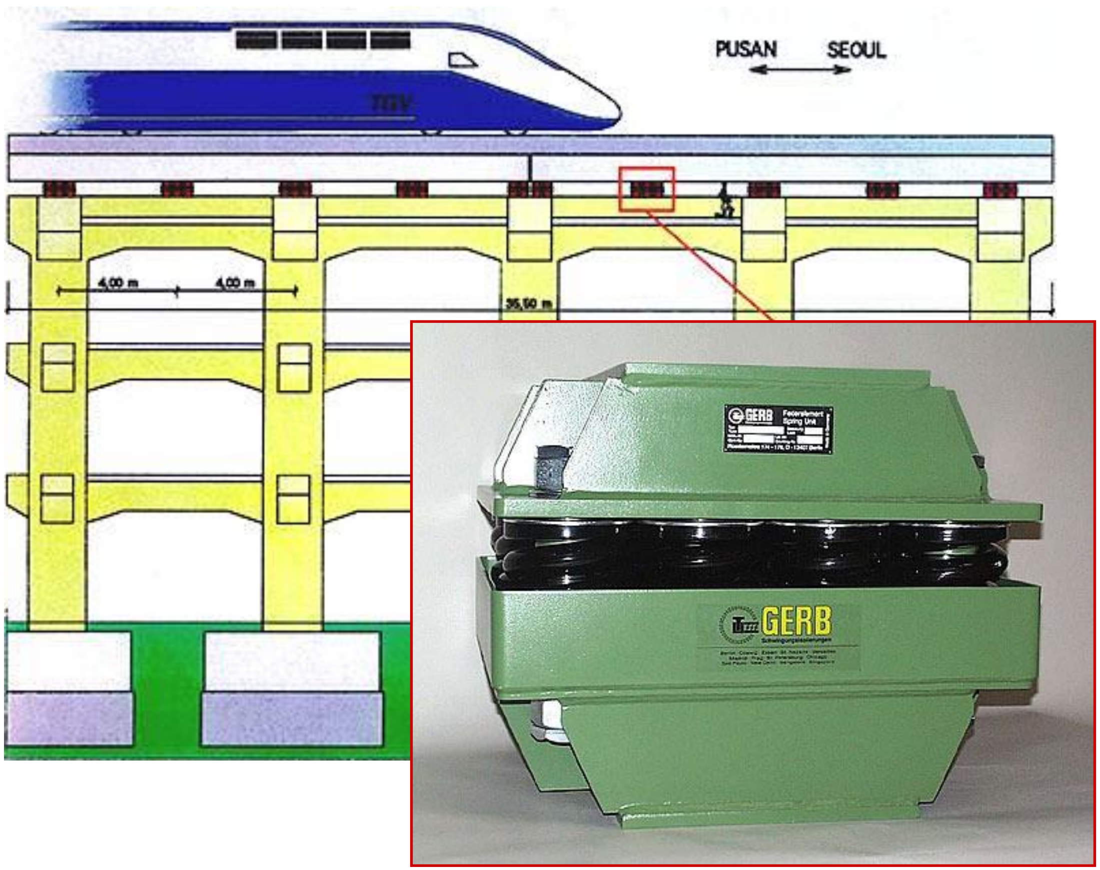
# VIBRATION ISOLATION

Vibration isolators (also known as “anti-vibration mounts”) are used for reducing the vibration transmitted from a source

They work by introducing flexibility between a device and its support




**Case (a)**

The aim is to reduce the force transmitted to the support

Examples are :-

a passing train that can produce ground-borne vibration and

a car engine that can transmit vibration to the body shell

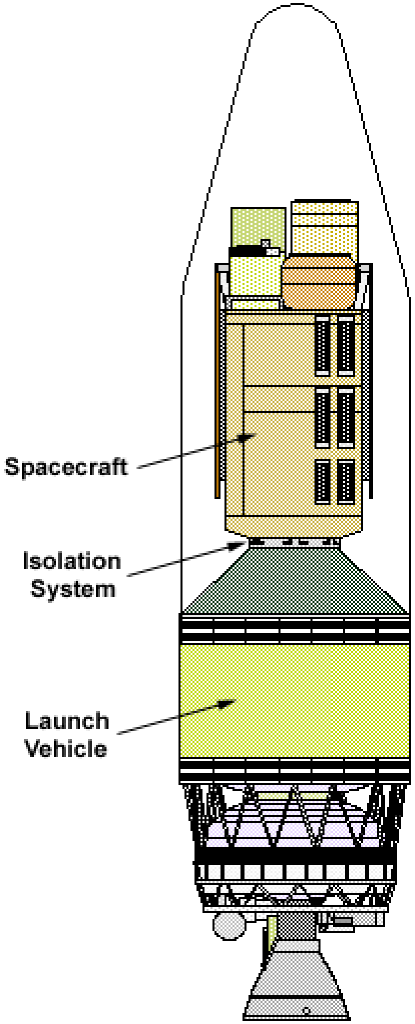
**Case (b)**

Here, the aim is to minimise the displacement transmitted to the device

Examples are :-

a satellite mounted in its launch vehicle or

the need to protect sensitive laser instruments from ground-borne vibration

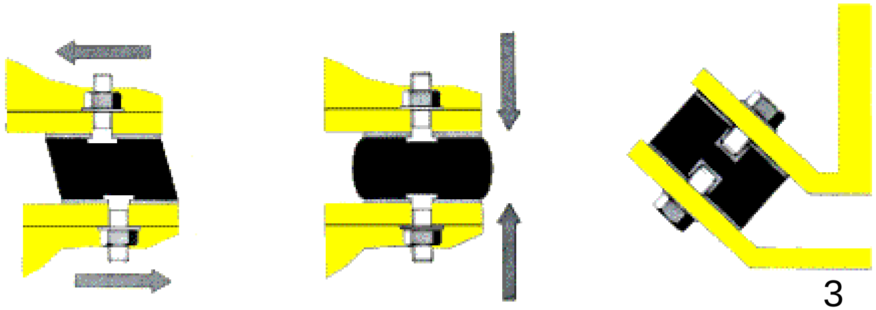


# TYPES OF ISOLATOR

## Elastomeric

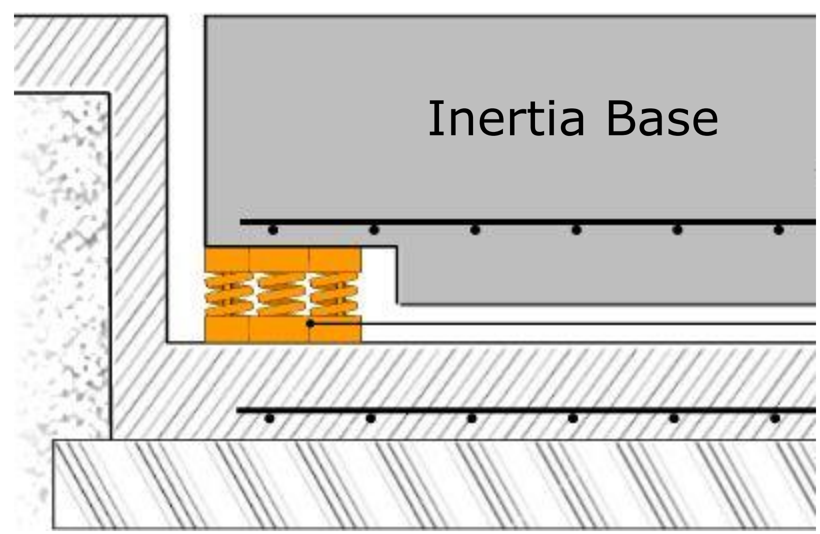
These are the most common type of isolator and there is a wide choice designs and load ratings


The mount may be loaded in shear, compression or a combination of both



## Pneumatic

## Coil spring



## TRANSMISSIBILITY ANALYSIS

In most cases, the isolators are much more flexible than the device they support

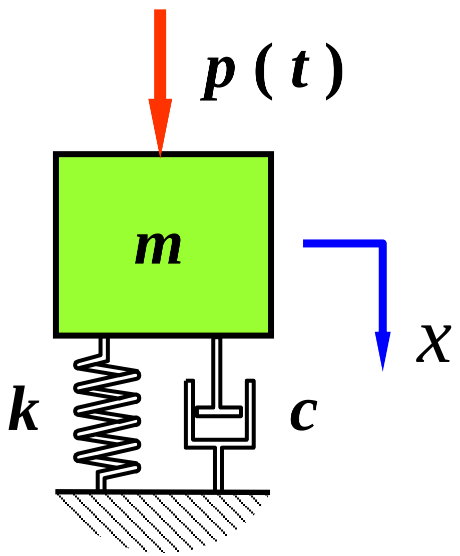
A good first approximation is to use a single-degree-of-freedom model in which

- the device to be isolated is treated as a rigid body
- the isolators are represented by a spring-damper combination
- steady-state harmonic response is used to characterise the isolation performance at different frequencies

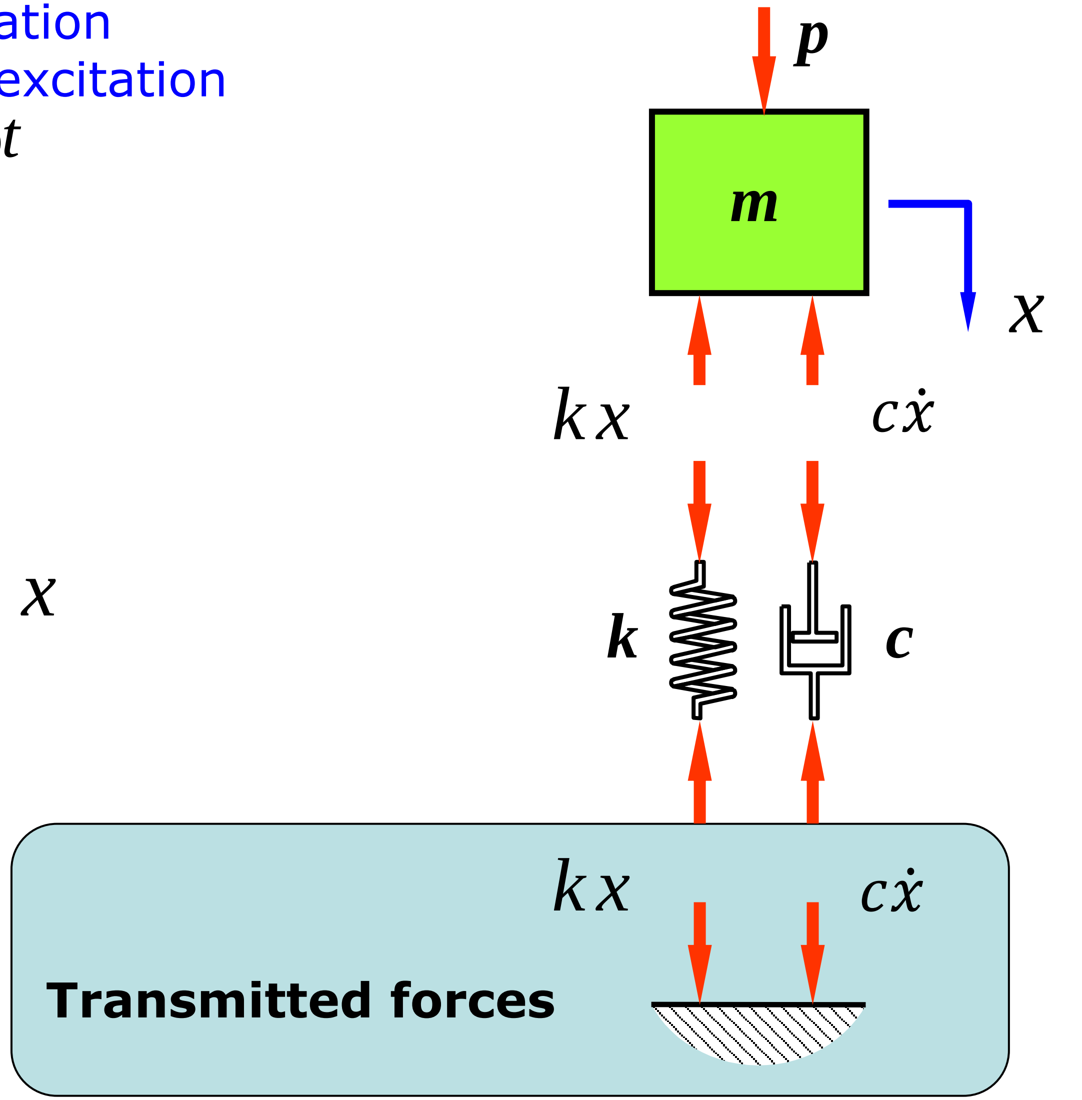
**Case (a)** Source of vibration within a device : **How much force is transmitted to the support?**

**STEP 1: Dynamic model**

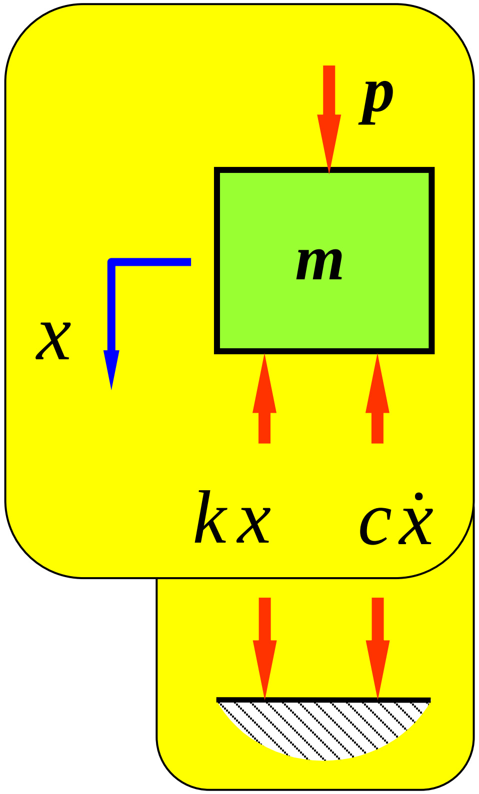
Assume that the vibration source generates an excitation force, $p(t) = P\cos\omega t$




**STEP 2: Free Body Diagram**



**STEP 3:** Equation of motion




For the device

$\downarrow x$

$$p - kx - c\dot{x} = m\ddot{x}$$

or

$$m\ddot{x} + c\dot{x} + kx = p \qquad (1)$$

Transmitted force

$$q(t) = kx + c\dot{x} \qquad (2)$$

Substitutions:

$$p(t) = P\,e^{i\omega t}, \quad q(t) = Q^*\,e^{i\omega t}$$

and

$$x(t) = X^* e^{i\omega t}$$
$$\dot{x}(t) = i\omega X^* e^{i\omega t}$$
$$\ddot{x}(t) = -\omega^2 X^* e^{i\omega t}$$

(1) ⇨ $X^* = \dfrac{P}{(k - m\omega^2) + \mathbf{i}\,c\omega}$

(2) ⇨ $Q^* = (k + \mathbf{i}\,c\omega)\,X^*$

$$X^* = \frac{P}{\left(k - m\omega^2\right) + \mathbf{i}\, c\,\omega} \qquad Q^* = \left(k + \mathbf{i}\, c\,\omega\right) X^*$$

Eliminating $X^*$ ⇨

$$\frac{Q^*}{P} = \frac{k + \mathbf{i}\, c\,\omega}{\left(k - m\omega^2\right) + \mathbf{i}\, c\,\omega}$$

For this application, only the magnitude of the transmitted force is of interest

We define **FORCE TRANSMISSIBILITY** as

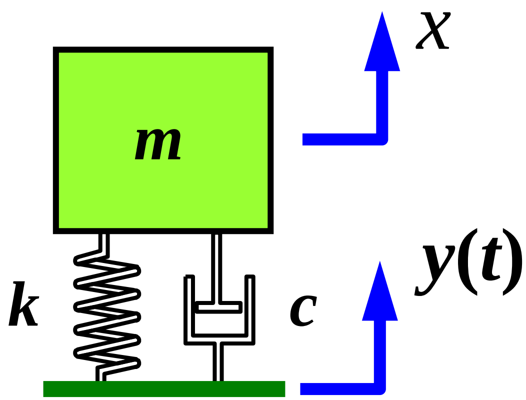
$$T_\mathrm{F} = \left|\frac{Q^*}{P}\right| = \sqrt{\frac{k^2 + c^2\omega^2}{\left(k - m\omega^2\right)^2 + c^2\omega^2}}$$

**Case (b)** Source of vibration from the support : **How much vibration is transmitted to the device?**

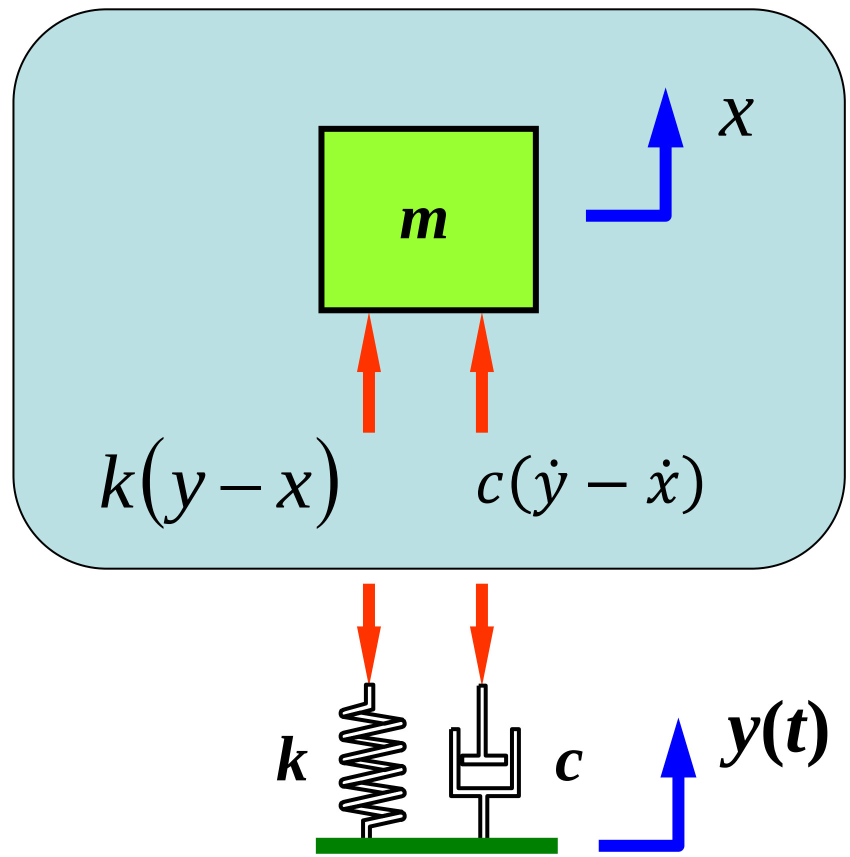
**STEP 1: Dynamic model**

The support vibration is defined by the **displacement**,

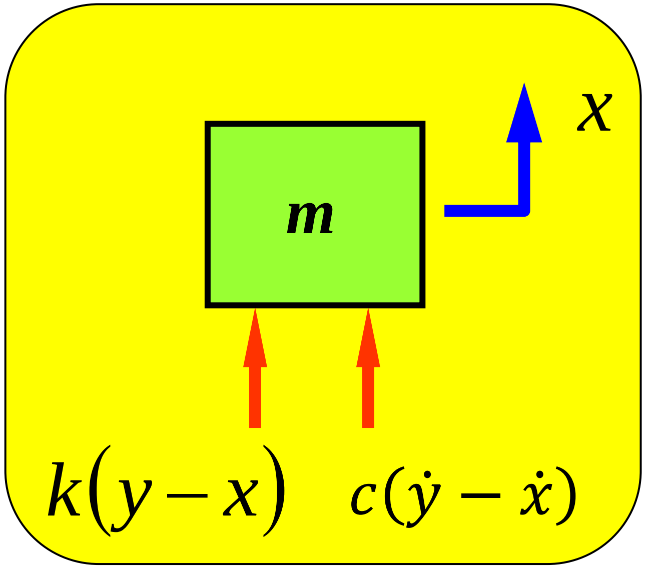
$$y(t) = Y\cos\omega t$$




**STEP 2: Free Body Diagram**



**STEP 3:** Equation of motion




↑ $x$ $\quad m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$

Substitutions: $y(t) = Ye^{i\omega t}$, $\dot{y}(t) = i\omega Y e^{i\omega t}$ and $x(t) = X^* e^{i\omega t}$, $\dot{x}(t) = i\omega X^* e^{i\omega t}$, $\ddot{x}(t) = -\omega^2 X^* e^{i\omega t}$

Hence,
$$X^* = \frac{(k + \mathbf{i}\, c\,\omega)Y}{(k - m\omega^2) + \mathbf{i}\, c\,\omega}$$

We define **DISPLACEMENT TRANSMISSIBILITY** as

$$T_{\mathrm{D}} = \left| \frac{X^*}{Y} \right| = \sqrt{\frac{k^2 + c^2\omega^2}{\left(k - m\omega^2\right)^2 + c^2\omega^2}}$$

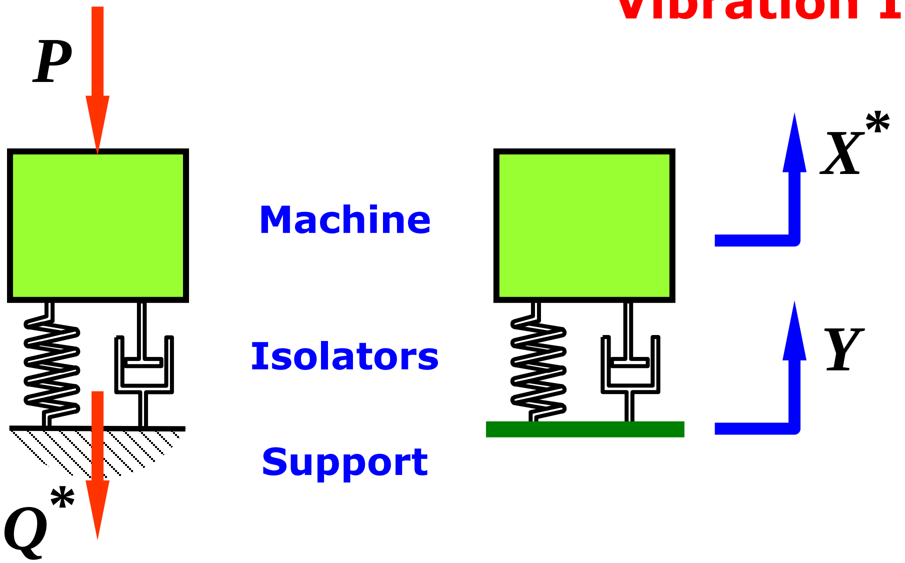
Note that the Force and Displacement Transmissibility expressions for these mass-spring-damper systems are identical

**!!!NOTE!!!**

**Other physical systems will have different transmissibility expressions.**

To be sure of your work it is best to derive $T_{D,F}$ every time.

# Vibration Isolation




Case (a)
Force
transmission

Case (b)
Displacement
transmission

The Force and Displacement Transmissibility expressions for this mass-spring-damper system are identical

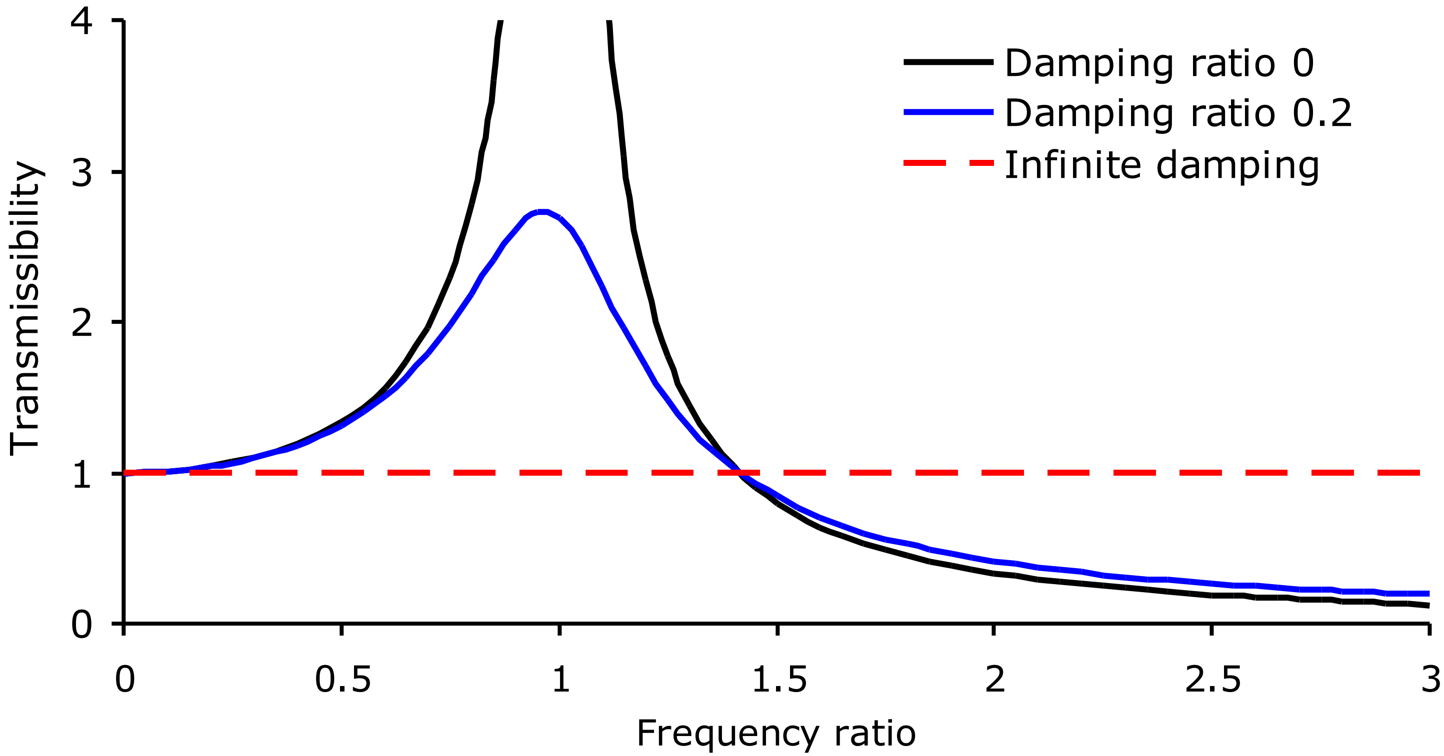
$$T_{D,F} = \sqrt{\frac{k^2 + c^2\omega^2}{\left(k - m\omega^2\right)^2 + c^2\omega^2}}$$

Divide top and bottom by $k^2$

$$T_{D,F} = \sqrt{\frac{k^2 + c^2\omega^2}{\left(k - m\omega^2\right)^2 + c^2\omega^2}} = \frac{\sqrt{1 + 4\gamma^2 \frac{\omega^2}{\omega_n^2}}}{\sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + 4\gamma^2 \frac{\omega^2}{\omega_n^2}}}$$

This expression is on the formula sheet

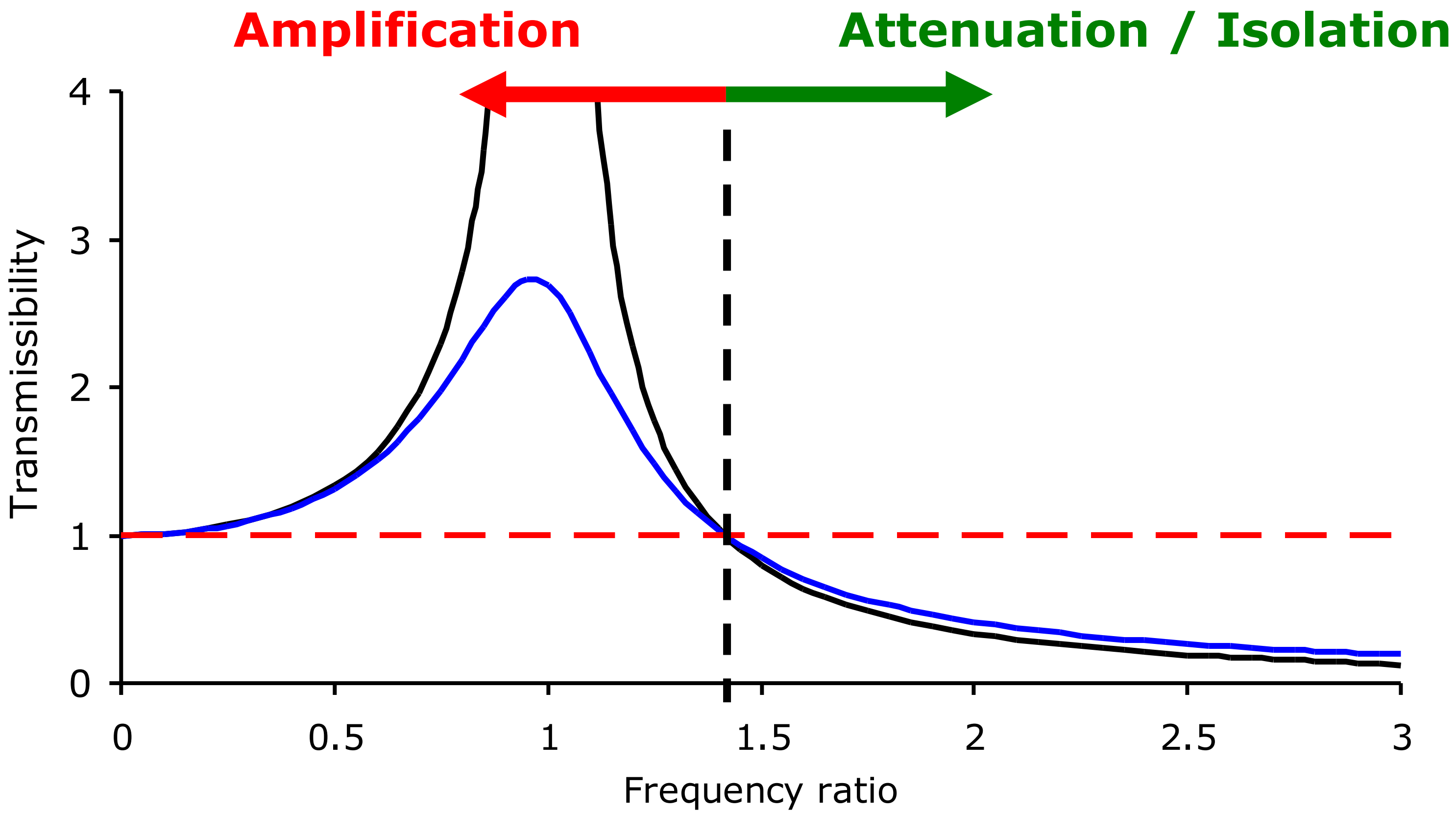
$$\omega_n = \sqrt{\frac{k}{m}} \text{ and } \gamma = \frac{c}{2\sqrt{km}}$$




Transmissibility curves show how excitation frequency affects the transmitted force or displacement

Damping has a significant effect near resonance, but little effect at high frequencies

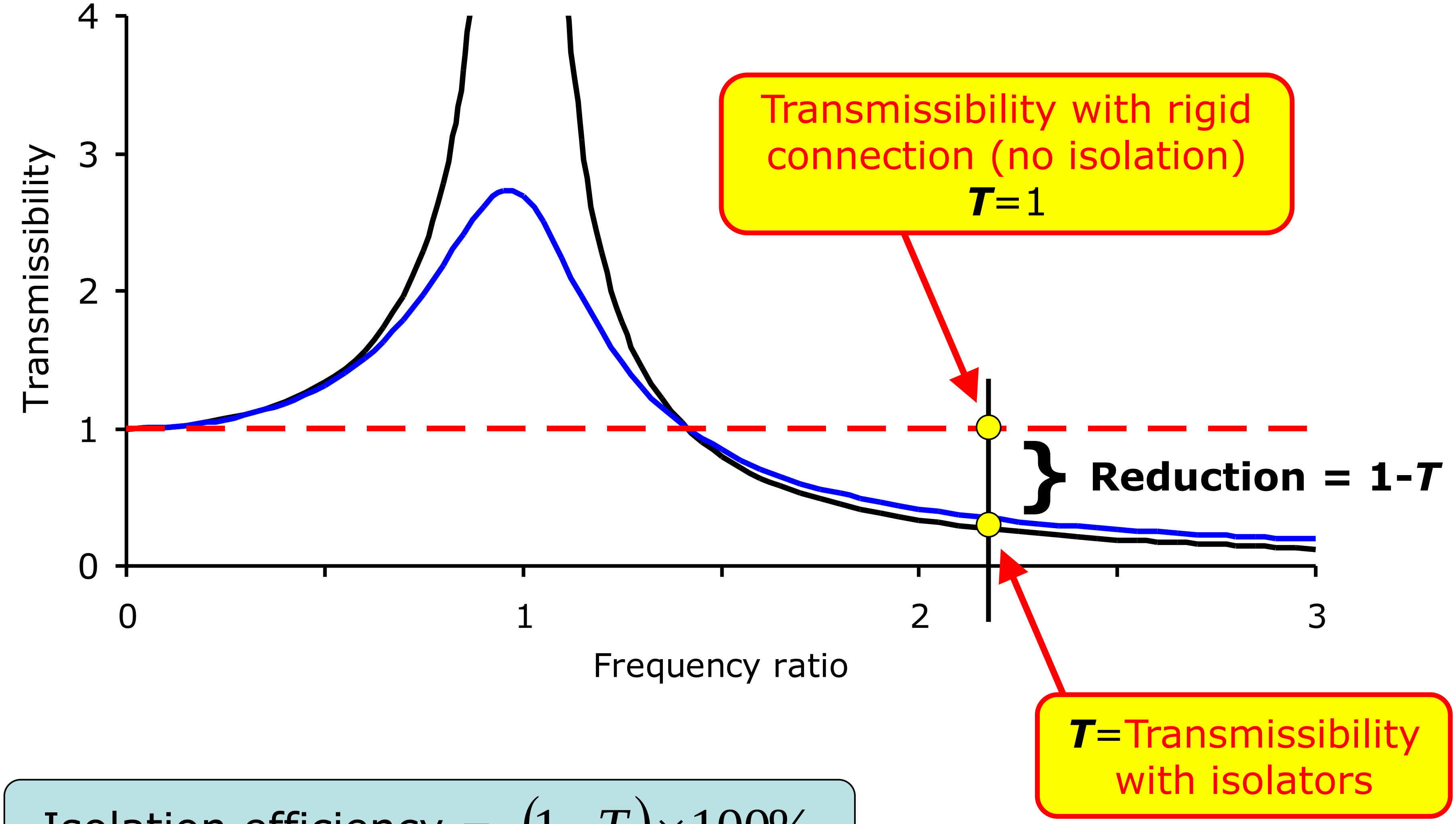
Infinite damping is a special case and corresponds to a rigid connection between the device and its support




It's easy to show that *T* = 1 when $\omega/\omega_n = \sqrt{2}$

**The aim in selecting isolators is to ensure that the system operates in the "isolation region"**

# Isolation Efficiency

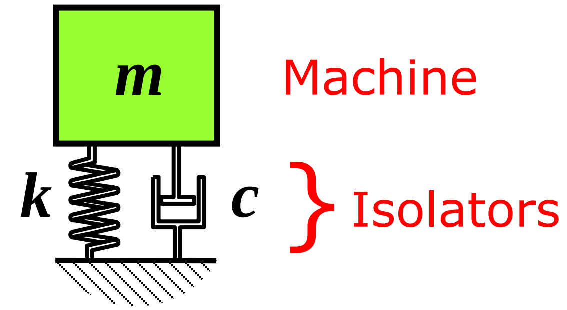


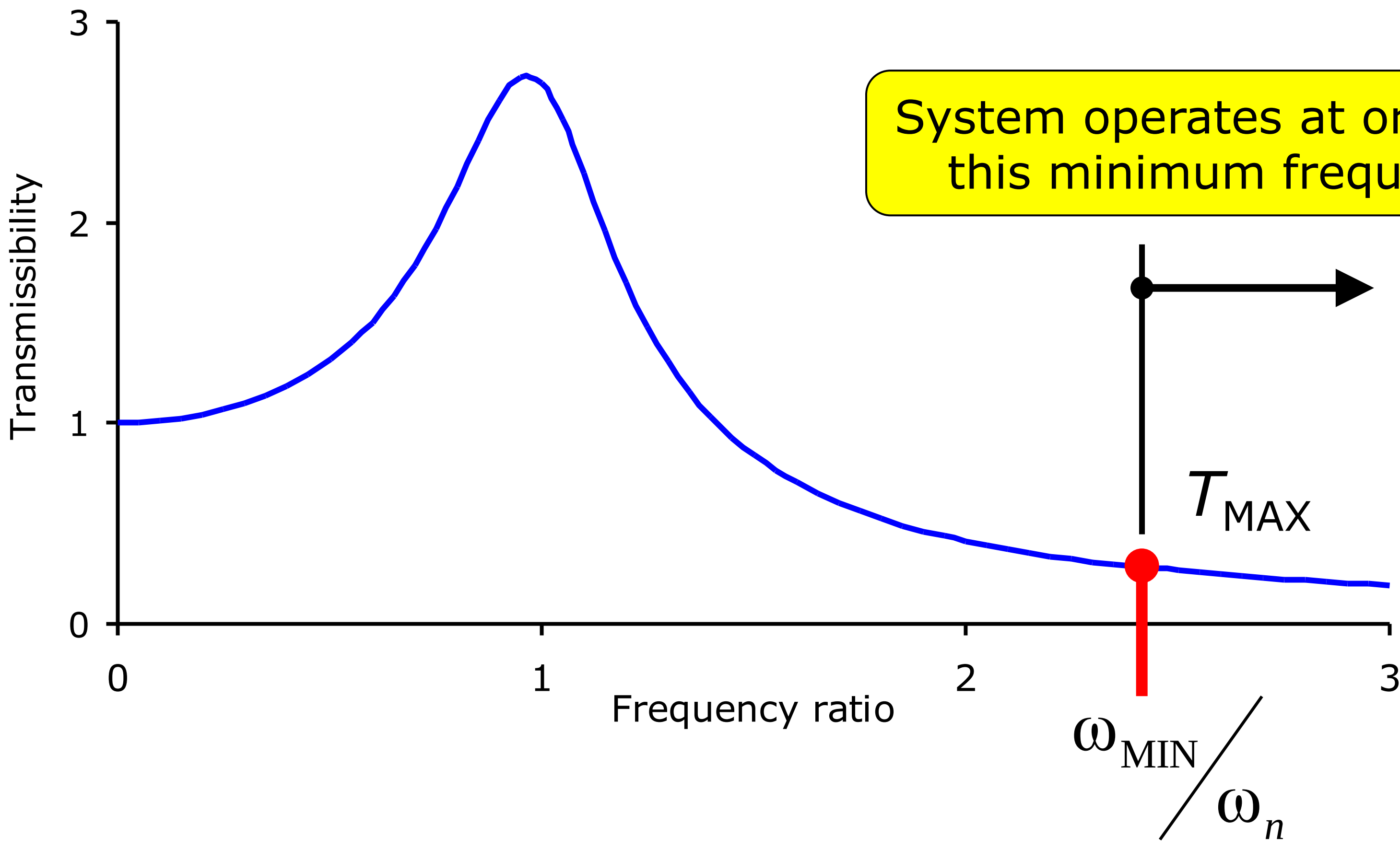

Isolation efficiency = $\left(1-T\right)\times 100\%$

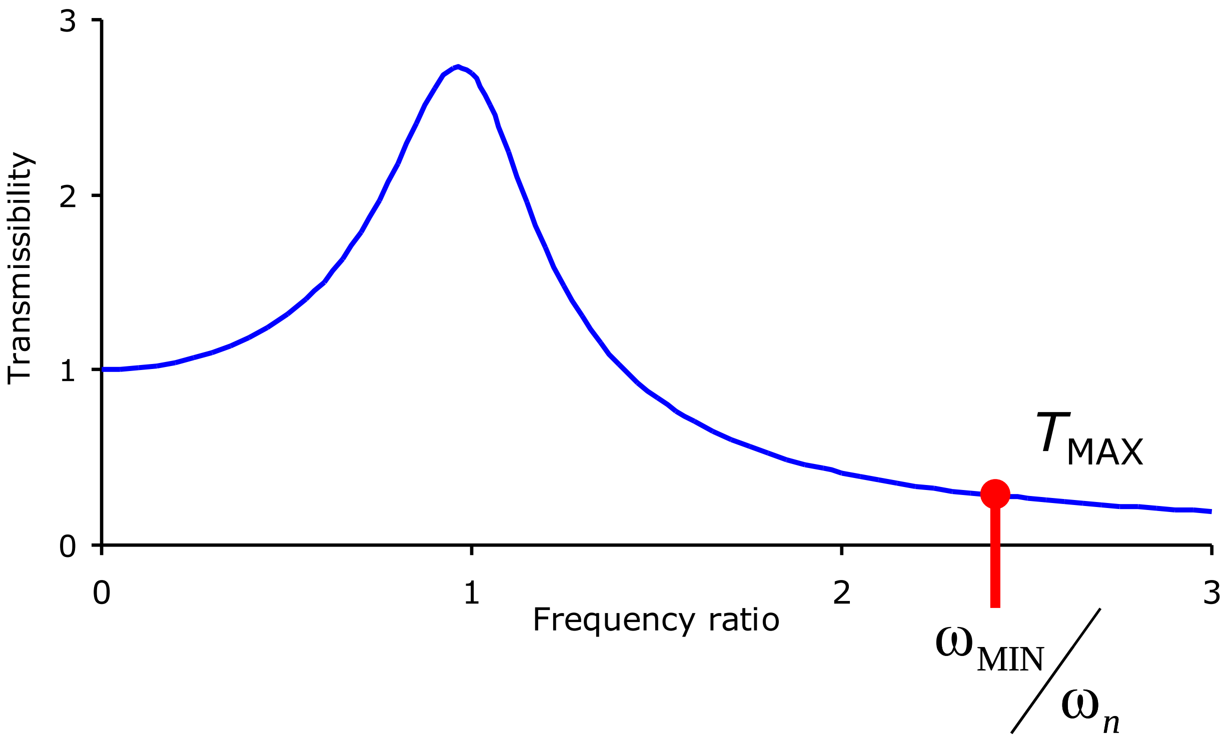
# Design Approach for Isolator Selection

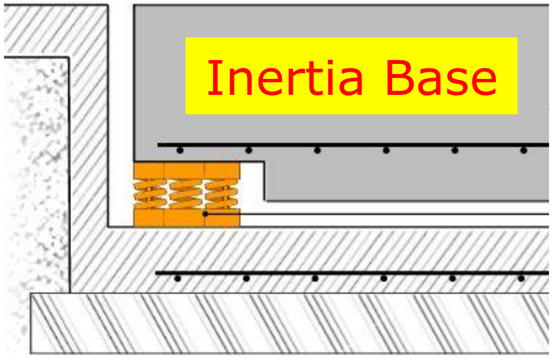
Two constraints for isolator selection:

- the lowest excitation frequency, $\omega_{MIN}$
- the maximum allowable transmissibility, $T_{MAX}$

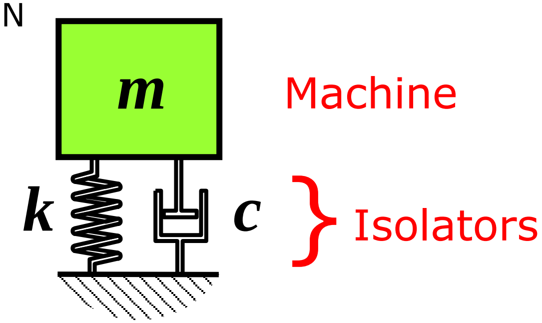


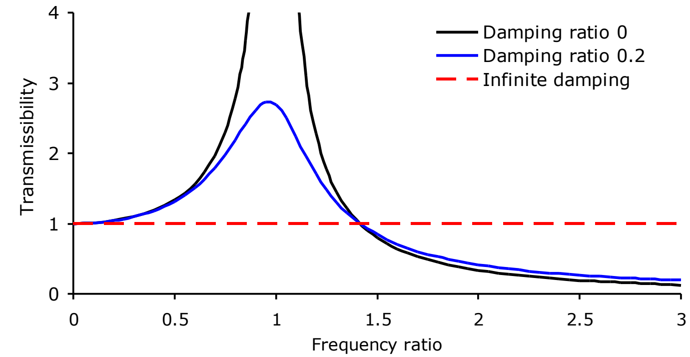



- For vibration reduction, $\omega_n$ must be well below $\omega_{MIN}$
- *m and k* together determine $\omega_n$
- The stiffness, *k*, is given by the selected isolators
- The mass supported by the isolators can be increased by mounting it on an inertia base. This will reduce $\omega_n$



In the isolation region, low damping gives the lowest transmissibility




For most commercial isolators, $\gamma < 0.1$

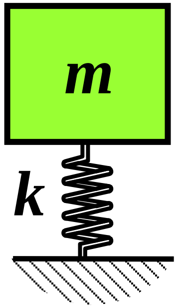
It is normal to **assume zero damping**

It is also normal to treat each isolator independently of the others

**In this case, *m* is the effective mass supported by the isolator in question**

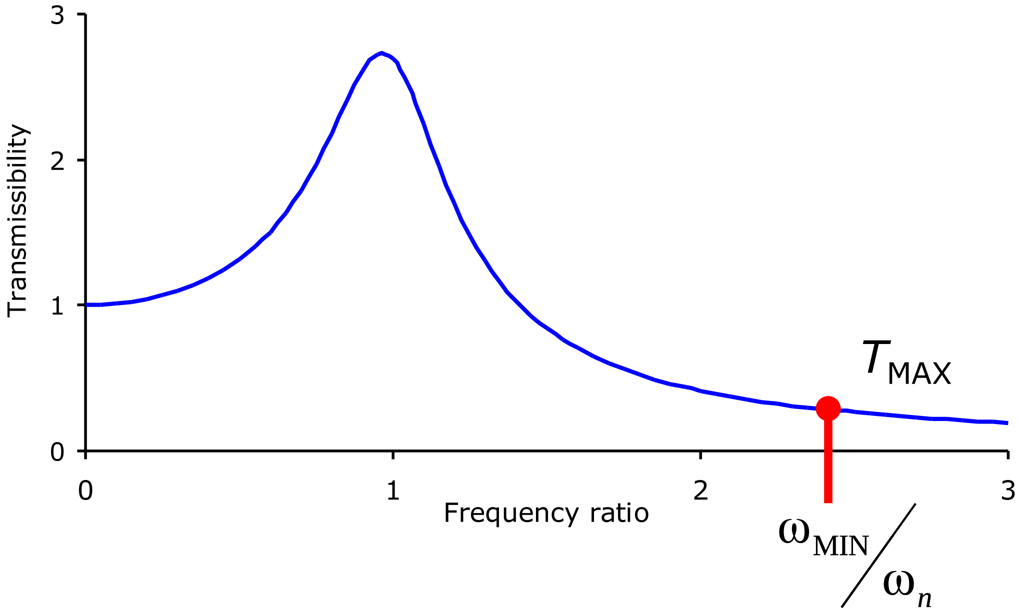
**For the simple mass-spring model with zero damping**

$$T = \sqrt{\frac{k^2}{\left(k - m\omega^2\right)^2}} = \left| \frac{1}{1 - \omega^2 / \omega_n^2} \right|$$




$$= \frac{1}{\omega^2 / \omega_n^2 - 1}$$

for the isolation region where $\omega > \omega_n$




$$T = \frac{1}{\omega^2/\omega_n^2 - 1}$$

If $T = T_{MAX}$ at $\omega = \omega_{MIN}$

$$\omega_n^2 = \frac{T_{MAX}\,\omega_{MIN}^2}{1+T_{MAX}}$$

Since $\omega_n^2 = \frac{k}{m}$ , the required isolator stiffness is

$$k = m\omega_n^2 = \frac{m T_{MAX}\,\omega_{MIN}^2}{1+T_{MAX}} \qquad (1)$$

Equation (1) is the ***maximum*** stiffness consistent with the design constraints

There are also constraints imposed by static considerations

Manufacturers often express these constraints by specifying a ***maximum static deflection***

The actual static deflection, $X_0$, is given by

$$X_0 = \frac{m\,g}{k_{\mathrm{ISOLATOR}}} \qquad (2)$$

Alternatively, combining (1) and (2) gives

$$X_0 = \frac{g}{\omega_{\mathrm{MIN}}^2}\left(1 + \frac{1}{T_{\mathrm{MAX}}}\right) \qquad (3)$$

This is the ***minimum* static deflection** consistent with the design constraints

## Design Procedure

1. Find the centre of mass of the machine
2. Select the number and position of attachment points for isolators
3. Estimate the load supported by each isolator
4. For each isolator position in turn,
    - 4.1 Calculate the maximum stiffness from equation (1)
    - 4.2 Select an isolator with a lower stiffness
    - 4.3 Check that this does not exceed any static deflection limit using equation (2).
    - 4.4 Repeat 4.2 and 4.3 with other isolators having even lower stiffness

**Example**

Machine mass 480 kg
Min. excitation frequency 10 Hz
Min. isolation efficiency 90%
Isolator stiffnesses available:
10, 30, 80 N/mm

With 4 isolators,
mass per isolator = 120 kg

$$\omega_{\text{MIN}} = 20\pi \text{ rad/s}$$

$$T_{\text{MAX}} = 0.1$$

From (1), the maximum isolator stiffness is

$$k_{\text{MAX}} = \frac{m T_{\text{MAX}} \, \omega_{\text{MIN}}^2}{1 + T_{\text{MAX}}}$$

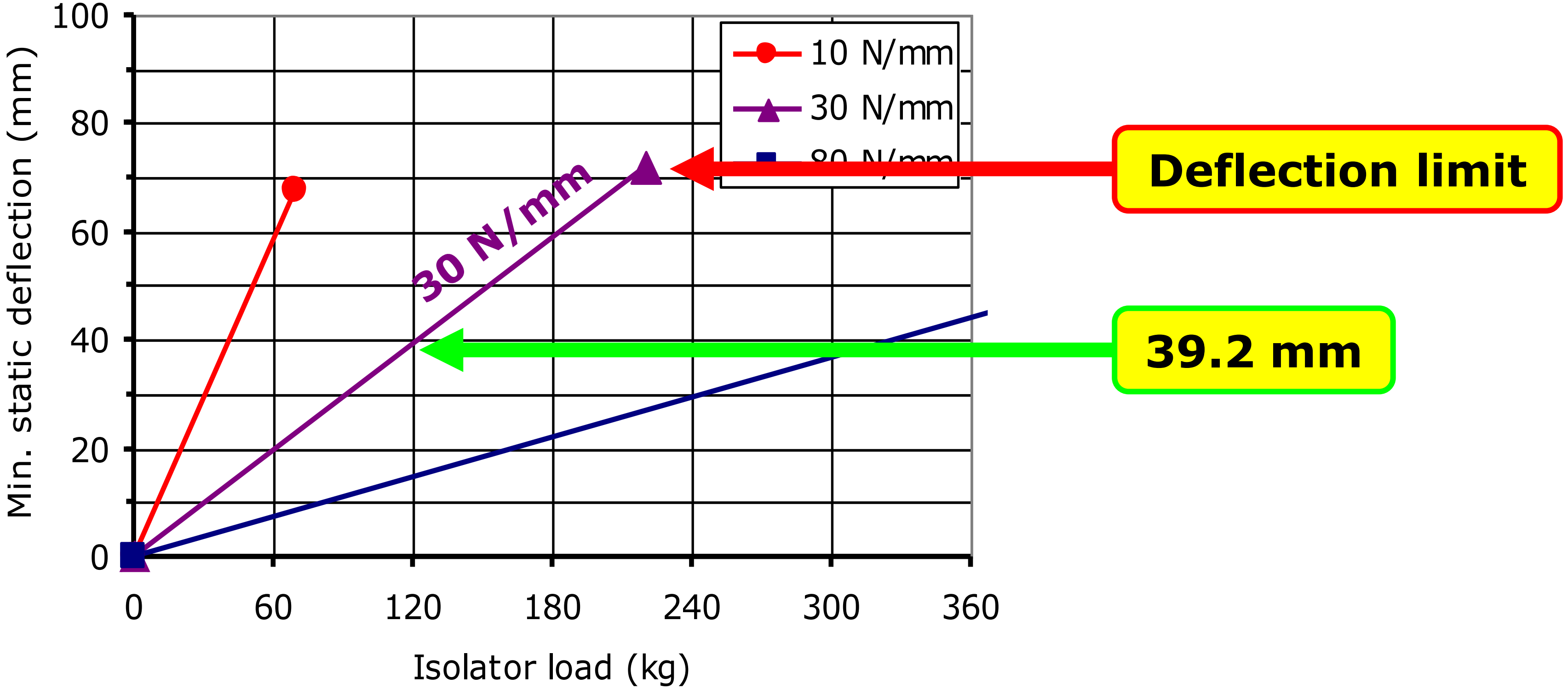
Therefore, choose 30 N/mm isolator

From (2), the actual static deflection, $X_0$, is

$$X_0 = \frac{m g}{k_{\text{ISOLATOR}}}$$

Need to check that this is within the allowable deflection range

Need to check that this is within the allowable deflection range



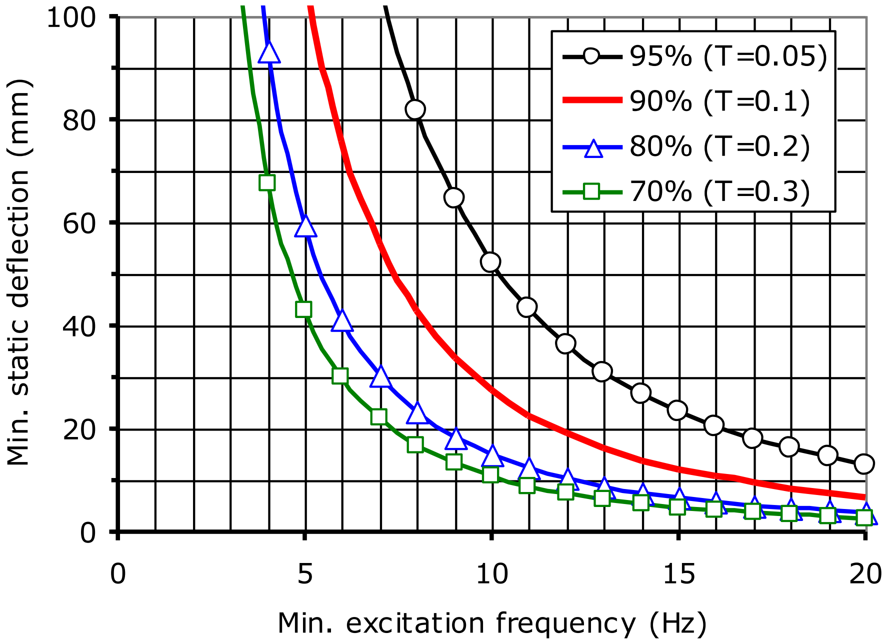

Here, the static deflection, $X_0$, is 39.2 mm

This is below the deflection limit for the 30 N/mm isolator, so the selection is acceptable

## Manufacturers' Charts

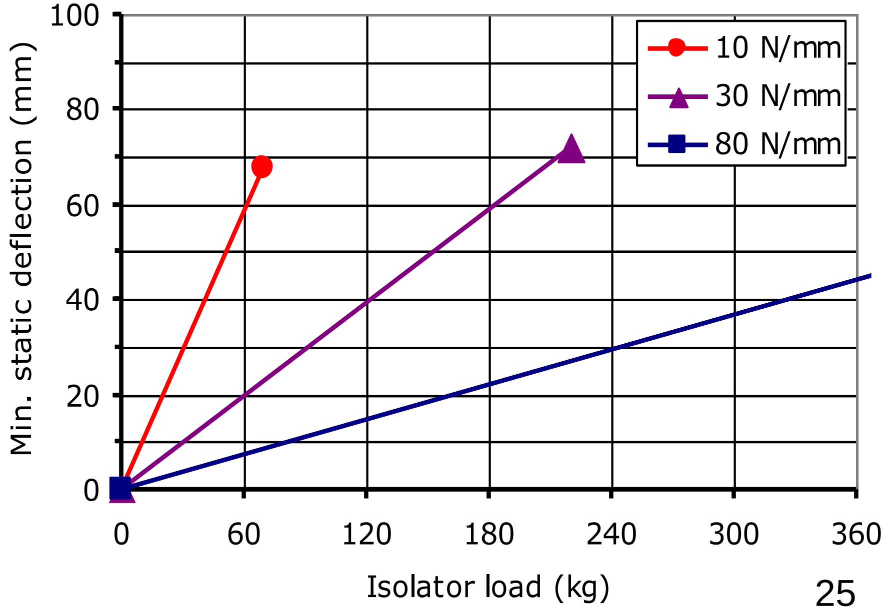
Upper graph uses equation (3)

$$X_0 = \frac{g}{\omega_{MIN}^2}\left(1 + \frac{1}{T_{MAX}}\right)$$




Lower graph uses equation (2)

$$X_0 = \frac{m\,g}{k_{ISOLATOR}}$$

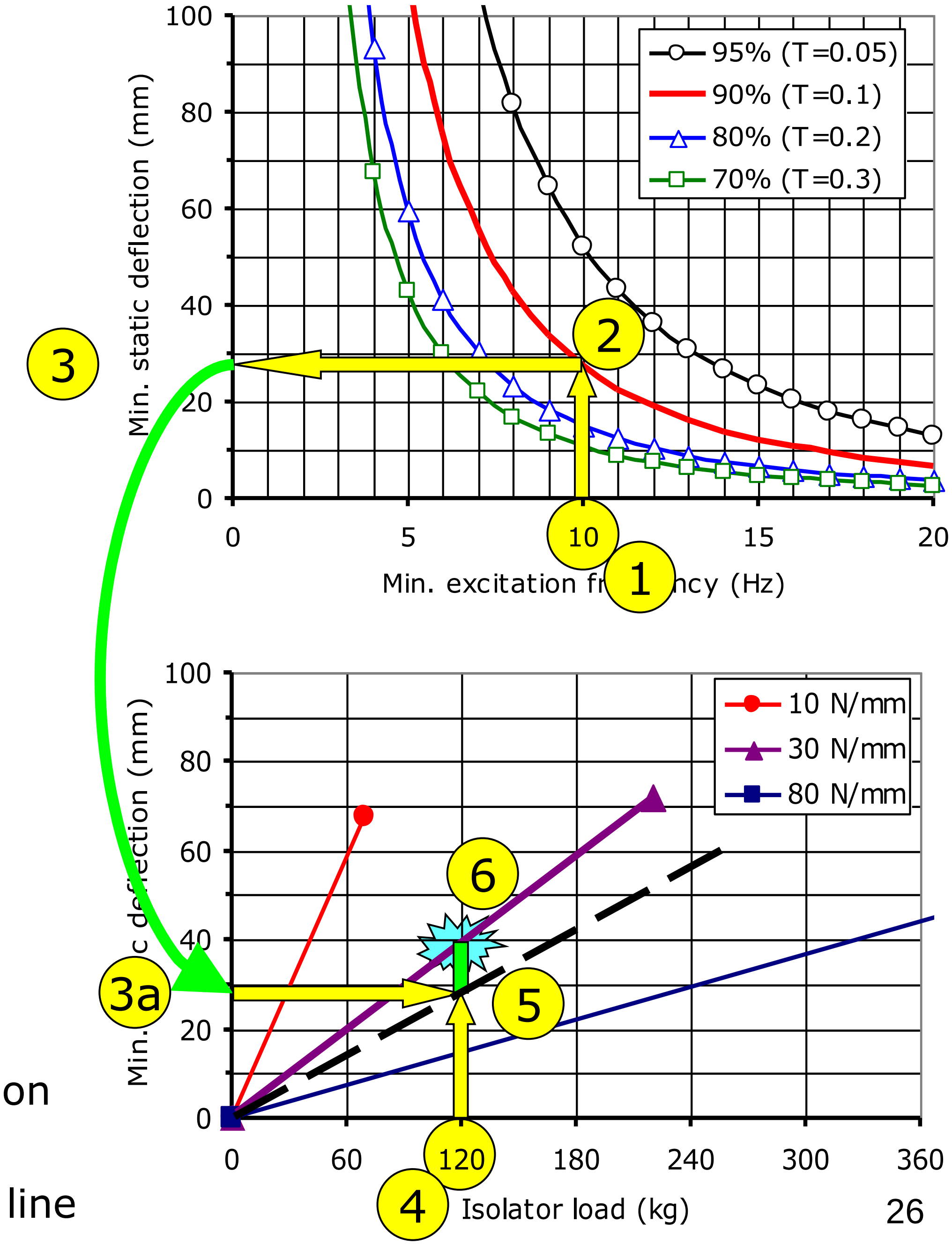


# Example 1

Machine mass 480 kg
Min. excitation frequency 10 Hz
Min. isolation efficiency 90%

Assume there are 4 isolators, so the mass per isolator = 120 kg

1. 10 Hz min. frequency
2. Intersection with 90% curve
3. Min. static deflection
3a. Transfer to lower chart
4. 120 kg per isolator
5. Intersection with min. deflection
6. Move up to intersect stiffness line



# Example 2

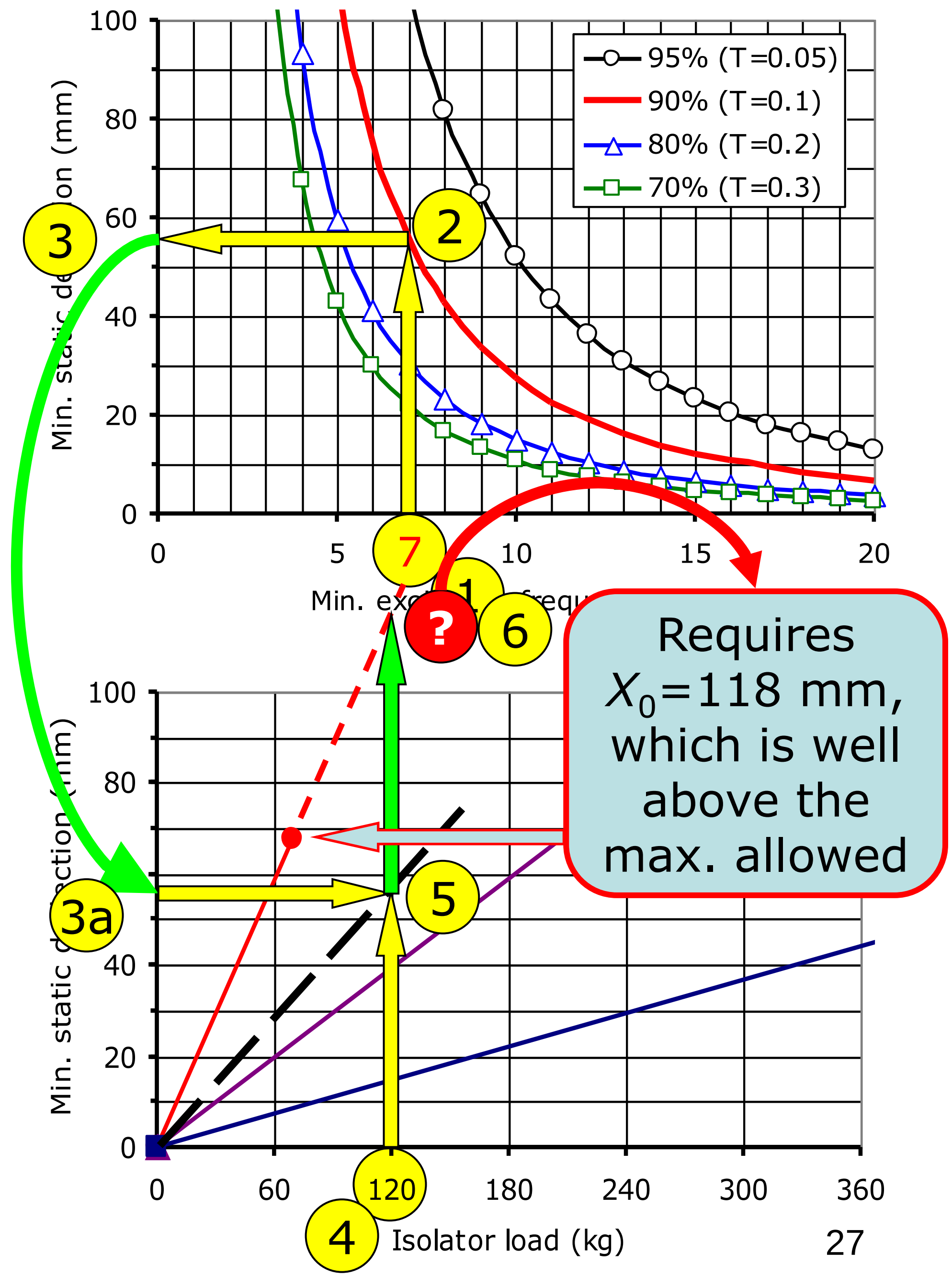
| | |
|---|---|
| Machine mass | 480 kg |
| Min. excitation frequency | **7 Hz** |
| Min. isolation efficiency | 90% |

With 4 isolators,
mass per isolator = 120 kg

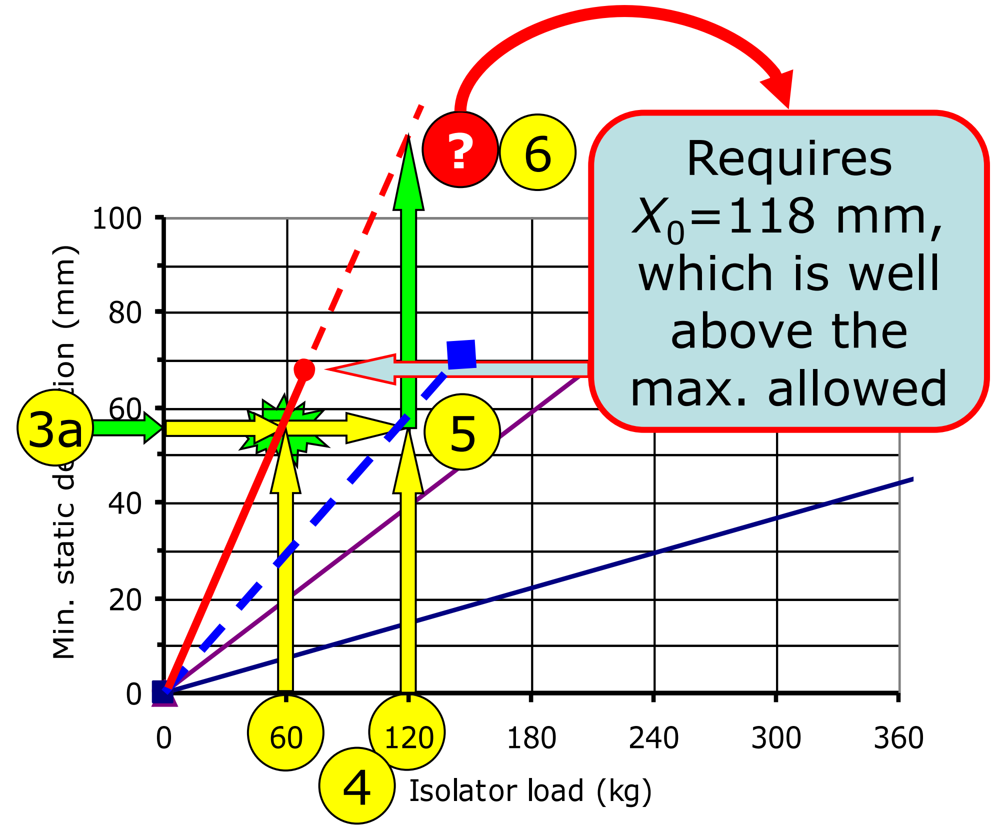
1. 7 Hz min. frequency
2. Intersection with 90% curve
3. Min. static deflection

3a. Transfer to lower chart

4. 120 kg per isolator
5. Intersection with min. deflection
6. Move up to seek stiffness line



**Q** What's the solution in this case




(a) Look for another isolator or another manufacturer

Something between 10 and 30 N/mm would be good

(b) Use more isolators – 6 or 8 instead of 4

e.g., with 8 isolators, $m$ = 60 kg per isolator

This give a satisfactory result using 10 N/mm isolators

NEXT