Solutions to Exercise Sheet 5

Q5.1 – see Seminar 3 slides for full details. Summary:

a) $f = 50$ Hz, $p = 2$ pairs of poles, $n_s = 60\,f/p = 60 \times 50/2 = 1500$ rev min^{-1}

b) Line-to-line voltage V_L is 400V. Motor is star connected so each phase sees line-to-phase (phase) voltage $V_P=V_L/\sqrt{3}=400/\sqrt{3}=230.9$ V

c) $R_R=4\Omega$, $X_R=16\Omega$ referred to stator. At onset of stall $s = a = R_R/X_R = 4/16 = 0.25$

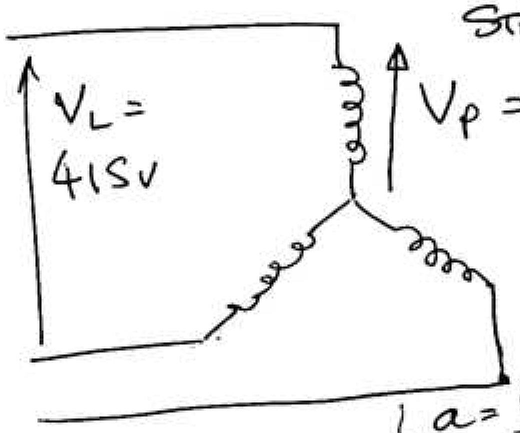
$$T = \frac{3p}{2\pi f}\times\frac{V^2 as}{X_R(a^2+s^2)} = \frac{3\times 2}{2\pi\times 50}\times\frac{230.9^2\times 0.25\times 0.25}{16(0.25^2+0.25^2)} = 31.8 \text{ Nm}$$

d) s = (1500-1430)/1500 = 0.0467

$$T = \frac{3p}{2\pi f}\times\frac{V^2 as}{X_R(a^2+s^2)} = \frac{3\times 2}{2\pi\times 50}\times\frac{230.9^2\times 0.25\times 0.0467}{16(0.25^2+0.0467^2)} = 11.49 \text{ Nm}$$

e) This point leads into Lecture 7 which includes discussion of variable-frequency inverters which change the synchronous speed of the motor and hence the running speed.

5.2



STAR CONNECTED SO:

$$V_P = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6\text{V}$$

$$f = 50\text{Hz}$$

$$a = \frac{R_R}{X_R} = \frac{0.9}{3.6} = 0.25$$

6 POLES $\Rightarrow$ 3 PAIRS OF POLES $\Rightarrow p = 3$

a) SLIP $= 0.04 = s$

$$n_s = \frac{60f}{p} = \frac{60\times 50}{3} = 1000 \text{ rev min}^{-1}$$

$$\Rightarrow n = n_s(1-s) = 1000(1-0.04) = 960 \text{ rev min}^{-1}$$

$$T = \frac{3p}{2\pi f}\times\frac{V^2 as}{X_R(a^2+s^2)}$$

$$= \frac{3\times 3}{2\pi\times 50}\times\frac{239.6^2\times 0.25\times 0.04}{3.6(0.25^2+0.04^2)}$$

$$= 71.3 \text{ Nm}$$

$$\omega = \frac{2\pi n}{60} = \frac{2\pi\times 960}{60} = 100.53 \text{ rad s}^{-1}$$

$$\Rightarrow P = T\omega = 71.3\times 100.53 = 7168\text{W}$$
$$= 7.17 \text{ kW}$$

b) $T = \frac{3p}{2\pi f} \times \frac{V^2 a s}{X_R(a^2+s^2)}$ AT STARTUP $n=0$ $\Rightarrow s = \frac{n_s - 0}{n_s} = 1$

$$= \frac{3\times 3}{2\pi\times 50} \times \frac{239.6^2 \times 0.25 \times 1}{3.6\times(0.25^2+1^2)}$$

$= 107.5$ Nm.

5.3 $V_L = 220V$, FIND V_P FOR STAR CONN.

$$\Rightarrow V_P = \frac{V_L}{\sqrt{3}} = \frac{220}{\sqrt{3}} = 127V$$

$p = 3$ (6 POLES $\rightarrow$ 3 PAIRS OF POLES), $f = 60Hz$

$$\Rightarrow n_s = \frac{60f}{p} = \frac{60\times 60}{3} = 1200 \text{ rev min}^{-1}$$

a) $n = 1160 \text{ rev min}^{-1} \Rightarrow s = \frac{n_s - n}{n_s} = \frac{1200-1160}{1200}$

$= 0.0333$

$R_R = 1.6\Omega, X_R = 16\Omega \Rightarrow a = \frac{R}{X_R} = \frac{1.6}{16} = 0.1$

$$\Rightarrow T = \frac{3p}{2\pi f} \times \frac{V^2 a s}{X_R(a^2+s^2)}$$

$$= \frac{3\times 3}{2\pi\times 60} \times \frac{127^2 \times 0.1 \times 0.0333}{16\times(0.1^2+0.0333^2)}$$

$= 7.22$ Nm

b) $T = \frac{3p}{2\pi f} \times \frac{V^2 a s}{X_R(a^2+s^2)}$ BUT $T = 10$ Nm

s IS UNKNOWN

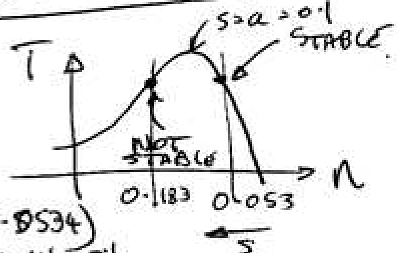
RE-ARRANGE AS QUADRATIC IN s:

$$2\pi f T X_R s^2 - 3pV^2 a s + 2\pi f T X_R a^2 = 0$$

$f = 60Hz$, $X_R = 16\Omega$, $T = 10$ Nm, $p = 3$, $a = 0.1$,
$V = 127V$

$$\Rightarrow 60318\, s^2 - 14516\, s + 603.2 = 0$$

$$\Rightarrow s = \frac{14516 \pm \sqrt{14516^2 - 4\cdot 60318 \cdot 603.2}}{2\times 60318}$$



$= 0.183$ OR 0.0534 OK

$s > a$ SO UNSTABLE

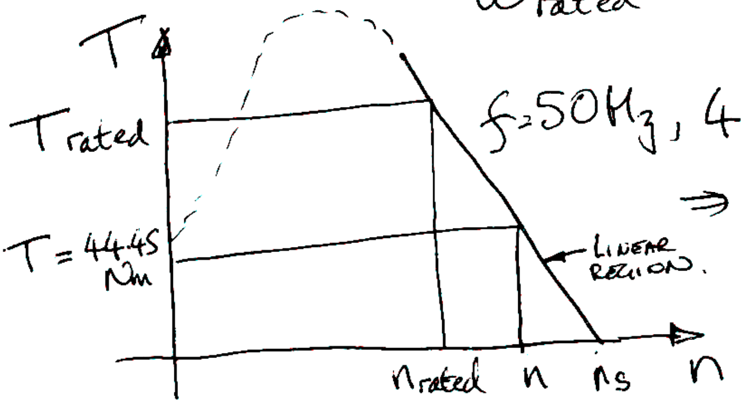
$\Rightarrow n = n_s(1-s) = 1200(1-0.0534)$
$= 1136 \text{ rev min}^{-1}$.

(3)

5.4 FULL-LOAD SPEED $= n_{rated} = 1425$ rev min^{-1}

$$\Rightarrow \omega_{rated} = \frac{2\pi \times 1425}{60} = 149.2 \text{ Nm}$$

$$\Rightarrow T_{rated} = \frac{P_{rated}}{\omega_{rated}} = \frac{9625}{149.2} = 64.51 \text{ Nm}$$



$f = 50$ Hz, 4-POLE so $p = 2$

$$\Rightarrow n_s = \frac{60f}{p} = \frac{60 \times 50}{2}$$
$$= 1500 \text{ rev min}^{-1}$$

$$n = n_s - \frac{T}{T_{rated}} \times (n_s - n_{rated})$$
$$= 1500 - \frac{44.45}{64.51} \times (1500 - 1425)$$
$$= 1448.3 \text{ rev min}^{-1}$$

(NOTE: COMPARE WITH "EXACT" PROBLEM IN Q5: SOL'N IS 1450 REV MIN^{-1})

5.5 $V = V_p = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6$ V

4-pole so $p = 2$
$$n_s = \frac{60f}{p} = \frac{60 \times 50}{2} = 1500 \text{ rev min}^{-1}$$

a) $n = 1425 \Rightarrow s = \frac{n_s - n}{n_s}$
$$= \frac{1500 - 1425}{1500} = 0.05$$

$$a = \frac{R_R}{X_R} = \frac{0.8}{4} = 0.2$$

$$\Rightarrow T = \frac{3p}{2\pi f} \times \frac{V^2 a s}{X_R(a^2 + s^2)} = \frac{3 \times 2}{2\pi \times 50} \times \frac{239.6^2 \times 0.2 \times 0.05}{4(0.2^2 + 0.05^2)}$$
$$= 64.5 \text{ Nm}$$

BUT $\omega = 1425 \times \frac{2\pi}{60} = 149.22$ rad s^{-1}

$$\Rightarrow P = T\omega = 64.5 \times 149.22 = 9624 \text{ W}$$

b) $n = 1450$ rev min^{-1} $\Rightarrow s = \frac{1500 - 1450}{1500} = 0.0333$

$$\Rightarrow T = \frac{3p}{2\pi f} \times \frac{V^2 a s}{X_R(a^2 + s^2)} = \frac{3 \times 2}{2\pi \times 50} \times \frac{239.6^2 \times 0.2 \times 0.0333}{4(0.2^2 + 0.0333^2)} = 44.45 \text{ Nm}$$

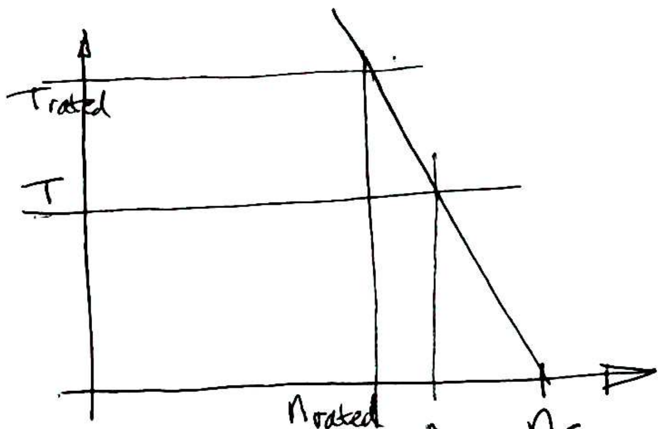
$\omega = 1450 \times \frac{2\pi}{60} = 151.84$ rad s^{-1} $\Rightarrow P = T\omega = 44.45 \times 151.84 = 6749$ W

5.6 a) $n_{rated} = 1450 \text{ rev min}^{-1}$

$$\Rightarrow \omega_{rated} = \frac{n \times 2\pi}{60} = \frac{1450 \times 2\pi}{60} = 151.84 \text{ Nm}$$

$$\Rightarrow T_{rated} = \frac{P_{rated}}{\omega_{rated}} = \frac{3000}{151.84} = 19.76 \text{ Nm}$$

b)



ASSUME $n_s = 1500 \text{ rev min}^{-1}$
SINCE 50 Hz & $n_{rated} = 1450 \text{ rev min}^{-1}$

$$n = n_s - \frac{T}{T_{rated}} \times (n_s - n_{rated}) \quad \text{BUT } T = 15 \text{ Nm}$$

$$= 1500 - \frac{15}{19.76} \times (1500 - 1450) = 1462 \text{ rev min}^{-1}$$

c) SEE NOTES NEAR END OF I.M. SECTION:

$f = f_{rated} = 50 \text{ Hz}$, $T = 15 \text{ Nm}$ AS ABOVE

$T_{rated} = 19.76 \text{ Nm}$ @ $1450 \text{ rev min}^{-1}$ AS ABOVE

PER PHASE: $V_{rated} = \frac{415}{\sqrt{3}} = 239.6$, $V = \frac{381}{\sqrt{3}} = 220 \text{ V}$

$$n \approx n_s - \frac{T}{T_{rated}} \times \left(\frac{V_{rated}}{V}\right)^2 \cancelto{1}{\left(\frac{f}{f_{rated}}\right)} (n_s - n_{rated})$$

$$= 1500 - \frac{15}{19.76} \times \left(\frac{239.6}{220}\right)^2 \times (1500 - 1450)$$

$$= 1455 \text{ rev min}^{-1}.$$