① Equation of motion from Sheet 1, Q1 (ii)

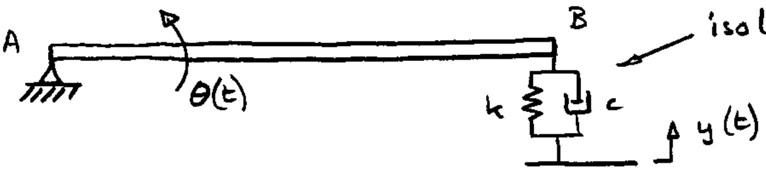
$$m\ddot{x} + c\dot{x} + (k+K)x = ky + c\dot{y}$$

Put $y(t) = Ye^{i\omega t}$ and $x(t) = X^* e^{i\omega t}$

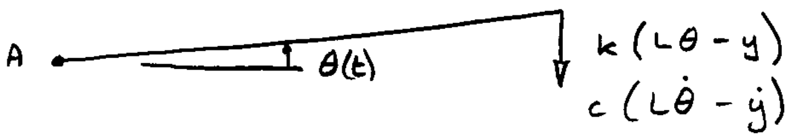
$$\left[(k+K) - m\omega^2 + i\omega c\right]X^* e^{i\omega t} = \left[k + i\omega c\right] Y e^{i\omega t}$$

Displacement transmissibility, $T_D = \left|\frac{X^*}{Y}\right|$

② Dynamic model:



Free body diagram



Equation of motion

$$A \circlearrowright) \quad -k(L\theta - y).L - c(L\dot{\theta} - \dot{y}).L = \tfrac{1}{3}mL^2\ddot{\theta}$$

$$\therefore \tfrac{1}{3}mL\ddot{\theta} + cL\dot{\theta} + kL\theta = ky + c\dot{y}$$

Put $y(t) = Ye^{i\omega t}$ $\theta(t) = \Theta^* e^{i\omega t}$

Displacement at B $= L\Theta^* e^{i\omega t}$. Hence the displacement transmissibility across the isolator at B is given by

$$T_D = \left|\frac{L\Theta^*}{Y}\right| = \left[\frac{k^2 + \omega^2 c^2}{(k - \frac{1}{3}m\omega^2)^2 + \omega^2 c^2}\right]^{1/2}$$

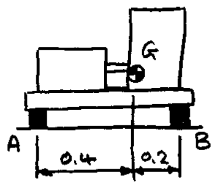
$$= 0.5775$$

Isolation efficiency $= (1 - T_D) \times 100\,\%$

$= 42.3\,\%$

7.2

③



Total mass 300 kg

Load distribution:

50 kg on each isolator at A

100 kg " " " " B

$$k_{max} = \frac{m\omega_{min}^2 T_{max}}{1 + T_{max}} \quad \text{(from notes)}$$

At A: $k_{max} = 95.8$ kN/m

Select $k = 70$ kN/m

Static deflection = 7.01 mm (< 10mm ∴ OK)

[For $k = 40$ kN/m, $X_o = 12.3$ mm > 10mm limit]

$$\text{Actual transmissibility} = \frac{1}{\frac{\omega^2 m}{k} - 1} = 0.203$$

At B: $k_{max} = 191.5$ kN/m

Select $k = 180$ kN/m

Static deflection, $X_o = 5.45$ mm (< 10mm ∴ OK)

Actual transmissibility = 0.277

[For $k = 110$ kN/m, $X_o = 8.92$ mm

This is also less than 10 mm, but gives little margin]