MMME2046 Dynamics and Control: Lecture 4

Planar Dynamics of Rigid Bodies

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Handouts Chapter III

Lecture objectives

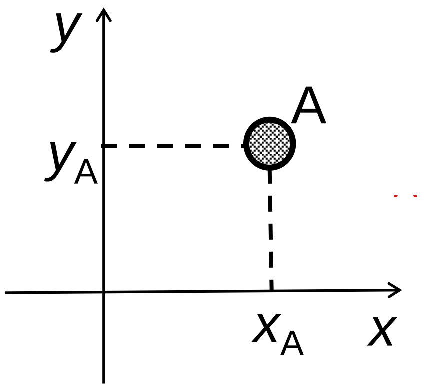
- Define degrees of freedom for rigid body motion
- Revise: particle dynamics and mass moments of inertia
- Introduce fundamental relationship for rigid body dynamics
- Formulate equations of motions
- Apply planar dynamics to the solution of basic problems

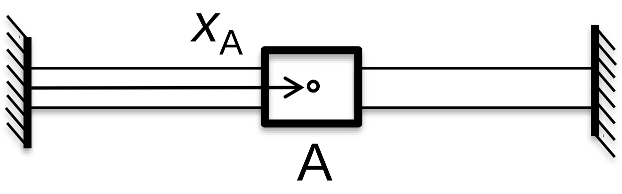
Degrees of Freedom

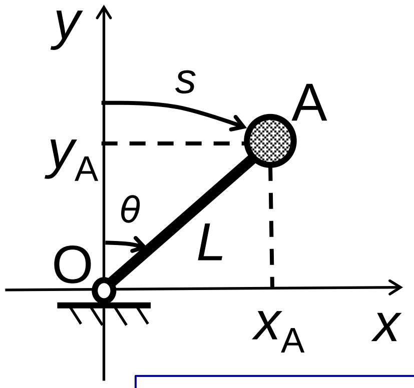
The **degrees of freedom** of a mechanical system in motion are the independent coordinates needed to uniquely specify the position of the system.

The **number of degrees of freedom** is the smallest number of different coordinates in a mechanical system that must be fixed in order to prevent the system from moving.

Quiz:

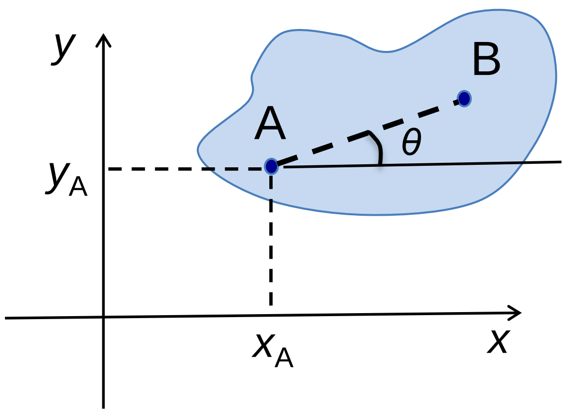


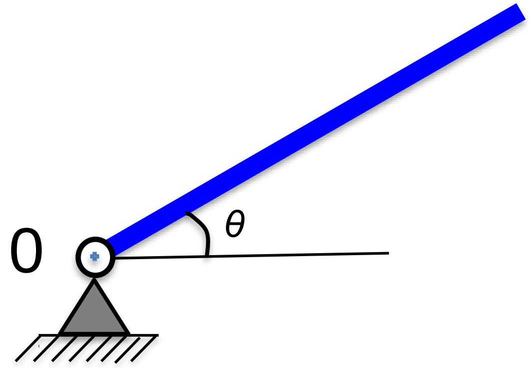


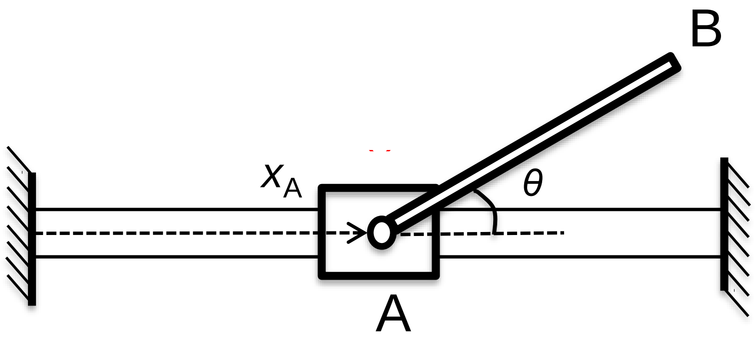


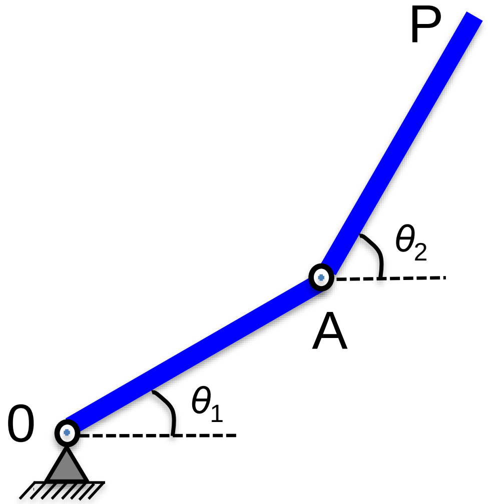
$$x_A(t)^2 + y_A(t)^2 = L^2$$

Degrees of Freedom

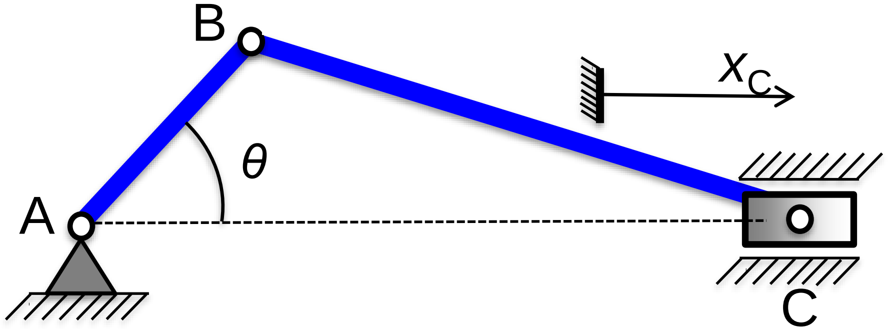






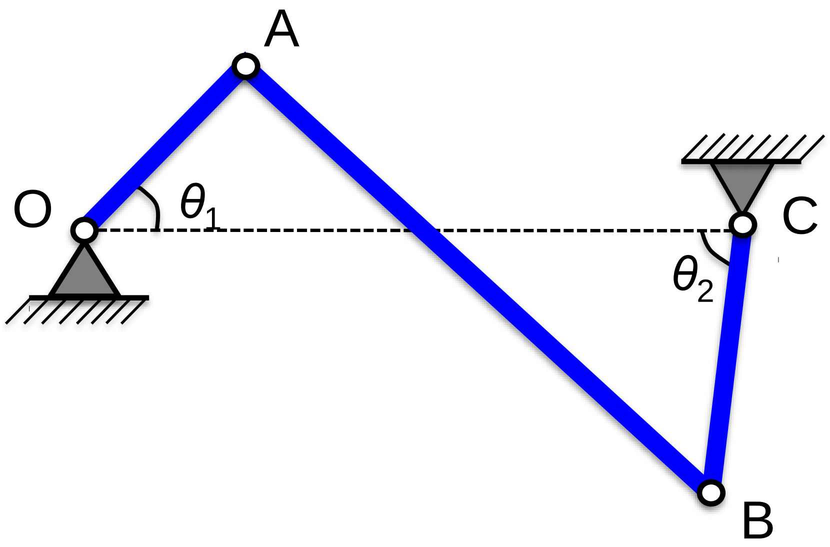


Degrees of Freedom



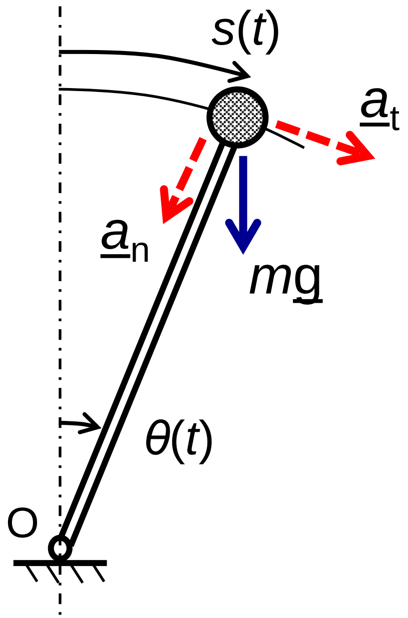
Unconstrained Particle: 2 E.o.M.

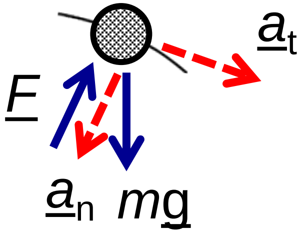
Unconstrained R.B.: 3 E.o.M.



Constrained Systems: some E.o.M. used for calculating reaction forces.

Example 1: Pendulum with a heavy bob (particle dynamics)





$$a_t(t) = \ddot{s}(t) = L\ddot{\theta}$$

$$a_n(t) = L\dot{\theta}^2$$

$$\searrow^{+}\ \Sigma F_t = ma_t\ :$$

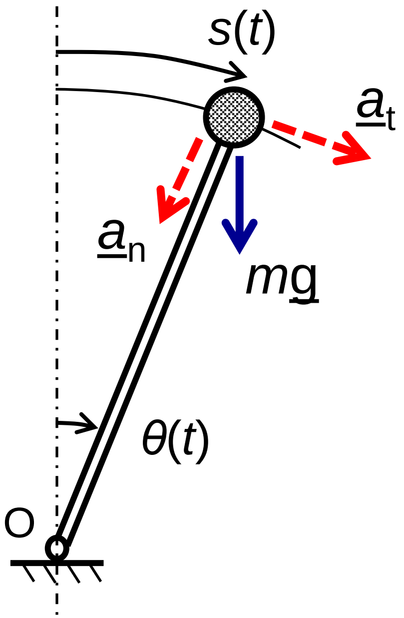
$$\swarrow^{+}\ \Sigma F_n = ma_n\ :$$

$$\ddot{\theta} = \frac{d\dot{\theta}}{dt}$$

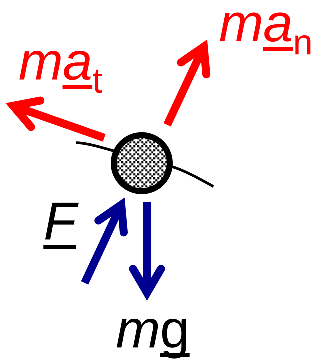
Integrating and assuming $\dot{\theta}(0) = 0$:

$$F =$$

Example 1: Pendulum with a heavy bob (particle dynamics)



d'Alembert's Principle:
transforms a dynamic system into an equivalent static system



"Inertia" forces = -ma

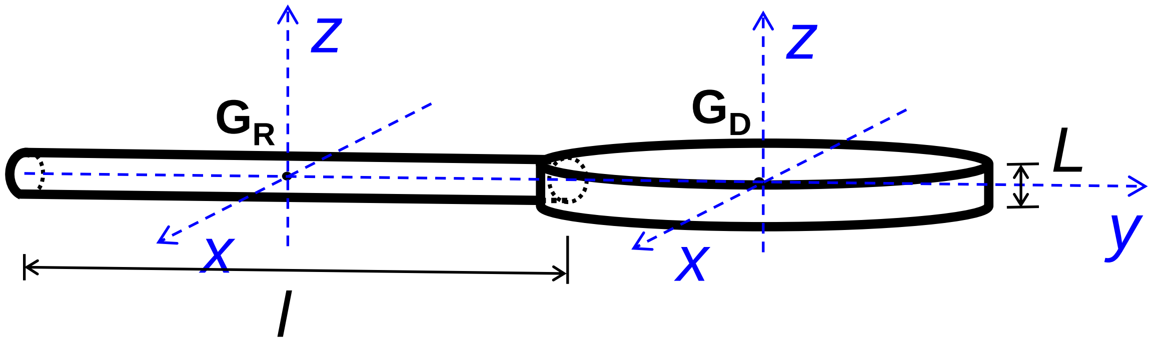
$$a_t(t) = \ddot{s}(t) = L\ddot{\theta}$$

$$a_n(t) = L\dot{\theta}^2$$

$$\searrow^{+} \ \Sigma F_t' = 0 \ : \quad mg\sin\theta - ma_t = 0$$

$$\swarrow^{+} \ \Sigma F_n' = 0 \ : \quad mg\cos\theta - F - ma_n = 0$$

Mass moment of inertia



l = 50 cm

r = 2 cm

L = 2 cm

R = 10 cm

Steel Rod $m_{\mathrm{R}} =$

$J_{\mathrm{GR},y} =$

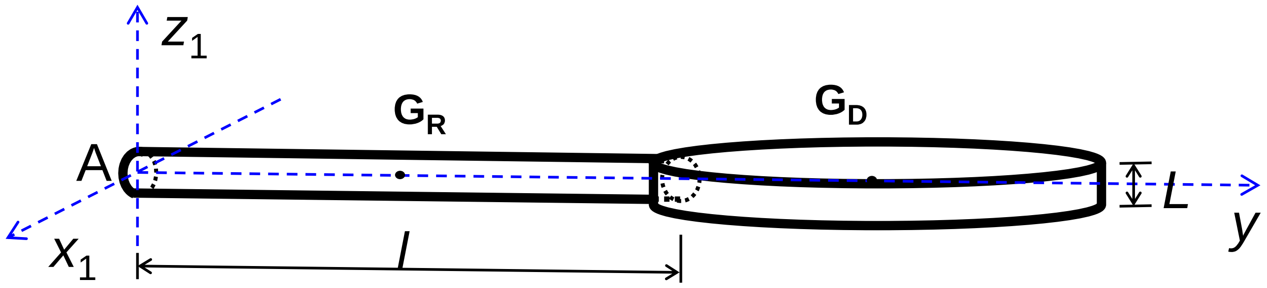
$J_{\mathrm{GR},x} = J_{\mathrm{GR},z} =$

Aluminium Disc $m_{\mathrm{D}} =$

$J_{\mathrm{GD},x} =$

$J_{\mathrm{GD},z} =$

Mass moment of inertia



l = 50 cm

r = 2 cm

L = 2 cm

R = 10 cm

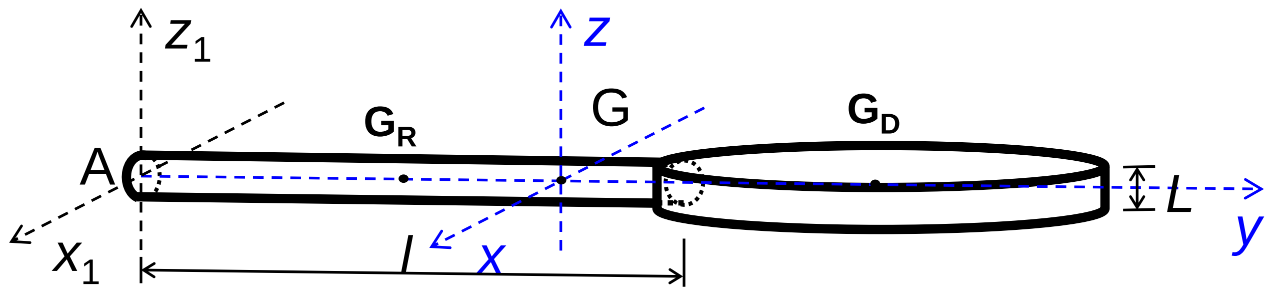
Combining the steel rod and cylinder together using <u>Parallel Axis theorem</u>:

$$J_{x1} = J_{\mathrm{GR},x} + m_{\mathrm{R}}\left(\frac{l}{2}\right)^2 + J_{\mathrm{GD},x} + m_{\mathrm{D}}(l+R)^2$$

$J_y =$

$J_{z1} =$

Mass moment of inertia



l = 50 cm

r = 2 cm

L = 2 cm

R = 10 cm

Finding the moment of inertia about the centre of mass:

$$m = m_{\mathrm{R}} + m_{\mathrm{D}} = 4.901 + 1.696 = 6.597\ \mathrm{kg}$$

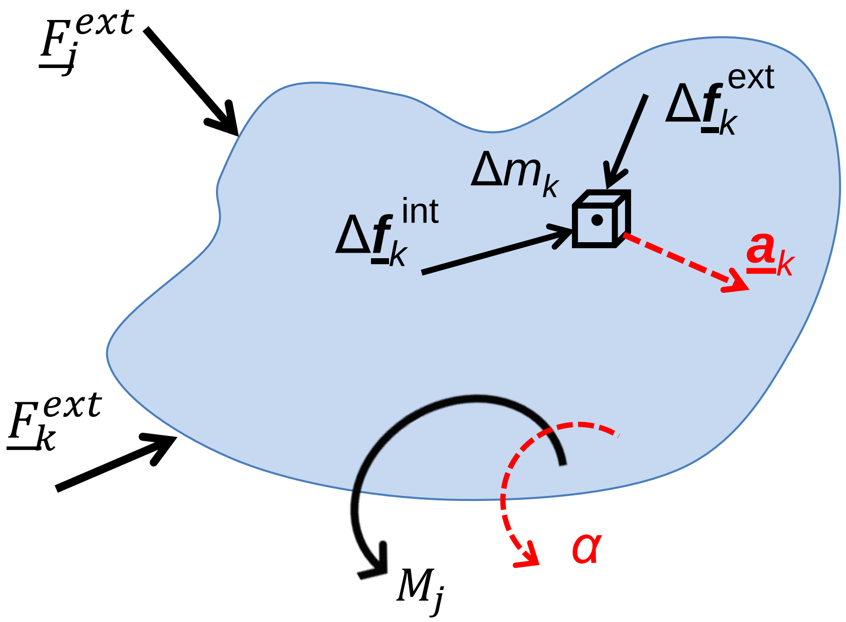
$$y_{\mathrm{G}} =$$

$$J_x =$$

$$J_y =$$

$$J_z =$$

Fundamental Laws of Rigid Body Motion



$$\underline{\boldsymbol{F}} = m\underline{\mathbf{a}}_G \qquad (1)$$

$\underline{\boldsymbol{F}}$: resultant of the external forces
$\underline{\boldsymbol{a}}_G$: acceleration of mass centre

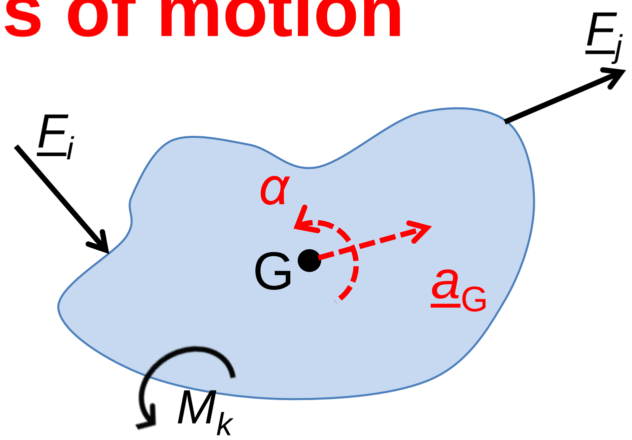
MMME1028 result:

$$M_G = J_G \alpha \qquad (2)$$

M_G : resultant of the applied moments about the axis of rotation
J_G : mass moment of inertia about the axis of rotation
α : angular acceleration of the rigid body

Fundamental Laws of Rigid Body Motion

Equations of motion

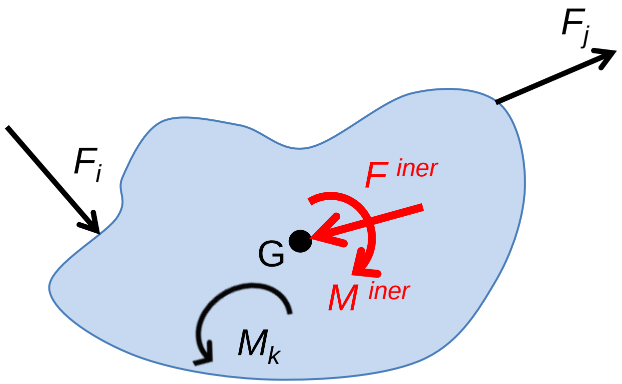


$$\rightarrow^{+}: \ \Sigma F_x = m_G a_{G,x}$$

$$\uparrow^{+}: \ \Sigma F_y = m_G a_{G,y}$$

$$\curvearrowleft^{+}: \ \Sigma M_G = J_G \alpha$$

D'Alembert's principle



$$\rightarrow^{+}: \ \Sigma F_x - F_x^{inertia} = \Sigma F_x - m_G a_{G,x} = 0$$

$$\uparrow^{+}: \ \Sigma F_y - F_y^{inertia} = \Sigma F_y - m_G a_{G,y} = 0$$

$$\curvearrowleft^{+}: \ \Sigma M_G - M^{inertia} = \Sigma M_G - J_G \alpha = 0$$

Fundamental Laws of Rigid Body Motion

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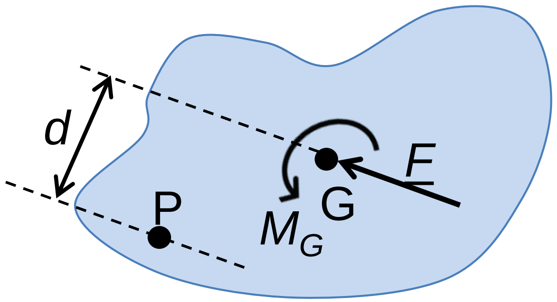
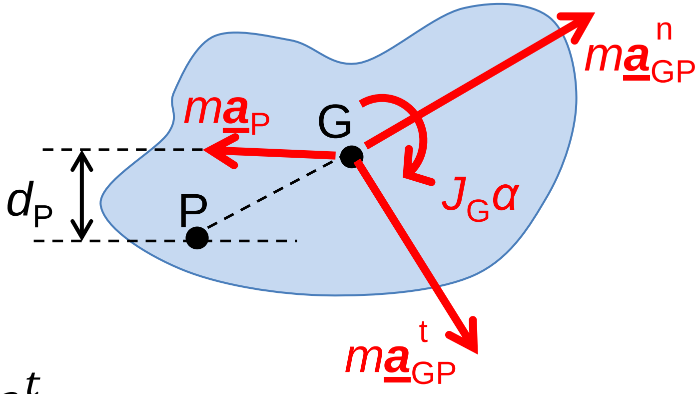
D'Alembert's principle

Given: arbitrary point P with known $\underline{\boldsymbol{a}}_P$





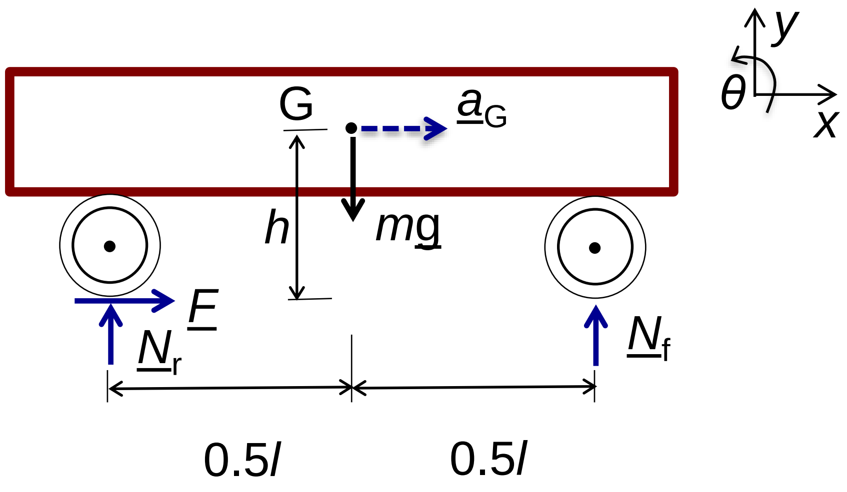
$$\underline{a}_G = \underline{a}_P + \underline{a}_{GP}^n + \underline{a}_{GP}^t$$

$$\circlearrowleft^+: \quad M_P' = M_G + Fd - J_G\alpha - mPG^2\alpha + ma_P d_P = 0$$

$$M_P + ma_P d_P = J_P\alpha \qquad (3)$$

Translation

Example 3: Rear-Wheel Drive Car, a_{max} = ? m = 1500 kg l = 2.5 m h = 0.5 m $\mu = 1.0$



$$\underline{\boldsymbol{F}} = m\underline{\mathbf{a}}_G \qquad (1)$$

$$M_G = 0 \quad \text{since} \quad \alpha = 0 \text{ and } \omega = 0 \qquad (2)$$

$$\rightarrow^+: \ \Sigma F_x = m a_{G,x} \quad \rightarrow \quad F = m a_{G,x}$$

$$\uparrow^+: \ \Sigma F_y = 0 \quad \rightarrow$$

$$\curvearrowleft^+: \ \Sigma M_G = 0 \quad \rightarrow$$

$$F \le F_{lim} = \mu N$$

$$F = \mu N_r$$

$$N_r =$$

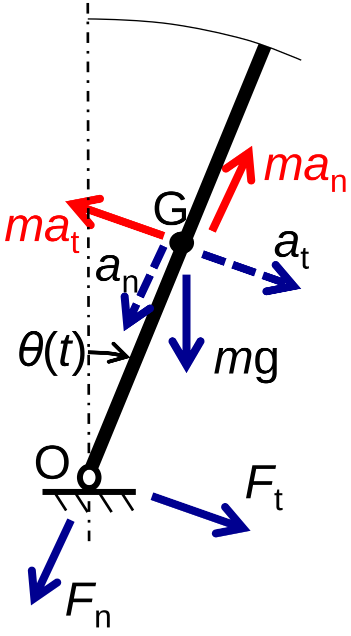
$$\mathrm{a}_{max} =$$

Rotation

Example 4: Pendulum Motion in a Vertical Plane

$\underline{\boldsymbol{F}} = m\underline{\mathbf{a}}_G$ (1)

$M_o = J_o\alpha$ (5) point O on the axis of rotation and $a_O = 0$



$$mg\frac{L}{2}\sin\theta = J_O\alpha$$

$J_O =$

$\ddot{\theta} =$

$\dot{\theta}^2 =$

$a_t =$

$a_n =$

$\searrow^+ \ \Sigma F_t' = 0$:

$\swarrow^+ \ \Sigma F_n' = 0$:

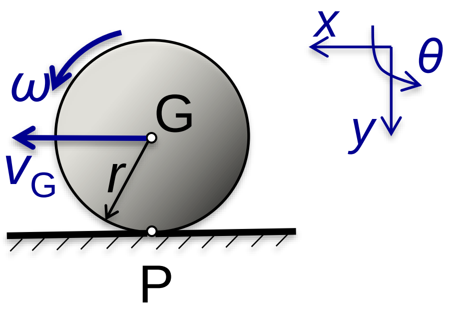
General planar motion

Example 5: Rear-Wheel Drive Car (cont'd)

m_w = 20 kg r = 0.3 m

Torque required to achieve a_{max} = ?

J_G = 1.35 kg.m^2

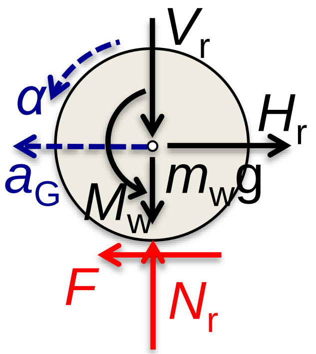


$$x_G(t) = \theta(t)r, \qquad \dot{x}_G(t) = \dot{\theta}(t)r, \qquad \ddot{x}_G(t) = \ddot{\theta}(t)r$$

$\rightarrow^+$:

$\uparrow^+$:

$\circlearrowleft^+$:



$V_r =$

$H_r =$

$M_W =$

For one tyre:

$$N_r = 4594 \text{ N} \quad \text{and} \quad F = 4594 \text{ N}$$

Lecture objectives

- Revise: particle dynamics and mass moments of inertia
- Define degrees of freedom for rigid body motion
- Introduce fundamental relationship for rigid body dynamics
- Formulate equations of motions
- Apply planar dynamics to the solution of basic problems

Next lecture

- Dynamics of a linkage mechanism
- Introduction to the coursework